Aerodynamic instability investigations of a novel, flexible and lightweight triple box girder design for long span bridges

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ABSTRACT

The present paper investigates the possibilities for avoiding classical flutter and static divergence for very long span suspension bridges with a novel, flexible and lightweight triple box girder. Previous studies have shown that the critical classical flutter wind speed tends to decline with the main span width. Other studies have shown that flutter can be avoided if the torsional frequency is lower than the vertical. The road to get there in practice, however, is complicated. The possibility for tuning the torsional natural frequency without affecting the vertical frequency is utilized in the present paper. The effect on aerodynamic stability is analyzed in detail for low torsional-to-vertical frequency ratios typical for very long span bridges with lightweight and flexible girders. The present study includes non-linear finite element analysis and static, free vibration and forced motion wind tunnel tests. Aerodynamic stability have

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been obtained in a section model test with a lightweight setup corresponding to a bridge
girder mass of only 6.38 t/m in full scale. Flutter was not observed for any torsional-
to-vertical frequency ratios in the range from \(0.97 \leq \gamma_\omega \leq 1.55\). Stability was observed
up to wind speeds of approximately 88 m/s in full scale for a 2125.2 m span. The aero-
dynamic stability obtained in the configurations of the present section at \(\gamma_\omega \approx 1.20\)
shows that this might be an economic feasible solution for future long span suspension
bridges because aerodynamic stability is achieved even though the torsional stiffness
and the mass of the deck is low.

**Keywords:** Bridge; Aeroelasticity; Flutter; Static Divergence; Triple-box gird-
ers; Lightweight bridges

**INTRODUCTION**

There is an ongoing demand for development of the infrastructure in many
countries around the world. This includes bridging wider and wider rivers and
straits, which means that suspension bridges are as a consequence expanding in
length on a regular basis. Recent examples are the ongoing fjord-crossing projects
in Norway (Dunham 2016) and other examples are the crossings of the Gibraltar-
, Sunda-, Messina- and Hainan Straits requiring main span lengths of up to
5km (Ge and Shao 2013). The investments required may postpone such projects
several times or even cancel them, however, a reduction of the bridge deck mass
is the key to minimize the total costs (Richardson 1981; Brown 1999). A decrease
in mass could cause the deck to be more easily excited by wind compared to a
heavier deck. While the mass of the deck is optimized, caution should be paid
to severe aeroelastic instability phenomena such as flutter and static divergence.
Aerodynamic instability is not allowed to occur below a desired critical wind
speed, typically 1.5 times the characteristic 10 minute mean velocity at bridge-
deck height (Dyrbye and Hansen 1997), which corresponds to a sufficient high
annual probability of failure.

Enhanced torsional stiffness of the girder is a known measure to avoid instabilities, but tend to increase the mass. Furthermore, the torsional girder stiffness is inverse proportional to the span length. If bridge decks can be designed aerodynamically stable with low torsional stiffness, the cross wind dimension, i.e. the depth, and thus the mass and the costs can be reduced considerably. The concept of obtaining aerodynamic stability against flutter by designing the bridge deck to have a lower fundamental torsional frequency than vertical has previously been suggested and studied in (Richardson 1981; Walshe and Wyatt 1992; Dyrbye and Hansen 1997; Bartoli et al. 2008; Bartoli et al. 2009; Andersen et al. 2014; Andersen et al. 2015). Most previous suggestions has been based on twin-boxes with the bridge girders located on the external side of the main cables, which makes the design prone to static rotations due to eccentric traffic loads. Many tests have verified that flutter is avoided when the torsional frequency is lower than the vertical, but that large static displacements tend to occur eventually followed by heavy vortex induced vibrations. The twin box designs where $\gamma_\omega < 1$ requires that the road boxes are located external to the cables, which tend to cause large static rotations due to eccentric traffic loads. In the present paper, a novel triple-box girder design is investigated where the traffic lanes are placed close to the elastic axis. The tuning of the torsional natural frequencies is conducted by adjusting the main cable-spacing and the mass distribution on the girder.

The objectives of the present study are to evaluate a novel triple-box girder design for long span suspension bridges with a free span of $L = 2125.2$ m by the means of finite element analysis (FEA) and wind tunnel tests. The span length was chosen to be suitable for crossings of Halsafjorden and Vartdalsfjorden in the Coastal Highway Route E39 project (Dunham 2016). The cross section of the triple box girder is illustrated in Figure 1.
Background

The background for the present design goes back to the introduction of perforated bridge decks (Walshe 1977) and to the concept of the twin suspension bridge (Richardson 1981). In (Richardson 1981), it was proposed to avoid coupled bridge deck flutter between the first torsional mode and the first vertical bending mode by making their frequencies identical. This concept was described as the non-flutter design principle in (Dyrbye and Hansen 1997). The shortcomings of the non-flutter twin box design first introduced in (Richardson 1981) was large static rotations due to eccentric traffic loading on a single lane as implied in (Walshe and Wyatt 1992) and again pointed out in (Larsen and Larose 2015). The non-flutter concept for twin boxes has experimentally been shown to withstand coupled flutter in (Bartoli et al. 2008; Bartoli et al. 2009; Andersen et al. 2015; Andersen et al. 2016) and for single boxes in (Nowicki and Flaga 2011; Johansson et al. 2013; Andersen et al. 2016). Single degree of freedom torsional flutter may occur for bluff single box sections as shown in (Nowicki and Flaga 2011). Multimodal flutter was found not to be an issue in the study of a 3.7 km long conceptual non-flutter twin box design in (Andersen et al. 2014).

