Correlations Between Interacting Rydberg Atoms

Asaf Paris-Mandoki$^{1,b}$, Christoph Braun$^{2,3}$ and Sebastian Hofferberth$^{2,a}$

$^1$Instituto de Física, Universidad Nacional Autónoma de México, Mexico City, Mexico
$^2$Department of Physics, Chemistry and Pharmacy, University of Southern Denmark, Odense, Denmark
$^3$5. Phys. Institut and Center for Integrated Quantum Science and Technology, Universität Stuttgart, Germany

$a)$Corresponding author: hofferberth@sdu.dk
$b)$asaf@fisica.unam.mx

Abstract. This paper is a short introduction to Rydberg physics and quantum nonlinear optics using Rydberg atoms. It has been prepared as a compliment to a series of lectures delivered during the Latin American School of Physics “Marcos Moshinsky” 2017. We provide a short introduction to the properties of individual Rydberg atoms and discuss in detail how the interaction potential between Rydberg atom pairs is calculated. We then discuss how this interaction gives rise to the Rydberg blockade mechanism. With the aid of hallmark experiments in the field applications of the blockade for creating correlated quantum systems are discussed. Our aim is to give an overview of this exciting and rapidly evolving field. The interested reader is referred to original work and more comprehensive reviews and tutorials for further details on these subjects.

INTRODUCTION

As soon as a physical system contains more than one constituting part, correlations can be studied between the parts. For example, in a perfect salt crystal, the positions of the ions are correlated, meaning that the knowledge of the position of one ion gives information about the positions of all other ions. This is an example of a classical correlation, however, when dealing with quantum systems a different kind of correlation may arise. A striking example of quantum correlations occurs with entangled systems, that is, systems where the total state cannot be written as a tensor product of the states of the subsystems. In this case, instantaneous correlations can result between two distant systems [1–4]. While it must be noted that these correlations do not allow information to be transmitted instantaneously, they are a valuable resource for future quantum technologies.

The key mechanism to create quantum correlations between systems is to have them interact. This has sparked a strong effort in the atomic physics community to increase and harness long-range interactions. As a result, the preparation of ultracold gases of molecules [5, 6] and atoms with a strong dipole moment [7–9] became an important milestone.

An alternative approach to achieve strong interactions is to take ground state neutral atoms, which would normally only interact weakly through collisions, and excite them to a high-lying state. These excited atoms, also known as Rydberg atoms [10], obey a strong scaling of the atomic polarizability $\alpha$ with principal quantum number ($\alpha \propto n^7$) resulting in a strong interaction between individual atoms at micrometer distances when excited to principal quantum numbers $n \lesssim 30$. This fact has established Rydberg atoms as a prominent tool for realizing strong interactions in a controlled manner.

This interaction between pairs of Rydberg atoms has proven to be a key feature for diverse applications in quantum information processing and quantum simulation [11]. A particularly important concept is the Rydberg blockade [12], where the excitation of two or more atoms to a Rydberg state is prevented due to the interaction. The Rydberg blockade of atomic ensembles [13] has been observed in ultracold atomic systems in the frozen Rydberg-gas regime [14, 15], both in bulk ensembles [16–27] and in small systems supporting only a single excitation [28–31]. More recently, experiments have also begun to probe Rydberg interaction effects in room-temperature thermal vapor [32–34]. Based on the Rydberg blockade, atomic two-qubit gates have been demonstrated [11, 35–37] as basic building blocks for large-scale neutral atom quantum registers [38–42]. In turn, the power-law decay of the Rydberg interaction provides interaction over long range and facilitates the extension to multi-qubit Rydberg-mediated gates in such
FIGURE 1. Energy structure of Rubidium. This energy structure is similar among Alkali atoms. Also depicted with arrows is a two-photon excitation scheme used to produce Rydberg atoms. Starting from an \( S \) state, \( S \) and \( D \) Rydberg states can be reached with this method. For large intermediate-state detuning \( \delta \) the three-level system behaves like a two-level system with effective Rabi frequency \( \Omega = \sqrt{\Omega_r^2/4\delta^2 + \Delta^2} \).

registers [43–45]. Such tailored atomic ensembles are also ideal for investigating processes such as excitation transfer [14, 46–49] or simulation of spin systems [50, 51].

The mapping of Rydberg interactions onto photons by means of electromagnetically induced transparency (EIT) [52] has emerged as a powerful approach to realizing few-photon optical nonlinearities [53–59], enabling a variety of optical quantum information applications such as highly efficient single-photon generation [60], entanglement generation between light and atomic excitations [61], single-photon all-optical switches [62] and transistors [63–65], single-photon subtraction [66], and interaction-induced \( \pi \)-phase shifts [67]. Additionally, Rydberg EIT provides access to novel phenomena such as attractive interaction between single photons [68], crystallization of photons [69], or photonic scattering resonances [70], as well as spatially resolved detection of single Rydberg atoms in a bulk medium [46, 71, 72]. Recent Rydberg EIT experiments, which simultaneously use Rydberg \( S \)- and \( P \)-states [73] or two different \( S \)-states [46, 63, 64] further increase the flexibility of manipulating weak light fields [74, 75].

