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Hidden Structure in a Lagrangian for Hyperfine Splitting of the Heavy Baryons

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Abstract

We investigate the hyperfine splitting of the heavy baryons in the bound-state approach. We start with an ordinary relativistic Lagrangian which has been extensively used to discuss finite mass corrections to the heavy limit predictions. It turns out that the dominant contribution arises from terms which do not manifestly break the heavy spin symmetry. The actual heavy spin violating terms are uncovered by carefully performing a $1/M$ expansion of this Lagrangian.

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1   Introduction

There has been a great deal of recent interest in studying heavy baryons in the bound-state picture [1, 2] together with heavy-quark spin symmetry [3]. This approach raises many fascinating questions which have been explored by several groups [4–10].

These models consist of a chiral Lagrangian for the light flavors and a Lagrangian, $\mathcal{L}_{\text{heavy}}$, which contains the heavy-meson multiplet H. The simplest choice for the latter is [11]

$$\mathcal{L}_{\text{heavy}}/M = iV_\mu \operatorname{Tr}\left[HD_\mu\overline{H}\right] + id\operatorname{Tr}\left[H\gamma_\mu\gamma_5 p_\mu\overline{H}\right] \ , \tag{1}$$

where V_μ is the four-velocity of the heavy particle and $D_\mu = \partial_\mu - iv_\mu$ is the covariant chiral derivative. Furthermore $v_\mu, p_\mu = (i/2)\left(\xi\partial_\mu\xi^\dagger \pm \xi^\dagger\partial_\mu\xi\right)$, wherein $\xi = \exp\left(i\phi/F_\pi\right)$ is the non–linear representation of the light pseudoscalar mesons ϕ. Finally M is the heavy meson mass while d is a heavy meson–light meson coupling constant. The light part of the Lagrangian allows for a soliton configuration ξ_{c}. In the bound state approach the heavy baryon then emerges as a heavy meson bound state in the background of ξ_{c}. The predictions are very simple in the limit where both N_{C} and M go to infinity. For example, the binding energy of the heavy baryon [4, 6, 8] is $(3/2)d\,F'(0)$ where $F'(0)$ is the slope of the soliton profile at the origin.

An immediate question is how to estimate what happens when we consider realistic values for M. In general this requires the addition of many unknown terms to Eq. (1). A predictive model for finite M corrections may be obtained by constructing a Lagrangian $\mathcal{L}$ [11, 12] of a heavy pseudoscalar meson P and a heavy vector meson Q_μ:

$$\begin{aligned}
\mathcal{L}(P, Q_\mu) &= -D_\mu P D_\mu \overline{P} - M^2 P\overline{P} - \frac{1}{2}Q_{\mu\nu}\overline{Q}_{\mu\nu} - M^{*2}Q_\mu\overline{Q}_\mu \\
&\quad + 2iMd\left(Pp_\mu\overline{Q}_\mu - Q_\mu p_\mu\overline{P}\right) - id'\epsilon_{\alpha\beta\mu\nu}\left(D_\alpha Q_\beta p_\mu\overline{Q}_\nu - Q_\alpha p_\beta D_\mu\overline{Q}_\nu\right) \ . \tag{2}
\end{aligned}$$

This reproduces Eq. (1) for large M when $d' = d$ and $M^* = M$. We have used $D_\mu\overline{P} = (\partial_\mu - iv_\mu)\overline{P}$ and $\overline{Q}_{\mu\nu} = \left(D_\mu\overline{Q}_\nu - D_\nu\overline{Q}_\mu\right)$. This model in particular allows for manifest breaking of the heavy spin symmetry by choosing $M^* \neq M$ and/or $d' \neq d$. The Lagrangian (2) represents the starting point for computing physical quantities along the lines of the original bound state approach [1] to strangeness in the Skyrme model [13, 14]. This requires the solutions to the equations of motion for P and Q_μ in the soliton background. The calculation [7, 10] exhibits sizable corrections for finite M. In addition, recoil effects (finite N_{C}) seem to be very important as well [9, 10]. When both these effects are taken into account it becomes difficult to fit the existing experimental data on the spectrum of the

heavy baryons. It was, however, noticed [8–10] that the inclusion of light vector mesons appreciably improves the situation.