Dynamic instability in terms of bridge deck flutter, is not the only disadvantage in the design of long span bridges. Torsional or static divergence may occur instead of flutter - in particular for very long span bridges. A nonlinear static divergence analysis method was introduced for cable-stayed bridges in (Boonyapinyo et al. 1994) and later for suspension bridges in (Cheng et al. 2002).

Reductions of the tension in the main cables were shown to reduce the torsional stiffness of the suspended girder in (Zhang et al. 2015; Zhang et al. 2013). This may, however, only be the case if the lift is positive upwards. The lift of the present bridge girder is pointing downwards and the steady aerodynamic moment is working in the counter clock wise direction (nose down). This may effectively
prevent torsional divergence.

If the torsional and vertical stiffness of the bridge girder are identical, e.g. zero, and the radius of gyration is equal to the distance from the vertically suspended hangers to the elastic axis, the torsional and vertical natural frequency will be equal (Gimsing and Georgakis 2012). In practice, the torsional girder stiffness is never zero and much larger than the vertical bending stiffness. This has pronounced effect on the torsional frequency and the torsional-to-vertical frequency ratio. The torsional stiffness contributions from the main cables become increasingly important for longer spans. In order to design $\omega_\alpha \leq 1$, most of the mass must be located outside the main cables. This causes an inherent lower torsional stiffness relative to configurations where $\omega_\omega > 1$ because the distance between the main cables needs to be considerably smaller than the total width of the cross section. The inherent lack of torsional stiffness is the Achilles’ heel of the non-flutter design principle. The non-optimal steady torsional load-to-stiffness ratio causes onset of static rotation at lower wind speeds. This may lead to severe oscillations (Andersen et al. 2016) or static divergence. Due to the reduced torsional stiffness in the non-flutter designs, the challenge is to reduce twisting moments from eccentric traffic loads and wind.

The key questions in the present research are:

- How can we design a suspension bridge where $\omega_\alpha < \omega_h$ without aerostatic issues and large rotations caused by eccentric traffic loads?
- Will such a cross section perform better in wind when $\omega_\omega \leq 1$ compared to more natural configurations for super long span bridges where $1.1 \leq \gamma_\omega \leq 1.5$?
- Can the mass of the bridge deck be reduced to ultra lightweight setups without compromising the structural safety?
This paper aims to answer these questions by presenting wind tunnel tests results of the present novel triple-box girder design. FEA of suspension bridges having dynamic similitude with free vibration wind tunnel tests are presented and the influence of structural mass on the static and dynamic stability is highlighted.

METHODS

The linear time-invariant model for a suspended bridge deck subjected to small displacements,

\[ M\ddot{u} + C\dot{u} + Ku = F_s(U, \bar{\alpha}) + F_m(U, \omega) \]  

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices is adopted. The displacement dependent steady mean wind load causing the static response is denoted, \( F_s(U, \bar{\alpha}) \) where \( U \) is the incoming mean wind speed and \( \bar{\alpha} \) is the mean effective angle of attack on the bridge deck. The motion induced unsteady forces, depending on the harmonic motion of the deck, is denoted \( F_m(U, \omega) \) where \( \omega \) is the harmonic motion frequency of the bridge deck. Buffeting forces due to turbulence will add a third term to the equations of motion, but they will not be dealt with here because they are of little relevance for the static and dynamic stability.

The static wind loads, \( F_s(U, \bar{\alpha}) \) changes with the static rotation of the deck. The structural stiffness matrix for a long span bridge, \( K \) relies on geometrical stiffness and thus highly displacement dependent. Large displacement nonlinear analyses are conducted for the dead load and steady wind load situations respectively. A non-linear iterative finite element method described in (Cheng et al. 2002) is used to calculate the static response from the steady wind loads.

The motion induced forces, \( F_m(U, \omega) \), have displacement and velocity dependent terms which are denoted aerodynamic stiffness and damping respectively.
These terms are described by 18 coefficients called flutter derivatives. They are either estimated by free vibration tests or by forced motion tests of section models (Scanlan and Tomko 1971; Sarkar et al. 1992; Poulsen et al. 1992; Matsumoto et al. 1993; Zasso 1996; Singh et al. 1996; Matsumoto 1996; Scanlan 1997; Brownjohn and Jakobsen 2001; Chen and Kareem 2004; Sarkar et al. 2009; Luca Caracoglia et al. 2009; Siedziako et al. 2017; Andersen et al. 2018). Single degree of freedom forced motion tests were conducted in the present study. State of the art frequency domain multimodal flutter analysis (Agar 1989; Jain et al. 1996) were conducted to estimate the critical flutter wind speed.

**Finite element methods**

Spine beam FE models (Xu 2013) representing different configurations of mass and stiffness were created. The main span of the suspended bridge was $L = 2125.2$ m, which comes close to the span length considered for some of the Fjord Crossings in the Ferry-Free E39 project along the Norwegian west coast. The cable sag was 10% of the span. An elevation drawing of the finite element model is shown in Figure 2.

The bridge girder and the main cables between the pylons were discretized in 86 elements each. A convergence study on the number of elements used along the bridge deck showed no change in the natural frequencies in still air or in the displacements due to static wind loading when the number of elements was increased to 173. The mass and stiffness of the triple-box bridge girder were modeled as a single girder by Timoshenko beam elements. Tension only spar elements were used for the hangers and main cables.

The mechanical properties of the bridge models are given in Table 1. Different configurations of the mass, mass moment of inertia, torsional stiffness and the main cables semi-distance were used. These intervals are shown within parenthe-
ses in Table 1. The values for the parameters were determined in order to obtain high torsional stiffness, low mass and low torsional-to-vertical frequency ratios.