In this manuscript we will first revise the properties of individual Rydberg atoms. Next, we consider a pair of Rydberg atoms and describe the nature of the interaction between them. The Rydberg blockade resulting from the interaction is then described in more detail. If an ensemble of atoms is fully contained inside a single blockade volume the entire system behaves like a single atom with enhanced coupling to the light. This superatom is discussed followed by a discussion on the consequences of implementing an EIT scheme involving a Rydberg state.

SINGLE RYDBERG ATOM BASICS

Alkalis are the most widely used species for exciting into a Rydberg state because of their simple level structure, requiring only a couple of laser frequencies for excitation. In these atoms all electrons occupy filled shells, except for a single valence electron. As a consequence, the Alkali spectra are qualitatively similar to that of Hydrogen, as long as one only considers transitions of the valence electron (excitations of closed shell electrons are in the UV to X-ray regimes).

The similarity to Hydrogen becomes more pronounced for Alkali Rydberg states, where the valence electron
is far away from the nucleus and the filled shells of other electrons. As presented in any basic quantum mechanics textbook, the energy levels for Hydrogen-like atoms are given by \( E_n = \frac{\hbar c R_\infty}{n^2} \), where \( c \) is the speed of light, \( R_\infty \) is Rydberg's constant and \( n \) is the principal quantum number of the electronic state. For Alkali Rydberg states the energy looks very similar to this, with (small) species-dependent corrections, which are accounted for by introducing the so-called quantum defect \( \delta_{n,l,j} \) resulting in an energy formula given by

\[
E_{n,l,j} = \frac{\hbar c R_\infty^*}{(n - \delta_{n,l,j})^2} = \frac{\hbar c R_\infty^*}{(n^*)^2},
\]

where \( n^* = n - \delta_{n,l,j} \) is the effective principal quantum number and \( R_\infty^* = R_\infty/(1 + \frac{m}{M_\text{amo}}) \) is the modified Rydberg constant for the specific atomic species. This formula is illustrated in Figure 1. It is worth noting that \( n^* \) is just defined as a convenience to express the energies while the quantum number describing the electronic state is still \( n \). This expression reflects the fact that the binding potential for the valence electron does not behave like a pure \( 1/r \) potential and thus, the \( l \)-degeneracy of the levels is broken. Furthermore, the dependence in \( j \) accounts for the fine-structure (relativistic) effects.

While the effective potential exerted on the valence electron is no longer the Coulomb potential, it is still a spherically symmetric potential which allows obtaining the wave function of the valence electron as a product of radial and angular wave functions. The angular part can be solved analytically and is given by spherical harmonics.

The radial wave functions for the Alkalis can be calculated to a very good approximation using model potentials for the radial and angular wave functions. The angular part can be solved analytically and is given by spherical harmonics. The spherically symmetric potential which allows obtaining the wave function of the valence electron as a product of radial and angular wave functions is far away from the nucleus and the filled shells of other electrons. As presented in any basic quantum mechanics textbook, the energy levels for Hydrogen-like atoms are given by \( E_n = \frac{\hbar c R_\infty}{n^2} \), where \( c \) is the speed of light, \( R_\infty \) is Rydberg's constant and \( n \) is the principal quantum number of the electronic state. For Alkali Rydberg states the energy looks very similar to this, with (small) species-dependent corrections, which are accounted for by introducing the so-called quantum defect \( \delta_{n,l,j} \) resulting in an energy formula given by

\[
E_{n,l,j} = \frac{\hbar c R_\infty^*}{(n - \delta_{n,l,j})^2} = \frac{\hbar c R_\infty^*}{(n^*)^2},
\]

where \( n^* = n - \delta_{n,l,j} \) is the effective principal quantum number and \( R_\infty^* = R_\infty/(1 + \frac{m}{M_\text{amo}}) \) is the modified Rydberg constant for the specific atomic species. This formula is illustrated in Figure 1. It is worth noting that \( n^* \) is just defined as a convenience to express the energies while the quantum number describing the electronic state is still \( n \). This expression reflects the fact that the binding potential for the valence electron does not behave like a pure \( 1/r \) potential and thus, the \( l \)-degeneracy of the levels is broken. Furthermore, the dependence in \( j \) accounts for the fine-structure (relativistic) effects.

While the effective potential exerted on the valence electron is no longer the Coulomb potential, it is still a spherically symmetric potential which allows obtaining the wave function of the valence electron as a product of radial and angular wave functions. The angular part can be solved analytically and is given by spherical harmonics. The radial wave functions for the Alkalis can be calculated to a very good approximation using model potentials for the valence electron [76–78]. The use of a model potential reduces the problem of finding the wave function of a full Alkali atom to solving the Schrödinger equation of a two-body problem which can be done numerically using standard methods [79]. The model potential \( V_{\text{mod}}(r) \) typically consists of three terms:

\[
V_{\text{mod}}(r) = V_c(r) + V_p(r) + V_{\text{so}}(r).
\]