In this note we will resolve an apparent puzzle which arises when calculating the corrections to the hyperfine splitting using Eq. (2).

2   An apparent puzzle

First let us consider the calculation of the hyperfine splitting in the heavy field approach. This, of course, arises at first sub-leading order in $1/M$ and violates the heavy spin symmetry. Thus we must add to Eq. (1) suitable heavy spin violating terms [5]:

$$\mathcal{L}'_{\text{heavy}}/M = \frac{M - M^*}{8} \text{Tr}\left[H\sigma_{\mu\nu}\overline{H}\sigma_{\mu\nu}\right] - \frac{i}{2}(d - d')\text{Tr}\left[Hp_\mu\overline{H}\gamma_\mu\gamma_5\right] + \cdots . \tag{3}$$

The first term has no derivatives while the second term has one derivative. The hyperfine splitting is related to a collective Lagrangian parameter (see section 4 for details) χ with a proportionality factor of the Δ-N mass difference:

$$m(\Sigma_Q^*) - m(\Sigma_Q) = [m(\Delta) - m(N)]\,\chi . \tag{4}$$

(At present only Σ_c is well established experimentally.) For Eq. (3) we have

$$\chi = \frac{M^* - M}{4dF'(0)} + \frac{d - d'}{4d} . \tag{5}$$

The first term was obtained in Ref. [5] while the second seems to be new. Notice that $(M^* - M)$ and $(d - d')$ behave as $1/M$. These quantities are the same as the ones appearing in the ordinary field Lagrangian (2). It would thus seem that $\mathcal{L}'_{\text{heavy}}$ in Eq. (3) neatly summarizes the heavy spin violation in Eq. (2).

Now let us consider the calculation of χ from Eq. (2) directly based on exact numerical solution of the associated coupled differential equations. We content ourselves with the graphical presentation of some results* and relegate the details to a forthcoming publication [15]†. Figure 1 shows χ plotted against M for three cases: i) $M^* = M$, $d' = d = 0.53$, ii) $M^* - M \simeq (0.258\text{GeV})^2/M$ (a fit to experiment), $d' = d = 0.53$, iii) $M^* = M$, $d' - d = (0.0991\text{GeV})/M$ (an arbitrary choice which sets the coupling constant splitting to be 10% at the D meson mass). We immediately notice that χ does not vanish when there

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* For the Skyrme model parameters we use the experimental value of F_π and $e_{\text{Sk}} = 6.0$. This results in a profile with $F'(0) = 1.20\,\text{GeV}$.

†Similar calculations were done in Ref. [7] but they did not consider the $M = M^*$, $d = d'$ case.
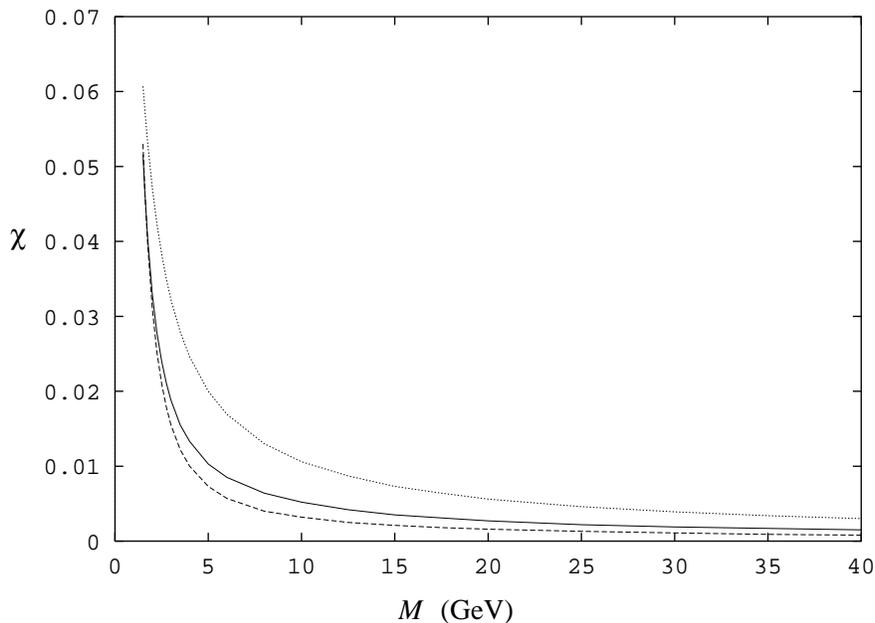
Figure 1: χ vs. M computed by numerical integration. Solid line $M^* = M$, $d' = d$; dotted line $M^* \neq M$, $d' = d$, dashed line $M^* = M$, $d' \neq d$.