**Modal analysis**

The mass matrix was obtained by a lumped mass approximation. Modal analyses for the dead load situation in still air were conducted by linear perturbation and Block Lanczos method. The equivalent mass and mass moment of inertia per unit length were obtained from the mode shapes. The mode shapes were scaled to unity modal mass, thus the equivalent mass, $m_e$ and mass moment of inertia, $I_e$ per unit length of the bridge deck were given by

$$m_e = \frac{1}{\int_0^L \phi_z(y)^2 dy}, \quad I_e = \frac{1}{\int_0^L \phi_\alpha(y)^2 dy}$$

(2)

where $\phi_\alpha(y)$ and $\phi_z(y)$ are a torsional mode shape and a vertical mode shape respectively. The equivalent inertia is larger than the inertia of the bridge deck only because the cables and pylons contributes to the modal mass.

Free vibration wind tunnel tests simulating the lowest torsional mode and the shape-wise similar vertical mode were scaled according to $m_e$ and $I_e$ for the FE models. These are reported in Table 2.

**Nonlinear aerostatic analyses**

The nonlinear aerostatic analyses were conducted as follows. First, the displacement vector for the bridge deck, $\mathbf{u}$ was found in still air. Next, the wind speed was increased to $U = 5$ m/s and the static wind load applied to the nodes along the bridge girder. The displacement vector, $\mathbf{u}$ were found.

Iterations were conducted until convergence of the displacement dependent wind loads. For each iteration step, $i$, the euclidean norms (Cheng et al. 2002), $\epsilon_j$ where $j = \{z, x, \alpha\}$ were calculated over the $m = 87$ bridge deck girder nodes. The steady wind loads applied in each bridge girder node, $n$, in the axial, normal and
torsional direction are denoted $F_{j,n}$. The convergence criteria were reached when 
$\epsilon_j < 0.005$ for $j = \{z, x, \alpha\}$. After convergence, the wind speed was increased by 
$\Delta U = 5\text{m/s}$.

$$\epsilon_j = \sqrt{\sum_{n}^{m} \frac{(F_{j,n,i} - F_{j,n,(i-1)})^2}{F_{j,n,(i-1)}^2}} \quad (3)$$

The convergence criteria were reached when $\epsilon_x < 0.005$, $\epsilon_z < 0.005$ and $\epsilon_\alpha < 0.005$. After convergence, the wind speed were increased with $\Delta U = 5\text{m/s}$. The calculations continued until $U = 100$ unless divergence or static rotations larger than $\alpha > 6^\circ$ occured before.

**Wind tunnel tests**

Wind tunnel tests were conducted in the boundary layer wind tunnel at
FORCE Technology, Denmark. The wind tunnel test section used were 2.6 m wide and 1.8 m high. Static, dynamic and forced motion tests were conducted at Reynolds numbers up to $Re \approx 5.5 \cdot 10^4$ based on the depth of the central box in model scale, $D = 0.04$ m.

The bridge section model spanned 2.55 m across the tunnel. The boundary layer of the tunnel sidewalls was assumed to limit end effects on the model. The large gap width between the central and external boxes were believed to reduce blockage effects. All tests were conducted in smooth flow with a measured turbulence intensity of $I_u \approx 0.1\%$. A pitot tube located 2.7 m from the center of the section model measured the pressure of the undisturbed incoming flow. The static pressure was measured analogue outside the tunnel. The density of the air, $\rho$, were determined based on the measured humidity, temperature and pressure of the air.
Bridge Section Models

Two different section models were tested; One without wind noses on the central box (Test series 8) and one with wind noses (Test series 9). The cross sections are shown in Figure 3. The geometric scale of the section models were \( \lambda_g = 1:50 \) relative to the full scale prototype. The central box was constructed as a square sandwich box by aluminum plates with a thickness of \( t = 0.5 \) mm glued to a foam core. Equilateral wind noses made out of foam were connected to the sides by screws and tape. The height of the section was 40 mm at the center of the central box and 36 mm at the edge of the sandwich box where the wind noses were connected. The cross girders were 1200 mm wide, 20 mm thick with varying depth. The cross girders were spaced with 510 mm along the span of the section model. The central and external boxes were glued to the cross girders. The slope of the cross girders were -3/100. At the inflection point at the bottom center of the cross girders, they were 15 mm high giving the section a locally total height of 55 mm. At the leading and trailing edges the cross girders were only 5 mm high. The sharp-edged rectangular external boxes, 7 mm high, made out of plywood, were glued on top of the cross girders. The railings, shown in Figure 1b, were 3D printed in model scale and glued to the external and central boxes respectively.

Test series

Additional wind tunnel tests of the section model without wind noses on the central box, shown in Figure 3a, were conducted. The tests included estimation of the flutter derivatives at different angles of attack, \( \alpha \), static tests and dynamic testings. The relevant results are reported when appropriate in the following. The section model without wind noses on the central box is referred to as test series 8 (S8), while the bridge section model including wind noses at the central box is referred to as test series 9 (S9).
STEADY FORCES

A static rig with strain gauge balances located outside the tunnel measured the steady pitching moment, $M$, normal and axial forces, $F_z$ and $F_x$ as well as the pitching angle, $\alpha$ simultaneously. The sign definitions used are shown in Figure 4.