The first term

\[
V_c(r) = -\frac{Z_0(r)}{r},
\]

with

\[
Z_0(r) = 1 + (Z - 1)e^{-\alpha r} - r(\alpha_3 + \alpha_4 r)e^{-\alpha_5 r},
\]

is the core potential including the effect of the screening of the nucleus by the core electrons. The polarizability term

\[
V_p(r) = -\frac{\alpha_0}{2r^4} \left[ 1 - e^{-(r/r_c)^2} \right],
\]

describes the fact that the core charge distribution can be deformed by the valence electron. Finally,

\[
V_{\text{so}}(r > r_c) = \frac{\alpha^2}{2r^4} \mathbf{L} \cdot \mathbf{S}
\]

is the spin-orbit coupling term, where \( \alpha \approx 1/137 \) is the fine-structure constant. The model parameters \( \alpha_{1,2,3,4,5} \) and \( r_c \) are chosen such that the obtained eigenvalues reproduce the experimentally measured energy levels. This form of \( V_{\text{so}}(r) \) is only valid for \( r > r_c \) and therefore should only be used to calculate high-lying states where this condition is satisfied \( (n > 20) \). A more general form of the spin-orbit term can be obtained from the Dirac equation [77].

The interaction of an atom with the electric field, both as a static field and as an electromagnetic wave is determined by the dipole Hamiltonian

\[
H_{\text{AF}} = -\mathbf{d} \cdot \mathbf{E}.
\]

Therefore, the knowledge of the dipole matrix elements is crucial for quantitatively understanding this interaction. Having obtained the radial wave functions, it is now possible to compute the dipole matrix elements using the following formula [80]

\[
\langle n,l,m|d_{lq}|n',l',j',m'\rangle = \frac{(-1)^{j+m}}{\sqrt{(2j+1)(2j'+1)(2l+1)(2l'+1)}} \times \sum_{l} \int_{0}^{\infty} R_{nlj} r R_{n'l'j'} e^{ilq} e^{-r} r^2 dr.
\]
where (:::) and {:::} are Wigner 3\textit{j} and 6\textit{j} symbols respectively.

The last integral in Eq. 4 depends on the spatial overlap of the involved states. As a result, the dipole matrix element between a low lying state and a Rydberg state with principal quantum number \( n \) has an \( n^{-3/2} \) scaling due to the normalization of the wavefunctions [10]. On the other hand, the dipole matrix elements between neighboring levels scales as the orbital radius, as \( n^2 \).

In principle, a Rydberg state can decay to the ground state through a forest of intermediate states. However, the most probable decay path is a direct decay to the lowest possible state. Following this argument, the scaling of the natural lifetime of a Rydberg state with small angular momoentum number \( \ell \) goes like \( \tau_{\text{Ryd}} \propto n^3 \). Nonetheless, spontaneous decay is not the only relevant loss mechanism for Rydberg state. As \( n \) increases the spacing between adjacent levels decreases while the transition elements between them remains large. This means that the blackbody radiation of an environment at 300 K can produce a significant coupling of a state to its neighbors. For example [81], for the 43\textit{S}_{1/2} state the natural lifetime is 80\,\mu s while coupling to the blackbody radiation of an environment at 300 K reduces this to 42\,\mu s.

Another relevant quantity is the polarizability of a Rydberg level. Following a simple second-order perturbation calculation, we can find the polarizability as the ratio of the square of the dipole matrix elements between adjacent levels (\( n^4 \)) and their energy spacing (\( n^{-3} \)) resulting in a scaling \( \alpha \propto n^7 \).

The following table summarizes the scaling of these relevant quantities with respect to principal quantum number:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>(-n^{-2})</td>
</tr>
<tr>
<td>Level spacing</td>
<td>(n^{-3})</td>
</tr>
<tr>
<td>Dipole Matrix Element ( \langle 5P</td>
<td>\hat{d}</td>
</tr>
<tr>
<td>Dipole Matrix Element ( \langle nP</td>
<td>\hat{d}</td>
</tr>
<tr>
<td>Natural Lifetime ( \tau_{\text{Ryd}} )</td>
<td>(n^3)</td>
</tr>
<tr>
<td>Polarizability ( \alpha )</td>
<td>(n^7)</td>
</tr>
</tbody>
</table>

Looking at these scaling laws, it becomes clear why circular Rydberg states have played such an important role in the quest for increasing the atom-photon coupling [82]. First of all, the decrease of the level spacing with \( n \) allows having transitions in the microwave regime where superconducting cavities are resonant. Moreover, the increase of dipole matrix elements between neighboring levels with \( n \) can be exploited to greatly increase the coupling.
RYDBERG INTERACTION

The physics of Rydberg interaction has been well-established for decades [83]. As long as the two atoms are well separated and their wave functions do not overlap, one needs to consider only the electrostatic interaction between two localized charge distributions, most conveniently utilizing the well-known electric multipole expansion in spherical coordinates [84–86]. The leading relevant term in this expansion is the dipole-dipole interaction [87] which, for unperturbed Rydberg atoms at large separation, results in the extensively studied van-der-Waals interaction [88–91]. More generally, as long as the interaction energies are small compared to the level spacing of the unperturbed Rydberg pair states, perturbative calculations offer a very convenient method for determining the radial [88] and angular [89, 90] behavior of the Rydberg potentials.