is no manifest heavy spin violation, i.e., $M = M^*$, $d = d'$. In fact the dominant part of the contribution to χ for realistic heavy meson masses is already present in this case. By subtracting out this piece we note that the signs of the contributions due to $M^* \neq M$ and $d' \neq d$ agree with those predicted in Eq. (5). It is interesting to note that all three curves in Fig. 1 fall off as $1/M$ for $M \geq 10\,\text{GeV}$. But our main task is to understand the source of the puzzling non-zero contribution in case i. It is clear that the ordinary field Lagrangian (2) must contain heavy spin violating pieces which are not manifest. We will now explore this in detail by rewriting Eq. (2) in terms of the "fluctuation field" H and expanding it in powers of $1/M$.

3 Expansion of Lagrangian

Since the effects of $M \neq M^*$ and $d \neq d'$ were taken into account in Eq. (5) it is sufficient to expand Eq. (2) with $M^* = M$ and $d' = d$. To describe the heavy particle moving with four–velocity V_μ, we introduce the factorization

$$P = e^{iMV \cdot x} P' , \qquad Q_\mu = e^{iMV \cdot x} \widetilde{Q}_\mu . \tag{6}$$

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P' is the pseudoscalar "fluctuation field". $\widetilde{Q}_\mu$ is not exactly the vector fluctuation field since $V \cdot \widetilde{Q}$ is not constrained to be zero. We therefore introduce the correct fluctuation field Q'_μ by

$$\widetilde{Q}_\mu = Q'_\mu - V_\mu V \cdot \widetilde{Q} \; , \tag{7}$$

which shows that $V \cdot Q' = 0$. Substituting Eqs. (6) and (7) into the Lagrangian (2) gives, in addition to the leading terms of order M, the presently interesting terms of order M^0:

$$
\begin{aligned}
\mathcal{L}(P,Q) &= (\text{order } M) + P'D^2\overline{P'} + Q'_\mu D^2\overline{Q'}_\mu - Q'_\mu D_\nu D_\mu \overline{Q'}_\nu \\
&\quad + id\epsilon_{\alpha\beta\mu\nu}\left(D_\alpha Q'_\beta p_\mu \overline{Q'}_\nu - Q'_\alpha p_\beta D_\mu \overline{Q'}_\nu\right) \\
&\quad + M^2 V\cdot\widetilde{Q}V\cdot\overline{\widetilde{Q}} - iM\left(D_\mu Q'_\mu V\cdot\overline{\widetilde{Q}} - V\cdot\widetilde{Q}D_\mu\overline{Q'}_\mu\right) \\
&\quad - 2iMd\left(P'V\cdot pV\cdot\overline{\widetilde{Q}} - V\cdot\widetilde{Q}V\cdot p\overline{P'}\right) + \cdots \; ,
\end{aligned}
\tag{8}
$$

where the three dots stand for terms of order $1/M$. In contrast to the massless fields P' and Q', $V \cdot \widetilde{Q}$ is seen to have the large mass M. We thus integrate it out using the equation of motion

$$V\cdot\widetilde{Q} = \frac{i}{M}D_\mu Q'_\mu + \frac{2id}{M}P'V\cdot p \; . \tag{9}$$