The force coefficients, $C_l(\alpha)$, $C_d(\alpha)$ and $C_m(\alpha)$, are defined by

$$C_d(\alpha) = \frac{F_d(\alpha)}{1/2 \rho U^2 B}, \quad C_l(\alpha) = \frac{F_l(\alpha)}{1/2 \rho U^2 B}, \quad C_m(\alpha) = \frac{M(\alpha)}{1/2 \rho U^2 B^2} \quad (4)$$

where the forces per unit length, $F_d$ & $F_l$ and $F_m$ are the mean values of the lift, drag and moment forces per unit length. The measured normal and axial forces were converted to lift and drag forces. All force coefficients are based on mean values of 60 seconds measurements sampled with $f_s = 200$ Hz. The steady force coefficients as a function of the mean pitch angle, $\alpha$, are shown in Figure 5.

It can be seen in Figure 5 that the drag forces were reduced with the wind noses, but that the negative lift and moment at $\alpha < 0^\circ$ were increased. Compared to the Messina strait bridge section in (Zasso et al. 2013), the present drag coefficients are smaller, the moment is negative and the slope of the moment, $\frac{\partial C_m}{\partial \alpha}$ is negative in the interval between $-6^\circ < \alpha < 0^\circ$ despite local nonlinearities at $-1^\circ < \alpha < 0^\circ$ and $-3^\circ < \alpha < -2^\circ$ where $\frac{\partial C_m}{\partial \alpha} > 0$ as shown in (Andersen and Brandt 2017). The negative lift of the present section is considerable larger than for the Messina section in (Zasso et al. 2013), which might be reduced by changing the shape of the wind noses. Lower negative lift might also cause a lower negative moment.

The almost flat moment coefficient shown in Figure 5 indicates that the aerodynamic center is located before and above the elastic axis because when the negative lift and the positive drag forces increases at $\alpha < 0^\circ$, the moment is
roughly unaffected. This indicates that the aerodynamic center is located before and above the elastic axis causing the drag force to turn the section nose up and the lift force to turn it nose down. This could effectively prevent torsional divergence.

**UNSTEADY FORCES**

Harmonic single degree of freedom forced motion tests were conducted for the lateral, vertical and torsional degrees of freedom, \( p, h \) and \( \alpha \) using a three degree of freedom forced motion actuator. The unsteady forces per unit length depending on small amplitude harmonic displacements and velocities of the bridge deck is described by \( \mathbf{F}_m(U, \omega) = \mathbf{C}^{ae}(U, \omega)\dot{\mathbf{u}} + \mathbf{K}^{ae}(U, \omega)\mathbf{u} \). The amplitudes for the lateral and vertical harmonic motion were 16 mm while the torsional motion amplitude was 2\(^{\circ}\). All tests were conducted at \( f = 0.7 \) Hz. The matrices, \( \mathbf{C}^{ae}(U, \omega) \) and \( \mathbf{K}^{ae}(U, \omega) \) are the aeroelastic damping and stiffness matrices which coefficients are given by the flutter derivatives as shown in Equation (5).

\[
\mathbf{C}^{ae}(U, \omega) = \frac{1}{2} \rho U BK \begin{bmatrix}
P_1^* & P_5^* & BP_2^* \\
H_5^* & H_1^* & BH_2^* \\
BA_5^* & BA_1^* & B^2A_2^*
\end{bmatrix}
\]

\[
\mathbf{K}^{ae}(U, \omega) = \frac{1}{2} \rho U^2 BK^2 \begin{bmatrix}
P_1^*/B & P_6^*/B & P_3^* \\
H_6^*/B & H_1^*/B & H_3^* \\
A_6^* & A_4^* & BA_3^*
\end{bmatrix}
\]

\[
K = \omega B/U; \quad \omega = 2\pi f
\]

Discrete time histories of the bridge deck motion and forces were measured. Thus, the input and the output are known. Digital filters were not applied to the measured forces, contrary to the results presented in (Andersen and Brandt 2017), but the measured motion in the non-excited degrees of freedom were replaced by
zero vectors to suppress the influence of measurement noise from these channels. The velocity time histories were obtained by Remez time differentiation method (Brandt 2011). Next, the displacements and velocities were organized in the input matrix, \( \mathbf{x} \), and the forces were organized in the output matrix, \( \mathbf{y} \). The total number of samples are denoted \( N \). A sampling rate of \( f_s = 200 \) were used and at least \( N = 12000 \) samples were recorded during each test.

\[
\mathbf{x} = \begin{bmatrix}
\dot{p}(1) & \dot{p}(2) & \cdots & \dot{p}(N) \\
\dot{h}(1) & \dot{h}(2) & \cdots & \dot{h}(N) \\
\dot{\alpha}(1) & \dot{\alpha}(2) & \cdots & \dot{\alpha}(N) \\
p(1) & p(2) & \cdots & p(N) \\
h(1) & h(2) & \cdots & h(N) \\
\alpha(1) & \alpha(2) & \cdots & \alpha(N)
\end{bmatrix}
\]

\[
\mathbf{y} = \begin{bmatrix}
F_p(1) & F_h(1) & F_\alpha(1) \\
F_p(2) & F_h(2) & F_\alpha(2) \\
\vdots & \vdots & \vdots \\
F_p(N) & F_h(N) & F_\alpha(N)
\end{bmatrix}
\]

An overdetermined set of linear equations with 18 unknowns are given by

\[
\mathbf{E}(U, \omega)\mathbf{x} = \mathbf{y}
\]

which is solved by the pseudo inverse matrix inversion method

\[
\mathbf{E}(U, \omega) = \mathbf{y}\mathbf{x}^+
\]

where \( + \) is the pseudo inverse operator. It is also possible to use least squares time domain methods to estimate \( \mathbf{E}(U, \omega) \) which is described in (Han et al. 2014;
Siedziako et al. 2017).