Furthermore, the theoretical description of the interaction between two Rydberg atoms can be greatly simplified using the Born-Oppenheimer approximation [92], where the nuclei positions are considered fixed and we only focus on the fast-moving electrons. We consider two neutral atoms separated by a distance r, each having an electron excited into a Rydberg state, as depicted in Figure 2. This system is described by the Hamiltonian

\[ \hat{H} = \hat{H}_1 \otimes \hat{H}_2 + \hat{H}_\text{int}(r), \]

where

\[ \hat{H}_1 = \sum_{n, l, j, m} E_{n, l, j} |n, l, j, m\rangle \langle n, l, j, m| \]

is the single-atom Hamiltonian with eigenvalues defined in Eq. 1 and \( \hat{I}_i \) is the identity operator acting on the Hilbert space of atom \( i \). Moreover, we assume the inter-atomic distance to be larger than the Le Roy radius, i.e.

\[ r > r_L = 2 \left( \frac{3}{i_L^3} \right)^{1/2} + 2 \left( \frac{3}{i_L^3} \right)^{1/2} \]

in which case the individual electronic wave functions do not overlap and the two valence electrons can be treated as distinguishable particles.

The interaction Hamiltonian can be obtained analogously to the classical case. The charge distribution of each atom can be written as a multipole expansion. The interaction potential is then given by the product of the two expansion series. For practical calculation, the interaction potential series is truncated at some point. Since the atoms are neutral, all terms with monopole moments vanish. Hence, the leading term that contributes to the interaction potential is the dipole-dipole term, which has the form

\[ \hat{H}_\text{int}(r) = \frac{\mathbf{d}_1 \cdot \mathbf{d}_2 - 3 (\hat{r} \cdot \mathbf{d}_1)(\hat{r} \cdot \mathbf{d}_2)}{r^3}. \]

Here, \( \hat{r} \) is a unit vector in the direction of \( \mathbf{r} \) and \( \mathbf{d}_1 \) and \( \mathbf{d}_2 \) are the dipole operators for atom 1 and 2 respectively.

As two atoms are involved in this problem, a sensible choice of basis is the pair-state basis made up of tensor products of eigenstates of the atomic Hamiltonian, i.e. kets of the form

\[ |n_1, l_1, j_1, m_1\rangle \otimes |n_2, l_2, j_2, m_2\rangle. \]

In this basis the the atomic Hamiltonians (Eq. 5) are diagonal. Moreover, when \( r \to \infty \) the total energy of such a state is given by \( E_{n_1,l_1,j_1,m_1} + E_{n_2,l_2,j_2,m_2} \). As these atoms come closer to each other the energy is modified by the interaction Hamiltonian. In the case where the interaction can be treated as a perturbation to the atomic energy levels, the interaction potential is given by the new perturbed levels as a function of the inter-atomic distance \( r \). According to perturbation theory the potential between two atoms in the pair state \( |k\rangle \) has the form

\[ V_{i,k}(r) = \langle k|\hat{H}_\text{int}(r)|k\rangle + \sum_{p \neq k} \frac{|\langle k|\hat{H}_\text{int}(r)|p\rangle|^2}{E_k - E_p}, \]

where the sum is taken over all other pair states \( |p\rangle \) and the potential has been shifted by a constant energy \( E_k \). The first term is zero because as seen in Equation 6 the interaction Hamiltonian is composed of dipole operators which can only couple states of opposite parity. This expression can be further simplified by noting that in most cases, out of
the infinite number of possible pair states \(|p⟩\), one state \(|p_0⟩\) will have an energy difference with respect to the \(|k⟩\) state \(Δ_E = E_k - E_{p_0}\) small enough to dominate over all other terms of the sum in Equation 8. In this case, the interaction potential can be written as a van der Waals potential of the form

\[
V_{\text{dd}}(r) = \frac{C_3}{r^3} + \frac{C_6}{r^6},
\]

\(V_{\text{dd}}(r) = \frac{C_3}{\Delta_E r^3} \approx \frac{C_6}{r^6}, \quad (9)
\]

where \(C_3/r^3 = ⟨k|\hat{H}_\text{dd}(r)|p_0⟩\) and \(C_6\) has been defined as \(C_3^2/\Delta_E\).

Due to the different polarizabilities of the pair states, it is possible to apply an electric field to bring two pair states into degeneracy. In this case, known as a Förster resonance, the second-order perturbation approximation applied in Eq. 8 is not valid. Instead, the interaction potential behaves like \(V_{\text{dd}}(r) = C_3/r^3\).

As noted by the previous discussion, many conditions have to be met in order for the interaction to be accurately described as a simple van der Waals potential. When these conditions are not met, a full diagonalization of the Hamiltonian must be carried out resulting in the complicated potential shown in Figure 3. In this case the potential is not a single energy curve but it must be described as multiple eigencurves associated to multiple eigenstates, all of which overlap with the \(r → \infty\) state.