Substituting Eq. (9) back into Eq. (8) gives

$$
\begin{aligned}
\mathcal{L}(P,Q) &= (\text{order } M) + P'D^2\overline{P'} + Q'_\mu D^2\overline{Q'}_\mu - iQ'_\mu F_{\mu\nu}(v)\overline{Q'}_\nu \\
&\quad - 2d\left(P'V\cdot pD_\mu\overline{Q'}_\mu + D_\mu Q'_\mu V\cdot p\overline{P'}\right) \\
&\quad + id\epsilon_{\alpha\beta\mu\nu}\left(D_\alpha Q'_\beta p_\mu\overline{Q'}_\nu - Q'_\alpha p_\beta D_\mu\overline{Q'}_\nu\right) \\
&\quad - 4d^2 P'\left(V\cdot p\right)^2\overline{P'} + \cdots \; ,
\end{aligned}
\tag{10}
$$

where $F_{\mu\nu}(v) = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu]$. In order to extract the heavy spin violating pieces it is convenient to rewrite the order M^0 Lagrangian in terms of the heavy multiplet field $H = \frac{i}{2}\left(1 - i\gamma\cdot V\right)\left(\gamma_5 P' + \gamma\cdot Q'\right)$. After some algebraic calculation we find

$$
\begin{aligned}
\mathcal{L}(H) &= \mathcal{L}_{\text{heavy}} - \frac{1}{2}\text{Tr}\left[HD^2\overline{H}\right] + i\frac{1}{8}\text{Tr}\left[[H,\gamma_\mu\gamma_\nu]\,F_{\mu\nu}(v)\overline{H}\right] \\
&\quad + d\Bigg[\frac{1}{2}\text{Tr}\left[D_\mu H\gamma_\mu\gamma_5(V\cdot p)\overline{H}\right] - \frac{i}{4}\text{Tr}\left[\gamma\cdot DH\gamma_\mu\gamma_5 p_\mu\overline{H}\right] \\
&\quad\quad - \frac{i}{4}\text{Tr}\left[\gamma\cdot DHp_\mu\overline{H}\gamma_\mu\gamma_5\right] + \frac{1}{8}\text{Tr}\left[\sigma_{\mu\nu}D_\alpha H\gamma_\alpha V\cdot p\gamma_5\sigma_{\mu\nu}\overline{H}\right] + \text{h.c.}\Bigg] \\
&\quad + d^2\left[\frac{1}{2}\text{Tr}\left[H\left(V\cdot p\right)^2\overline{H}\right] + \frac{1}{4}\text{Tr}\left[\sigma_{\mu\nu}H\sigma_{\mu\nu}\left(V\cdot p\right)^2\overline{H}\right]\right] + \cdots \; ,
\end{aligned}
\tag{11}
$$

where $\mathcal{L}_{\text{heavy}}$ is given in Eq. (1). At this stage we see that Eq. (11) actually contains pieces which are not manifestly invariant under the heavy spin transformations $H \to SH$, $\overline{H} \to \overline{H}S^\dagger$. These pieces involve two derivatives.

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4 Hyperfine splitting

We now sketch the computation of the portion of χ in Eq. (4) which results from the "hidden" heavy spin violation in Eq. (2) that has been made explicit in Eq. (11). For this purpose one needs the collective Lagrangian of the quantum variable $A(t)$ which is obtained after substituting

$$\xi(\boldsymbol{x}, t) = A(t)\xi_{\rm c}(\boldsymbol{x})A^{\dagger}(t) , \qquad \overline{H}(\boldsymbol{x}, t) = A(t)\overline{H}_{\rm c}(\boldsymbol{x}) , \qquad (12)$$

(where $\xi_{\rm c}(\boldsymbol{x})$ is the classical Skyrme soliton and $\overline{H}_{\rm c}(\boldsymbol{x})$ is the heavy meson bound-state wave function) and integrating over d^3x. The key dynamical variable is the "angular velocity" $\boldsymbol{\Omega}$ defined by $A^{\dagger}\dot{A} = \frac{i}{2}\boldsymbol{\tau} \cdot \boldsymbol{\Omega}$. The bound-state wave function may be conveniently presented in the rest frame where

$$\overline{H} \to \begin{pmatrix} 0 & 0 \\ \overline{h}_{lh}^{a} & 0 \end{pmatrix} , \qquad (13)$$

with a, l, h representing respectively the iso-spin, light spin and heavy spin bivalent indices. We write [9]