The coefficients in the $3 \times 6$ matrix, $E(U, \omega)$, describes the transfer functions between the measured input motion $x$ and output forces $y$ which include contributions from both inertia forces and motion induced forces. Still air harmonic forced motion tests were used to estimate the transfer functions for the inertia forces, $E(0, \omega)$. Forced motion tests in wind with identical motion time history were conducted and the motion induced force transfer functions, i.e. the flutter derivatives, were estimated by subtracting the inertia force transfer functions. The dimensional flutter derivatives given by,

$$\begin{bmatrix}
C^{ae}(U, \omega) \\
K^{ae}(U, \omega)
\end{bmatrix} = E(U, \omega) - E(0, \omega)$$

were used to express the non-dimensional flutter derivatives, $P_i^*, H_i^*, A_i^*$ where $i = 1 - 6$, shown in Figure 6. The normalization with $B, \omega, U$ and $\rho$ follows from Equation (5).

**FINITE ELEMENT RESULTS**

The bridge girder equivalent mass and stiffness of the lowest torsional and vertical modes obtained from modal analysis of the FE models investigated in the present paper are shown in Table 2.

**Modal analysis**

An example of the mode shapes of the bridge deck and the corresponding natural frequencies are shown for configuration 9E in Figure 7.
Static response

With the development of stronger and lighter materials, lightweight bridge girders might be feasible for future long span bridges. The present FE models constitutes a set of lightweight bridge girders with low vertical and torsional stiffness. Furthermore, the cross sectional area of the main cables are low relative to the span. Due to the low masses and stiffnesses, however, the bridges become more flexible.

The static response of the present set of bridge models at $U = 70 \text{ m/s}$ are shown for the rotational, vertical and lateral degrees of freedom, $\alpha$, $z$ and $x$ in Table 3. The static displacements may seem large due to the low mass and stiffness. Nevertheless, static divergence were not observed. It is explained by the negative lift and moment of the present section. As the section rotates nose down the downward lift force increases the tension in the main cables and thus the geometrical stiffness. This effect is opposite to the torsional stiffness reduction caused by positive moments and positive upward lift forces described in (Zhang et al. 2013).

The smallest static displacements were obtained for series 9E with a maximum static rotation, $\bar{\alpha} = -3.2(\degree)$ (nose down) at $U = 70 \text{ m/s}$. The static response along the span at different mean wind speeds is shown in Figure 8.

Case study on torsional stiffness

A case study was done where the torsional constant of the girder was increased to $GI_t = 3.9365 \times 10^{11} \text{ Nm}^2$ and the main cable semi-distance to $e_k = 26.5 \text{ m}$. In this case, the static rotation was reduced to $\bar{\alpha} = -2.2(\degree)$ at $U = 70 \text{ m/s}$ while the vertical displacement was reduced to $\bar{z} = -3.0 \text{ m}$.
**Case study on bridge girder mass**

A case study on the effect of lower girder mass was conducted. The torsional stiffness of the girder, $GI_t$, and the main cable semi-distance, $e_k$, were identical to the case study on torsional stiffness. The low mass caused lower tension in the main cables which reduced the geometrical stiffness. This was observed by calculating the equivalent stiffnesses, $k_{eq} = \omega^2 m_{eq}$ of the bridge girder in the dead load situation in still air. The displacements, relative to the dead load situation, increased. Similar observation can be done by comparing configuration 9F with 9G in Tables 2 and 3.

**Multimodal flutter analysis**

State-of-the art multimodal flutter analysis (Agar 1989; Jain et al. 1996) was conducted for the models including the first 10 still air modes and the curve fit to the 18 flutter derivatives shown in Figure 6. The equivalent still air stiffness matrix was obtained from the undamped natural frequencies and the identity mass matrix. The equivalent damping matrix was obtained by assuming a damping ratio of $\zeta = 0.5\%$ for all modes.

**FREE VIBRATION MODEL TESTS**

The free vibration tests were conducted to explore any aeroelastic phenomena between the torsional and vertical degree of freedom and to observe the static displacements in model scale. The lateral degree of freedom of the section model was restrained by dragwires during the tests. If static divergence or flutter occurred, the critical wind speed would have been experimentally observed.

**Scaling**

Dynamic similitude between the lowest torsional mode of the bridge deck in the finite element models and the shape-wise similar vertical mode were obtained by making the torsional-to-vertical frequency ratio, $\gamma_\omega$ identical. Furthermore,
the equivalent mass per unit length, $m_e$ and mass moment of inertia per unit length, $I_e$, were scaled by $\lambda_m = \lambda_g^2 = 1^2 : 50^2 = 1 : 2500$ and $\lambda_I = \lambda_g^4 = 1 : 6250000$ respectively. The reduced wind speed, $U_r = U/(f_z B)$, was used to convert the tunnel wind speed to full scale. Thus, the wind scale was given by $\lambda_w = \lambda_g f_z^p / f_z^m$ where $f_z^p$ is the frequency of the vertical mode in the FE model and $f_z^m$ is the vertical frequency of the elastically suspended bridge section model in the dynamic rig.

**Results**

Summarized results from the stability limit tests conducted in the dynamic rig are reported in Table 4. The highest reduced wind speeds obtained during the test series are denoted $U_{max}$. The equivalent mass, $m_e$ and mass moment of inertia, $I_e$, were obtained from the natural frequencies of still air free decay tests and the aprior known dynamic stiffness of the rig. This was determined from series of free decay tests of the section model exposed to a wide range of dummy mass configurations. All reduced wind speeds are defined by $U_r = U/(f_z B)$ where $f_z$ is the still air frequency of the vertical mode. The torsional-to-vertical frequency ratios, $\gamma_\omega$, between the vertical and torsional still air mode were adjusted by the torsional stiffness of the dynamic rig given by the distance between the suspending springs.