A sophisticated experiment [93] in which two microtraps, holding individual atoms, can be arbitrarily positioned at will has allowed measuring the angular dependence of the interaction between two Rydberg atoms [94]. In the vicinity of a Förster resonance various angular momentum states mix and strongly anisotropic interaction can occur. Here, two single \(^{87}\text{Rb}\) atoms were prepared in their ground state in two tightly focussed optical tweezers. Both the distance \(R\) between the two atoms and the angle \(θ\) between the interatomic axis and the external fields could be precisely tuned. Using a two-photon excitation scheme, both atoms were excited to the \([59D_{3/2}, m_j = 3/2]\) state by applying a \(π\)-pulse. In the pair state basis and at zero electric field, the state \(|dd⟩ = [59D_{3/2}, m_j = 3/2; 59D_{3/2}, m_j = 3/2]\) is detuned by 8.69 MHz from the state \(|pf⟩ = [61P_{1/2}, m_j = 1/2; 57F_{5/2}, m_j = 5/2]\). Due to the different polarizabilities of the states, both pair states could be tuned into degeneracy by applying a weak electric field of 34.3 mV/cm. With this approach, Ravets et al. could map out the angular shape of the electric dipole-dipole interaction between the two atoms [94].

More specifically, the strength of the interaction was measured by letting the two-atom system evolve after the Rydberg excitation and in the presence of the electric field. After a variable hold time, a second optical \(π\)-pulse coupling to the \(|dd⟩\) state was employed to bring the atoms back to their ground state. By measuring the ground-state population after the full sequence, the time-evolution of the \(|dd⟩\) Rydberg pair state population could be reconstructed. Performing this experiment for various angles \(θ\) and fixed distance \(R = 9.1\ \mu\text{m}\) resulted in the beautiful dipole-dipole pattern of the interaction shown in Figure 4.
A more detailed discussion on the calculation of Rydberg interaction potentials can be found in the tutorial of Reference 78.

**RYDBERG BLOCKADE**

Naïvely, the simplest scheme to excite an atom into a Rydberg state may seem to use a laser to directly drive the transition from the atomic ground state to a target Rydberg state. However, transitions into a Rydberg state are in the UV (297 nm for rubidium) which makes this approach technically and economically challenging. An alternative approach is a two-photon excitation scheme as shown in Figure 1. In many experiments the laser for the first transition is set to be far detuned to minimize spontaneous scattering of photons via the intermediate level. The second laser can compensate for this detuning in order to drive the transition to the Rydberg state with no or little detuning. The intermediate level can be eliminated (i.e. it doesn’t play a role in the dynamics) for large enough detuning $\delta$. In this case the remaining effective two-level system has a total coupling strength $\Omega_R = \sqrt{\Omega_r^2 + \Delta^2}$, which results in an additional advantage of using a two-photon excitation scheme, as opposed to a direct one, since the coupling strength of an individual photon can be greatly enhanced by using a strong laser to drive the other transition.

A very important feature of Rydberg atoms is that their excitation is strongly modified by interactions with neighboring Rydberg atoms. Considering two atoms separated by a distance $r$, their possible pair-state energies are depicted in Figure 5. When the atoms are far away from each other, where the interaction is negligible, excitation of the pair into the $|rr\rangle$ state is possible using the same laser frequencies for both atoms. However, as the atoms get closer together the interaction shifts the $|rr\rangle$ level away precluding this state from being reached during the second excitation step. This defines a blockade radius $r_b = (C_6/\hbar \Omega_R)^{1/6}$ below which a second excitation cannot occur.

In the late 1990s intense experimental work with ultracold Rydberg atoms began and first hints of the Rydberg blockade were observed in the frozen Rydberg gas regime [14, 15]. Prompted by the applications of an interaction-induced blockade to quantum information processing [13] further evidence establishing the Blockade mechanism was found [16] followed by the indirect observation of coherent collective effects induced by the blockade [20]. Further on, very conclusive observations of the blockade mechanism were carried out by Urban et. al. [35] and Gaëtan et. al. [36] in 2009. In these experiments, two individual atoms where held in two distinct trapping sites as shown in Figure 6. In the experiment by Urban et. al. the two sites could be addressed independently and it was shown that when a Rydberg atom was excited in one of the sites, the excitation for the atom in the other site was suppressed. In contrast, in the work by Gaëtan et. al. both sites were addressed simultaneously by the same lasers and the separation between the two sites could be varied. They first showed that the simultaneous excitation of the two atoms is possible when the
FIGURE 5. Rydberg blockade mechanism. When two atoms are far apart they can both be excited into a Rydberg state. As the atoms get closer together, the interaction between them shifts away the states with more than one excitation preventing a second excitation from occurring.

FIGURE 6. Sketch of experiments demonstrating the Rydberg blockade. In these experiments two individual atoms (green) are trapped in two independent optical traps (grey). (a) Experiment by Urban et. al. [35] where the two atoms separated by 10 µm can be addressed independently by the excitation lasers. (b) Experiment by Gaëtan et. al. [36] where the atoms are addresses simultaneously by the excitation lasers. In this setup the distance between the atoms \( R \) can be varied.

atoms are far apart (18 µm) but when moved close together (3.6 µm) it is strongly suppressed. Moreover, they observe that when the two atoms are close together, the probability to excite only one atom oscillates with a factor of \( \sqrt{2} \) faster than if only a single atom is present. This enhancement of the Rabi frequency is a clear signature that this process is a collective oscillation between the ground state \( |gg\rangle \) and the entangled state \( |W\rangle = (|rg\rangle + |gr\rangle) / \sqrt{2} \).