$$\overline{h}_{lh}^{a} = \frac{u(r)}{\sqrt{4\pi M}} \left(\widehat{\boldsymbol{x}} \cdot \boldsymbol{\tau} \right)_{ad} \psi_{dl,h} , \qquad (14)$$

where $u(r)$ is a radial wave function (assumed very sharply peaked near $r = 0$ for large M) and, to leading order in M, the "angular part" of the ground state wave function is [8, 9]

$$\psi_{dl,h}^{(1)} = \frac{1}{\sqrt{2}}\epsilon_{dl}\delta_{2h} . \qquad (15)$$

The specific value of the index h results from the choice $G_3 = G = 1/2$ where G is the "grand spin". To next leading order in M the ground state wave function receives a heavy spin violating admixture of

$$\psi_{dl,h}^{(2)} = \sqrt{\frac{2}{3}}\delta_{d1}\delta_{l1}\delta_{h1} + \frac{1}{\sqrt{6}} \left(\delta_{d2}\delta_{l1} + \delta_{d1}\delta_{l2} \right) \delta_{h2} . \qquad (16)$$

Finally, the hyperfine splitting parameter χ is recognized by expanding the collective Lagrangian [1], in powers of $\boldsymbol{\Omega}$ and picking up the linear piece $L_{\rm coll} = (\chi/2)\Omega_3 + \cdots$. Noting that the Δ–nucleon mass difference is given by the moment of inertia, which relates the angular velocity to the spin operator [14], this piece of the Lagrangian yields Eq. (4) after canonical quantization of the collective coordinates [1]. There are two types of contribution to χ. The first type, from the heavy spin violating terms proportional to d in Eq. (11), corresponds to the evaluation of heavy spin violating operators in the ground state (15). The second type corresponds to the evaluation of heavy spin conserving operators in the

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ground state which includes an admixture of Eq. (16) due to the $\mathrm{Tr}\left[\gamma_\mu \gamma_\nu H F_{\mu\nu}(v)\overline{H}\right]$ term in Eq. (11). The net result for the "hidden" part of χ is

$$\chi = \frac{F'(0)}{4M}\left(d - \frac{1}{2d}\right) \ . \tag{17}$$

This equation is expected to hold for large M. To this should be added the "manifest" part given in Eq. (5).

It is important to compare Eq. (17) with the result for χ obtained by the exact numerical solution for the model based on Eq. (2). This is gotten as an integral over the properly normalized radial functions $\Phi(r), \ldots, \Psi_2(r)$ which appear in the P–wave solution of the bound state equation [10]:

$$
\begin{aligned}
P^\dagger &= A(t)\frac{\Phi(r)}{\sqrt{4\pi}}\,\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}\rho e^{i\epsilon t}, \qquad Q_4^\dagger = \frac{i}{\sqrt{4\pi}}A(t)\Psi_4(r)\rho e^{i\epsilon t}, \\
Q_i^\dagger &= \frac{1}{\sqrt{4\pi}}A(t)\left[i\Psi_1(r)\hat{r}_i + \frac{1}{2}\Psi_2(r)\epsilon_{ijk}\hat{r}_j\tau_k\right]\rho e^{i\epsilon t} \ .
\end{aligned}
\tag{18}
$$

The spinor ρ labels the grand spin of the bound heavy meson. The choice $G_3 = +1/2$ corresponds to $\rho = (1,0)^\dagger$. The heavy limit bound state wave function in Eq. (15) corresponds to the special choice

$$\Phi(r) \propto u(r) \ , \quad \Psi_1(r) = -\Phi(r) \ , \quad \Psi_2(r) = -2\Phi(r) \quad \text{and} \quad \Psi_4(r) = 0 \ . \tag{19}$$

The numerical solution to the bound state equations exactly exhibits these relations for $M, M^* \to \infty$ [10].