**Series 8A to 9A**

Large amplitude oscillations were observed for series 8A at $\gamma_\omega = 0.96$. The wind noses attached to the central box did reduce the amplitudes, but severe vibrations were present in 9A also between $6 \leq U_r \leq 9$. The severe oscillations are illustrated by a large peak in the standard deviation of the measured vertical and torsional signals shown in Figure 9. The vortex induced vibration amplitude was reduced when the torsional stiffness was increased.
Series 9B to 9D

In series 9B, the torsional stiffness was enhanced and the frequency ratio increased to $\gamma_\omega = 1.03$. Results for series 9B at $U_r < 9$ are not shown in Figure 9 because only free decay tests were conducted here. The severe oscillations observed for series 9A did not occur during the free decay tests at similar reduced wind speeds for series 9B. The torsional stiffness was enhanced in series 9C and again in series 9D, which decreased the mean displacements as well as the standard deviations. The maximum equivalent full scale wind speed reached in series 9D were $U = 63$ m/s assuming a vertical still air frequency, $f_z = 0.082$ Hz, of the full scale structure. Series 9B, 9C and 9D were not prone to aerodynamic instabilities.

Series 9E to 9G

Prior to test series 9E, a softer rig was installed in the wind tunnel in order to reduce the natural frequencies and thereby obtain higher reduced wind speeds. Test series 9E ($\gamma_\omega = 1.21$) reached a reduced wind speed $U_r = 17.82$ corresponding to an equivalent full scale wind speed of $U = 88$ m/s. At the high reduced wind speeds, the section had very high levels of aerodynamic torsional and vertical damping which can be seen in the free decay tests at $U_r = 14.53$ and $U_r = 17.03$ shown in Figure 9. The initial impulse is damped out immediately after its first cycle of oscillation at $U_r = 17.03$. The remaining part of the signal is dominated by buffeting due to signature turbulence. Static divergence or flutter was not observed. The second order effects due to negative angle of attacks mainly caused larger downward displacement. The static rotations were merely caused by first order effects in the interval between $-6(^\circ) < \alpha < 0(^\circ)$ where $\frac{\partial C_m}{\partial \alpha} \approx 0$ as shown in Figure 5.

Test series 9F and 9G had similar torsional stiffness, but series 9G had significantly less mass and mass moment of inertia giving higher vertical and torsional
frequencies. Both sections had aerodynamically stable behavior at the highest wind speeds reached in the tunnel.

Conversion of displacements from model scale to full scale

Static displacements in the free vibration model scale wind tunnel tests can be converted to full scale maximum displacements using the geometric scale, $\lambda_g$. However, the static stiffness in the finite element models is higher than in the scaled section model tests because only two modes are represented here. This tends to cause small errors on the conservative side which can be seen by comparing the torsional and vertical displacements at the reduced wind speeds for $U = 70$ m/s reported in Table 3 with the displacements for the same reduced wind speed in the model tests of series 9A to 9D in Figure 9.

Flutter analysis

Classical bimodal flutter analysis (Dyrbye and Hansen 1997; Simiu and Scanlan 1986; Theodorsen 1934) for the lowest torsional mode and a shape-wise identical vertical mode was conducted for configuration 9A to 9F. The critical wind speeds, $U_{cr}/(f_z B)$ versus the torsional-to-vertical frequency ratio, $\gamma_\omega$ are shown in the upper left plot in Figure 10. The masses and the vertical frequencies correspond to Table 2. The crosses shown in Figure 10 represent the calculated bimodal flutter wind speeds for the scaled free vibration model tests with the masses and stiffnesses given in Table 4. There is an excellent agreement between the bimodal approach (Dyrbye and Hansen 1997) assuming shape-wise identical modes and with the multimodal flutter approach (Agar 1989; Jain et al. 1996) considering the different degrees of similarity between the first 10 still air mode shapes. This agrees with the findings reached in (Larsen 2014).

The bimodal flutter wind speeds using the flutter derivatives for the present section is approximately 210 % higher than for equivalent flat plates (Theodorsen...
1934) having identical masses, stiffnesses and widths at frequency ratios \( \gamma \omega > 1.4 \).

It is seen in the upper left plot in Figure 10 that the flutter curves obtained by the present flutter derivatives do not show asymptotic behavior at the lowest frequency ratios contrary to the curves obtained by flat plate derivatives also shown in the upper left plot in Figure 10 below the flutter curves for the present section.

The converged values for the frequency and damping of the vertical and torsional dominated poles obtained using an iterative bimodal are shown with solid lines in the upper right and lower left plots in Figure 10, respectively. The circular and triangular markers are results obtained from operational modal analysis of the free decay wind tunnel tests. There seems to be good agreement until \( U_r = 5 \), but significantly higher damping values are observed at higher wind speeds indicating a considerable amount of positive aerodynamic damping. The flutter analysis predicted a critical flutter wind speed \( U_{cr}/(f_zB) = 13.6 \) for series 9E, but dynamically stable tests were conducted at \( U_r = 17.07 \) as shown in Figure 9.