The entanglement created by exciting a \( |W\rangle \) state has a lifetime limited by the Rydberg state which is in the order of 100 µs to 1 ms. However, by transferring the Rydberg population into an hyperfine ground state long-lived Bell states have been created [37]. Moreover, the Rydberg blockade effect has also been used to implement a controlled-NOT gate using two individual atoms in the same two-site setup presented before [95]. The implementation of this controlled-NOT relied in first implementing a controlled-phase gate by exploiting the blockade physics which can subsequently be converted into a controlled-NOT gate by applying a series of rotations. Isenhower et. al. describe the implementation of their controlled-phase gate as follows [95]:

...excitation of a control atom to a Rydberg level with principal quantum number \( n = 90 \) prevents subsequent excitation of a target atom in a neighboring site separated by \( R = 10 \) µm. Excitation and deexcitation of the target atom corresponds to a \( 2\pi \) rotation of an effective spin \( 1/2 \) which therefore imparts a \( \pi \) phase shift to the wave function of the target atom. If the control atom blocks the target excitation then the rotation does not occur and there is no phase shift of the target wave function. The result is a \( CZ \) controlled phase operation.
This proof-of-principle implementation of a CNOT gate achieved an operating fidelity \( F = 0.58 \). By technical improvements such as achieving lower temperatures and better control of the atomic motion by means of optical lattices this number could be increased. More details on quantum computation and simulation with Rydberg atoms can be found in Reference 96.

**RYDBERG SUPERATOMS**

Strong interactions between Rydberg atoms can turn the quantum behavior of an individual atom into a collective phenomena. For example, when an ensemble of atoms is contained within a blockade volume, it will behave like a single quantum two-level system with an enhanced coupling to the light field. This collective system is known as a Rydberg superatom.

In general, for an \( N \)-atom system, the laser excitation is described by the collective dipole operator
\[
\hat{d}_N = \hat{d} \otimes \hat{I}_2 \otimes \cdots \otimes \hat{I}_N + \cdots + \hat{I}_1 \otimes \cdots \otimes \hat{I}_{N-1} \otimes \hat{d},
\]
where \( \hat{d} \) is the single-atom dipole operator such that the single-atom Rabi frequency is given by
\[
\Omega_0 = -\langle \hat{d} | \hat{r} \rangle E_0 / \hbar,
\]
with \( E_0 \) being the electric field amplitude and \( |g\rangle \) and \( |r\rangle \) the ground and Rydberg states. If the ensemble is fully blockaded, the coupling between the ground state \( |G\rangle = |g \cdots g\rangle \) and the collective excited state
\[
|W\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |j\rangle,
\]
with \( |j\rangle = |g \cdots g \underbrace{r \cdots g} \cdots g\rangle \) the \( j \)th atom

\[
\langle G|\hat{d}_N|W\rangle = \sqrt{N}\langle g|\hat{d}|r\rangle
\]
where the \( \sqrt{N} \) collective enhancement becomes evident. Direct measurements of this enhancement have been carried out using well defined, fully blockaded ensembles of atoms prepared in an optical lattice[31] or in an array of optical micro-traps[51] and their coupling to an optical field was measured through Rabi oscillations. The results by Zeiher et. al. [31] are shown in Figure 8.

The collective enhancement can be pushed to an extent where a few photons are sufficient to drive Rabi oscillations. In the work presented in Reference 97 an ensemble of \( N \approx 10^4 \) atoms interacts with a pulse of 10 to 100 photons.
Arrays of well defined numbers of atoms $N$ are prepared in an optical lattice. These atoms couple collectively to the light enhancing the coupling by a factor of $\sqrt{N}$. Adapted from Reference 31.

The ability to have sizable interaction effects with such a small number of free-space (as opposed to cavity) photons allows observing the Rabi oscillations in the shape of the outgoing pulse as well as creating measurable correlations between the photons leaving the system as seen in Figure 9.

For larger cloud sizes, where multiple Rydberg excitations can be hosted, the collective coherent behavior breaks down \[30, 31, 51\]. In this case, different excitations can appear in the medium at random times and positions, without any phase relation between them whatsoever.