Equation (17) has an interesting d-dependence and vanishes at $d = 1/\sqrt{2}$, which actually is not too far from the experimental value of this quantity. In Fig. 2 we compare the d-dependence of the exact numerical calculation with the perturbative result of Eq. (17). It is seen that the large M perturbation approach works reasonably well and the gross structure of the hyperfine splitting is reproduced. For a detailed comparison of the two treatments it is important to note that for fixed $M = M^*$ the binding of the heavy meson increases with d. In particular this implies that the wave function is only reasonably localized for large enough d. As a strong localization is a basic feature of the perturbative approach it is easy to understand why this calculation does not yield the exact (numerical) result for small d. In fact, as d increases the agreement expectedly improves. However, upon further increase of d (at finite M, M^*), the numerical solution to the bound state equations shows noticeable deviations from the heavy limit relations (19), which causes the moderate differences at larger d.
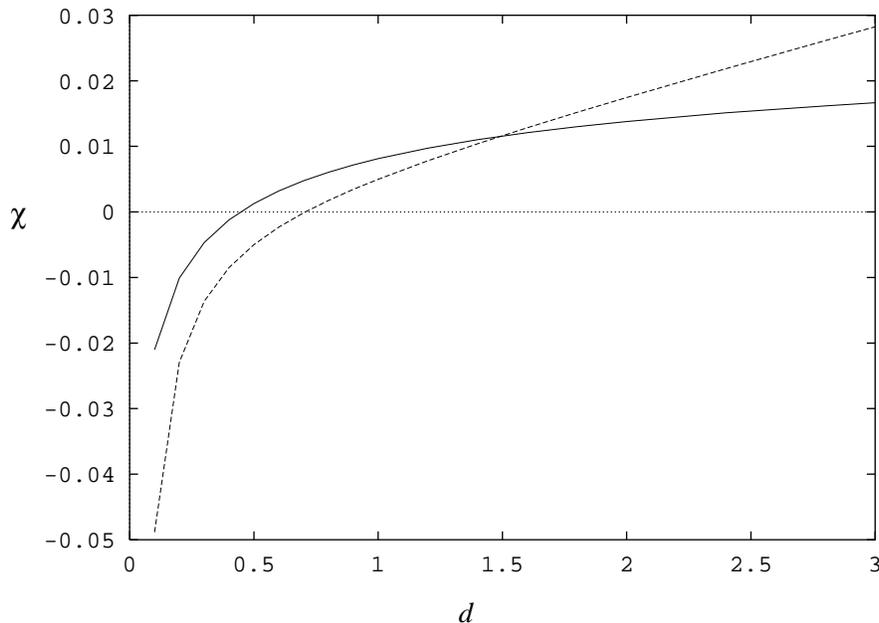
Figure 2: The d dependence of χ for $M = M^* = 30\,\text{GeV}$ and $d = d'$. Solid line is the exact numerical calculation. Dashed line is the large M perturbation formula given in Eq. (17).

5 Discussion

We have solved the apparent puzzle associated with the use of a model Lagrangian containing ordinary fields for computing the hyperfine splitting parameter χ by carefully expanding the Lagrangian in powers of $1/M$. The key point was the need to preserve the constraint $V \cdot Q' = 0$ for the heavy vector fluctuation field.

Of course, such a model Lagrangian (which has been used in many calculations) is not exactly QCD. Nevertheless it seems reasonable since it automatically has the correct relativistic kinematics and satisfies the heavy spin symmetry at leading order. We have seen (Eq. (11)) that at next order in $1/M$, it predicts the coefficients of many terms which otherwise would be unspecified by heavy spin symmetry (even if reparameterization invariance [16] were taken into account).

It is amusing to note that these $1/M$ suppressed terms involve two derivatives and are actually more important for the computation of χ than the zero derivative term in Eq. (5). This is readily understandable since the dynamical scale in this calculation is the binding energy, $m(B) + m(N) - m(\Lambda_b) \simeq 620\,\text{MeV}$ which is rather large for neglecting light vector mesons, higher derivatives etc. [See, for example, Ref. [17].]

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We are regarding the Lagrangian (2) as an illustrative model rather than as a realistic one for comparison with experiment. As indicated earlier it seems necessary to include, in addition to finite M corrections, the effects of light vector mesons as well as nucleon recoil. The discussion of χ in this more complicated model and further details of the present calculation will be given in a forthcoming publication [15].

Acknowledgements

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