It is known that the flutter derivatives change with the mean angle of attack, \( \bar{\alpha} \) (Zhang et al. 2002; Diana et al. 2013; Zasso et al. 2013). Different flutter derivatives were obtained at different angles of attack for the present test series 8 as shown in Figure 6. In (Zasso et al. 2013) it is reported that the critical flutter wind speed, denoted \( V_{fl} \) in (Zasso et al. 2013), decreased with nose up static rotations. Nose down rotations increased \( V_{fl} \) from \( V_{fl} = 87 \text{ m/s} \) at \( \bar{\alpha} = 0(\degree) \) to \( V_{fl} = 98 \text{ m/s} \) at \( \bar{\alpha} = -2(\degree) \). At \( \bar{\alpha} = -4(\degree) \) the flutter wind speed increased to infinity, \( V_{fl} \to \infty \) (Zasso et al. 2013). It is therefore plausible that the nose down static rotations of the present section saved it from flutter.

**CONCLUSIONS**

The findings in the present study shows that light-weight bridge girders can
be aerodynamically stably despite of limited torsional girder stiffness. This could
be a sustainable and cost effective solution for future long span bridges because of
large material savings in the bridge girder, cables and pylons.

The novel triple-box girder, presented in this paper, has a negative moment
coefficient causing the bridge deck to rotate with the nose down. In the interval
between $-6° < \alpha < 0°$, an almost flat curvature for the moment coefficient is
obtained. The effect of increased drag, causing a moment in the nose up direction,
and negative lift, causing a moment in the nose down direction, balances. This is
an effective measure to avoid torsional divergence. Degradation of the torsional
stiffness provided by the main cables, in wind, is not an issue contrary to bridge
decks with positive lift and moment. Instead, the maximum allowable cable
tension could become the limiting design factor.

Seven different configurations of masses and stiffnesses of the triple-box girders
were analysed by non-linear static finite element analyses and wind tunnel tests.
Static divergence were avoided in all cases, but naturally, the configurations hav-
ing least torsional stiffness experienced the largest displacements. A configuration
where the torsional frequency was lower than the vertical was studied. The mean
displacements was larger than for configurations with higher torsional stiffness
and wind tunnel tests revealed severe steady-state oscillations at reduced wind
speeds $6 < U/(f_z B) < 10$. Wind noses were shown to reduce these vibrations,
but the best measure was to increase the torsional stiffness and the torsional-to-
vertical frequency ratio.

Eighteen flutter derivatives for the present sections were obtained by forced
motion tests. Flutter analysis showed that the critical flutter wind speed versus
the torsional-to-vertical frequency ratio, $\gamma_\omega$ do not show asymptotic behavior for
the present section (with wind noses) at $\gamma_\omega \approx 1$ contrary to the classical flat plate.
Multimodal and bimodal flutter analysis were shown to give similar results if the
bimodal analysis were conducted with the lowest anti-symmetric vertical and
torsional modes assuming that they are shape-wise identical. Compared to the
flat plate flutter wind speeds, the present section reaches more than 200% higher
critical flutter wind speeds. The critical flutter wind speed was not observed
during the free vibration tests. It is plausible that the static rotation of the
section increased the critical flutter wind speed considerably more than 200%.

The triple box girder (with wind noses) in the present paper were shown to
be aerodynamically stable despite its lightweight setup relative to the long span
of $L = 2125.2$ m. This indicates that aerodynamic stability for very long span
suspension bridges can be achieved despite low mass of the bridge girder which
is an economic advantage. Thus, there will be other design obstacles than the
aerodynamics, that govern the lower limit of the girder mass.

The torsional stiffness of the girder in the free vibration tests and finite element
analyses conducted in the present study were considered to be very low. There
is no reason, however, to artificially reduce the torsional stiffness of the present
section in order avoid classical flutter by designing $\gamma_{\omega} \leq 1$ since the section showed
no vulnerabilities to flutter at several torsional-to-vertical frequency ratios slightly
above $\gamma_{\omega} = 1$.

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REFERENCES


Theodorsen, T. (1934). “General theory of aerodynamic instability and the mech-


APPENDIX I. FIGURES AND TABLES

(a) Cross section.

(b) Railings of the external boxes (left) and central box (right).

FIG. 1: Cross sectional drawing of triple box girder suspension bridge studied in the present paper. Different configurations of the cable semi-distance, $e_k$ are investigated.

FIG. 2: Elevation of the finite element models of the conceptual prototype suspension bridges studied in the present paper.

(a) Bridge cross sectional model, test series 8 (S8).

(b) Bridge cross sectional model, test series 9 (S9).

FIG. 3: Bridge section models investigated by wind tunnel tests. All dimensions are in mm.
FIG. 4: Definition of the mean wind; $U$, body axes; $x, z$, wind loads per unit length; $F_x, F_d, F_z, F_l$ and $F_m$, displacements; $x, p, z, h$ and $\alpha$. The elastic axis is denoted EA.

Steady force coefficients

FIG. 5: Force coefficients. The coefficients are normalized with the full width, $B = 1.2$ m in model scale corresponding to $B = 60$ m in full scale. Open square markers represent experimental results for test series 8 (S8) and the solid circular markers represent results for test series 9 (S9).