In the Hilbert space spanned by the set $\{|j\rangle\}_{j=1}^{N}$ there are other $N-1$ states which are orthogonal to $|W\rangle$. We can easily show that any one of these states are dark states, i.e. they are not coupled to the ground state by the light. If we consider any one of these states $|D\rangle = \sum_{j=1}^{N} \alpha_j |j\rangle$, by being orthogonal to $|W\rangle$ it fulfills

$$0 = \sqrt{N} \langle W|D \rangle = \sum_{j=1}^{N} \sum_{k=1}^{N} \langle j|k \rangle \alpha_k$$

$$= \sum_{j=1}^{N} \alpha_j.$$

The dipole coupling of the state $|D\rangle$ to the ground state is then

$$\langle G|\hat{d}_N|D \rangle = \sum_{j=1}^{N} \langle G|\hat{d}_N|j \rangle \alpha_j$$

$$= \sum_{j=1}^{N} \langle g|\hat{d}|r \rangle \alpha_j = 0,$$

since $\langle g|\hat{d}|r \rangle$ does not depend on $j$. Therefore, all $N-1$ states orthogonal to $|W\rangle$ are dark states, leaving only a two-level system consisting of the states $|G\rangle$ and $|W\rangle$. Nonetheless, the additional structure provided by the dark states results in additional degrees of freedom to realize systems fundamentally different than an individual atom [98]. One example of this is a single-photon absorber [66]. Here, inhomogeneous dephasing between the atoms, caused by spatially and temporally varying level shifts, can transfer the superatom state from $|W\rangle$ into the manifold of dark states. In this case, a photon absorbed as a Rydberg excitation will be transferred to a dark state where the light is prevented from driving it back to the ground state, while still blocking the medium from absorbing further photons.

**RYDBERG EIT**

Another powerful tool for interfacing and manipulating light down to the few-photon level results from realizing electromagnetically induced transparency (EIT) [99] while using a Rydberg state. EIT is a phenomenon that occurs when two long-lived states are coupled via an intermediate short-lived state using two optical fields: a strong coupling...
field and a weak \textit{probe} field. Various level configurations can be used to implement EIT as shown in Figure 10. In this manuscript we will focus on a ladder arrangement where the upper-most state is a Rydberg state.

For any EIT system, the transmission of the probe beam is modified by the control field. The optical response of the medium can be obtained by finding the steady-state solution to the master equation describing the time evolution of the system’s density matrix \cite{99}.

The polarization of the system provides the link between the internal state of the atoms and the optical response of the medium. Concerning the optical response of the medium to a weak probe ($\rho_{11} \approx 1$), the polarization density of the medium $P$ is given by

\[ P(\mathbf{\sigma} + \mathbf{E}) = \rho \langle \hat{d} \rangle = \rho \mu_{13} \rho_{13} = \epsilon_0 \chi E(\mathbf{\sigma} + \mathbf{E}), \]

where $\rho$ the atomic density, $\mu_{13} = \langle 1 | \hat{d} | 3 \rangle$ the dipole matrix element, $\epsilon_0$ the vacuum permittivity, $\chi$ is the susceptibility of the medium and $P^+(\mathbf{\sigma})$ and $E(\mathbf{\sigma})$ are the positive-rotating components of the polarization density and electric field respectively. Using these relations and the steady-state density matrix $\rho_{13}$, we can obtain the susceptibility as \cite{99}

\[ \chi = \frac{\mu^2 \rho}{\epsilon_0 \hbar} \times \left[ \frac{4\Delta(\Omega_c^2 - 4\Delta \delta) - 4\delta \gamma^2_3}{\Omega_c^2 + (\gamma_{31} + i2\delta)(\gamma_{21} + i2\Delta)} \right] + \frac{8\delta^2 \gamma_{31} + 2\gamma_{31} \left( \Omega_c^2 + \gamma_{31} - \gamma_{21} \right)}{\Omega_c^2 + (\gamma_{31} + i2\delta)(\gamma_{21} + i2\Delta)}, \tag{10} \]

where $\gamma_{ij}$ are the coherence decay rates between the states $i$ and $j$.

Having obtained the optical susceptibility of the medium, the index of refraction can be well approximated by $n \approx 1 + \chi/2$. In Figure 11 the imaginary and real parts of the index of refraction are shown for different control laser powers. In the imaginary (absorptive) part of the index of refraction it can be noted that as the control power is increased, a transmission feature appears at the two-photon resonance position ($\delta = 0$). However, what makes EIT most striking are the dispersive features observed in the real part of the refractive index. Here, the presence of a control field allows controlling the on-resonance slope of the index of refraction. Furthermore, as the control power becomes vanishingly small, the slope gets arbitrarily big. An interesting consequence is that the group index of refraction is given by

\[ n_g = n + \omega \frac{dn}{d\omega} \]

and thus, by making $dn/d\omega$ arbitrarily large, the group velocity inside the medium can be strongly reduced to the extent where a light pulse can be completely stopped inside the medium \cite{100, 101}.

\[ \text{FIGURE 9. Few-photon Rabi oscillations.} \] A square pulse (dashed gray) containing only a few impinges on a superatom. Rabi oscillations are observable in the Rydberg population (orange) and in the outgoing light (blue). Points average values measured and solid lines are produced with a theoretical model. Adapted from Paris-Mandoki et. al. \cite{97}.\]
FIGURE 10. Level schemes for Electromagnetically Induced Transparency. EIT had been initially implemented with a so called A-scheme (a) employing two hyperfine ground states and an excited state. However, due to their long lifetime it is also possible to implement it in a ladder arrangement (b) involving a Rydberg state.

EIT by itself provides a versatile mechanism to develop media with radically new optical properties. By adding Rydberg states into the mix [52, 53] it becomes possible to achieve nonlinear effects down to the few-photon level (see Figure 12).

As the slow light moves through the medium it travels as a so-called polariton [102]. These are superposition states of the electromagnetic field and atomic coherences. In the special case of Rydberg polaritons the interaction between the atoms results in an effective interaction between polaritons and ultimately between photons. Exactly on EIT resonance the interactions have a dissipative effect [52]. This means that when two polaritons are too close to each other, i.e. closer that the blockade radius, the EIT condition is broken and the polaritons are converted into scattered photons [55]. This process imposes a limit on the rate at which photons can be transmitted through the medium. A Rydberg EIT scheme was first realized by Pritchard et. al. [54] and later shown to result in nonlinearities at the few photon level by Peyronel et. al. [58].

An application of the strong nonlinearities realized with Rydberg EIT is the implementation of all-optical switches [62] and transistors [63–65], and interaction-induced π-phase shifts [67]. The working principle of the optical transistors is sketched in Figure 12. First a source optical field can be transmitted through the medium under EIT conditions. However, if a gate photon is first stored in the medium as a stationary Rydberg excitation it will cause the traveling source photon to crash into a blockaded volume. Thus, hundreds [65] of source photons can be blocked by storing a single gate photon.

The field of Rydberg quantum optics is undergoing a rapid expansion and several lines of research are opening up. An up to date review of the current state of the field can be found in Reference 52.

CONCLUSIONS & OUTLOOK

In these notes we have reviewed some basic properties of Rydberg atoms, their interaction and the blockade effect that arises when optically exciting them. We have discussed how the blockade effect can facilitate the creation of correlated atomic and optical quantum states. We have also introduced some concepts of electromagnetically induced transparency, in particular, when implemented involving a Rydberg level. Few-photon transistors are presented as an application of Rydberg EIT.

Experiments exploiting the Rydberg interacting are developing at rapid speed in many exciting directions. As one prime example, the technique used to arbitrarily position two micro-traps has been recently extended to create arbitrary two-dimensional micro-trap arrays [93]. Moreover, these types of arrays can be manipulated by means of movable optical tweezers to deterministically create atomic arrays without defects [41, 42]. Combining this bottom-up approach for creating arrangements of atoms with the strong Rydberg interaction opens up novel promising paths to create and study many-body quantum systems. We hope these notes motivate the reader to explore the topic of interacting Rydberg atoms in more detail.
FIGURE 11. Typical index of refraction of an EIT medium. Imaginary (left) and real (right) parts of the index of refraction under EIT conditions. As the control Rabi frequency $\Omega_c$ is increased the transmission window widens and the speed of light inside the medium increases.

FIGURE 12. Rydberg EIT. (top) Under EIT conditions probe light is transmitted through the medium. (middle) For higher photon rate, the Rydberg interaction breaks the EIT condition causing overlapping polaritons to scatter as photons out of the medium. (bottom) If a photon is stored as a stationary Rydberg excitation in the medium, this system can be used as an optical transistor.

ACKNOWLEDGMENTS

The authors would like to thank the organizers of the Latin American School of Physics “Marcos Moshinsky” 2017 for organizing a highly exciting summer school and for their generous hospitality. We thank Sebastian Weber, Christoph Tresp, Ivan Mirgorodskiy and Florian Christaller for discussions and comments on this manuscript.
REFERENCES

G. Günter, M. Robert-de-Saint-Vincent, H. Schempp, C. S. Hofmann, S. Whitlock, and M. Weidemüller, 

D. Maxwell, D. J. Szwer, D. Paredes-Barato, H. Busche, J. D. Pritchard, A. Gauguet, K. J. Weatherill, M. P. A. Jones, and C. S. Adams, 

W. Li, D. Viscor, S. Hofferberth, and I. Lesanovsky, 

Y.-M. Liu, X.-D. Tian, Y. Zhang, C.-L. Cui, and J.-H. Wu, 

M. Marinescu, H. R. Sadeghpour, and A. Dalgarno, 

C. E. Theodosiou, 

S. Weber, C. Tresp, H. Menke, A. Urvoy, O. Firstenberg, H. P. Büchler, and S. Hofferberth, 


A. Paris-Mandoki, H. Gorniaczyk, C. Tresp, I. Mirgorodskiy, and S. Hofferberth, 

R. Löw, H. Weimer, J. Nipper, J. B. Balewski, B. Butscher, H. P. Büchler, and T. Pfau, 

J. M. Raimond, M. Brune, and S. Haroche, 

H. Margenau, 

P. R. Fontana, 

D. Comparat and P. Pillet, 

M. Born and R. Oppenheimer, 

F. Nogrette, H. Labuhn, S. Ravets, D. Barredo, L. Béguin, A. Vernier, T. Lahaye, and A. Browaeys, 

S. Ravets, H. Labuhn, D. Barredo, T. Lahaye, and A. Browaeys, 


M. Saffman, 


J. Honer, R. Löw, H. Weimer, T. Pfau, and H. P. Büchler, 

M. Fleischhauer, A. Imamoglu, and J. P. Marangos, 

L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, 

C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, 

M. Fleischhauer and M. Lukin, 