<table>
<thead>
<tr>
<th>Structural part</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridge deck</td>
<td>Width, $B$</td>
<td>60 m</td>
</tr>
<tr>
<td></td>
<td>Height, $D$</td>
<td>2 m</td>
</tr>
<tr>
<td></td>
<td>Mass per unit length, $m_d$</td>
<td>$(6.38 - 13.94) \times 10^3$ kg/m</td>
</tr>
<tr>
<td></td>
<td>Mass moment of inertia per unit length, $I_d$</td>
<td>$(2275 - 6107) \times 10^3$ kgm$^2$/m</td>
</tr>
<tr>
<td></td>
<td>Vertical bending stiffness, $EI_z$</td>
<td>$1.092 \times 10^{11}$Nm$^2$</td>
</tr>
<tr>
<td></td>
<td>Lateral bending stiffness, $EI_x$</td>
<td>$4125 \times 10^{11}$Nm$^2$</td>
</tr>
<tr>
<td></td>
<td>Torsional stiffness, $GI_t$</td>
<td>$(1.21 - 3.07) \times 10^{10}$Nm$^2$</td>
</tr>
<tr>
<td>Main cables</td>
<td>Tensional stiffness, $AE_c$</td>
<td>$4.5258 \times 10^{10}$ N</td>
</tr>
<tr>
<td></td>
<td>Mass per unit length, $m_c$</td>
<td>1.89 t/m</td>
</tr>
<tr>
<td></td>
<td>Semi-distance, $e_k$</td>
<td>(19.25-24.5) m</td>
</tr>
<tr>
<td>Hangers</td>
<td>Tensional stiffness ($AE_h$)</td>
<td>$28.16 \times 10^8$ N</td>
</tr>
<tr>
<td></td>
<td>Mass per unit length</td>
<td>148 kg/m</td>
</tr>
</tbody>
</table>

TABLE 1: Mechanical properties of the bridge models in the present paper.
FIG. 6: Flutter derivatives obtained by forced motion tests. The derivatives are normalized with the full width, $B = 1.2$ m in model scale corresponding to $B = 60$ m in full scale.
FIG. 7: Spanwise static displacements relative to still air shown in the upper plots and lower left plot in wind speed intervals \( \Delta U = 5 \text{ m/s} \). Static displacements at the mid span are shown in the lower right plot.

<table>
<thead>
<tr>
<th>Test series</th>
<th>( \gamma_{\omega_{\omega}} )</th>
<th>( m_e )</th>
<th>( I_e )</th>
<th>( f_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9A</td>
<td>0.97</td>
<td>18.92</td>
<td>7888</td>
<td>0.082</td>
</tr>
<tr>
<td>9B</td>
<td>1.03</td>
<td>18.92</td>
<td>8438</td>
<td>0.082</td>
</tr>
<tr>
<td>9C</td>
<td>1.09</td>
<td>19.02</td>
<td>8500</td>
<td>0.082</td>
</tr>
<tr>
<td>9D</td>
<td>1.15</td>
<td>19.02</td>
<td>8490</td>
<td>0.082</td>
</tr>
<tr>
<td>9E</td>
<td>1.21</td>
<td>19.02</td>
<td>8062</td>
<td>0.082</td>
</tr>
<tr>
<td>9F</td>
<td>1.10</td>
<td>19.08</td>
<td>8060</td>
<td>0.082</td>
</tr>
<tr>
<td>9G</td>
<td>1.55</td>
<td>11.60</td>
<td>2812</td>
<td>0.080</td>
</tr>
</tbody>
</table>

TABLE 2: FE model properties of suspension bridges.
FIG. 8: Mean values, $\mu$, and standard deviations, $\sigma$, of the vertical and torsional response from test series 9A-9D. The legends shown in the upper right plot applies to all plots.

Table 3: Mean static response displacements at $U = 70$ m/s at the mid-span of the bridge deck relative to still air
FIG. 9: Free decay tests for test series 9E showing that the aerodynamic damping and the static displacements increases with the reduced wind speeds.

<table>
<thead>
<tr>
<th>Test series</th>
<th>B</th>
<th>$\gamma_\omega$</th>
<th>$m_e$</th>
<th>$I_e$</th>
<th>$f_z$</th>
<th>$U_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
<td>$\omega_e/\omega_z$</td>
<td>kg/m</td>
<td>kgm$^2$/m</td>
<td>Hz</td>
<td>$\frac{U}{Bf_\omega}$</td>
</tr>
<tr>
<td>8A</td>
<td>1.2</td>
<td>0.96</td>
<td>7.41</td>
<td>1.27</td>
<td>1.36</td>
<td>12.36</td>
</tr>
<tr>
<td>9A</td>
<td>1.2</td>
<td>0.97</td>
<td>7.57</td>
<td>1.27</td>
<td>1.35</td>
<td>12.54</td>
</tr>
<tr>
<td>9B</td>
<td>1.2</td>
<td>1.03</td>
<td>7.57</td>
<td>1.35</td>
<td>1.35</td>
<td>12.51</td>
</tr>
<tr>
<td>9C</td>
<td>1.2</td>
<td>1.09</td>
<td>7.61</td>
<td>1.36</td>
<td>1.34</td>
<td>12.57</td>
</tr>
<tr>
<td>9D</td>
<td>1.2</td>
<td>1.15</td>
<td>7.62</td>
<td>1.36</td>
<td>1.34</td>
<td>12.76</td>
</tr>
<tr>
<td>9E</td>
<td>1.2</td>
<td>1.21</td>
<td>7.63</td>
<td>1.29</td>
<td>0.89</td>
<td>17.82</td>
</tr>
<tr>
<td>9F</td>
<td>1.2</td>
<td>1.10</td>
<td>7.63</td>
<td>1.29</td>
<td>0.89</td>
<td>17.17</td>
</tr>
<tr>
<td>9G</td>
<td>1.2</td>
<td>1.55</td>
<td>4.64</td>
<td>0.45</td>
<td>1.14</td>
<td>13.40</td>
</tr>
</tbody>
</table>

TABLE 4: Mechanical configuration of free vibration section model tests in the dynamic rig.
FIG. 10: Upper left plot: Theoretical flutter curves. Upper right plot: Theoretical frequency curves and experimentally estimated values. Lower left plot: Corresponding damping curves and experimentally values. Lower right plot: Static rotations/displacements from tests.