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Bastien, Diane; Athienitis, Andreas

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Passive thermal energy storage, part 2: design methodology for solaria and greenhouses

Diane Bastien*, Andreas K. Athienitis

Dept. of Building, Civil and Environmental Engineering, Concordia University, 1455 Maisonneuve W., Montreal, Quebec, Canada H3G 1M8

Abstract

This paper presents a methodology for sizing passive thermal energy storage (TES) systems in solaria and greenhouses. Six different configurations are investigated, which encompass the most frequent cases. These configurations are studied with two complementary approaches: frequency response (FR) and finite difference thermal network (FD). FR models are used for sensitivity studies under short periodic design sequences while FD models are used in full-year performance assessments with real weather data. The most relevant performance variables for characterizing TES in solaria and greenhouses were identified in Part 1. In this paper, simulation results of these key variables are presented under varying conditions.

A methodology for sizing TES in solaria and greenhouses is presented along with design recommendations. The energy balance equations for six different configurations are included in order to make the methodology applicable to a variety of designs. The methodology is based on a FR model with a simulation design period of five cold sunny days followed by five cold cloudy days; this sequence is representative of the most extreme conditions in a year and thus provides a good basis for the comparative assessment of design improvements.

Keywords: sunspace, isolated gain, solaria, thermal mass, frequency domain, finite difference method

*Corresponding author +1 514 848-2424 x7080
Email address: solarbuildings@dianebastien.ca (Diane Bastien)
1. Introduction and overview

Most building types may benefit from having passive thermal energy storage (TES) systems with potential thermal comfort improvement (Berroug et al., 2011; Di Perna et al., 2011; Gagliano et al., 2014; Nemecek and Kalousek, 2015) and reduced heating and cooling energy requirements (Bojic and Loveday, 1997; Yang and Li, 2008; Aste et al., 2009; Williamson, 2011; Al-sanea et al., 2012; Csaky and Kalma, 2015). Different objectives may be pursued by incorporating TES in buildings; they are reviewed in Part 1 (Bastien and Athienitis, 2016), where the most relevant performance variables for solaria and greenhouses have been identified as follows:

- the absorbed solar radiation - storage temperature time lag \( (\tau_{Q_a-T_s}) \);
- the daily operative temperature swing;
- the average operative temperature;
- the minimum and maximum operative temperature;
- the space heating and cooling requirements.

The aim of this contribution is to present a methodology for assisting in the design of passive TES for solaria and greenhouses. Six general solarium/greenhouse designs are investigated in this study and presented in section 2, which encompass the most frequent configurations. These configurations are analyzed with two complementary numerical modelling methods: one based on a frequency response (FR) approach for the analysis of design sequences and one based on a thermal network model solved with the finite difference (FD) method for yearly analysis. Simulation results for the key performance variables identified in Part 1 are presented.

With the FR method presented in section 3, different design periods are examined along with their impact on the optimal thickness of thermal storage. The effect of varying floor area and aspect ratio, varying thermal resistance of the insulation layer, varying thermal storage material and different design periods on the main performance variables is investigated.

The FD method, which requires spatial discretization of the thermal mass, is detailed in section 4. First, results obtained with the FR model are used to assist in the determination of important parameters needed for the FD model, such as the number of nodes needed for the discretization of thermal mass. The effect of using constant, linear or non linear radiative and convective coefficients is assessed. Then, the FD method is using real weather data for analyzing the behavior of the investigated configurations for a complete
The amount of storage needed for reaching different design goals is investigated. Both heated and unheated spaces are analyzed. The performance is evaluated for two different years and Canadian cities, which allows the assessment of the sensitivity of the TES design to varying climatic conditions.

Finally, the proposed methodology for sizing TES, based on the FR model, is detailed in section 5 and design recommendations are presented in section 6.

2. Investigated solarium configurations

The six solarium configurations investigated in this paper are represented in figure 1. This research is focused on building-integrated passive TES (BITES); thus, only cases where a TES is fully covering a surface are considered. All configurations are oriented with their roof facing south. Their main characteristics are:

- F0: Fully glazed, thermal mass on the floor
- F1: Insulated north wall, thermal mass on the floor
- N1: Insulated north wall, thermal mass on the north wall
- N2: Insulated north, east and west walls, thermal mass on the north wall
- FN1: Insulated north wall, thermal mass on the floor and the north wall
- FN2: Insulated north, east and west walls, thermal mass on the floor and north wall

All configurations have an RSI 20 (R 114) insulation layer behind the storage mass, a 35° sloped roof, a 3 m high north wall, a 2.4 m width and 10 m length, except otherwise specified. When insulated, east and west walls have an outer layer with a thermal resistance of RSI 20 and gypsum boards as inner layer. The rationale behind the choice of such a large value for the thermal resistance layer is explained in section 3.2.4.
(a) Configuration F0 – TM on floor, all glazed

(b) Configuration F1 – TM on floor, opaque N, E/W glazed

(c) Configuration N1 – TM on N, E/W glazed

(d) Configuration N2 – TM on N, E/W opaque

(e) Configuration FN1 – TM on floor and N, E/W glazed

(f) Configuration FN2 – TM on floor and N, E/W opaque

Figure 1: Investigated solaria configurations – TM=Thermal mass, N=highest partition (north facing, on the left), E/W= East/West partitions.
3. Frequency response model

3.1. Model description

The first numerical method is based on fundamental network concepts used with Laplace transforms in the frequency domain. Here an analytical solution is provided: no spatial discretization is required, since the thermal mass is modelled as a two-port distributed element. The thermal network of a given configuration must first be defined, identifying all conductances and heat source elements. The admittance matrix is then defined based on this network, allowing to solve $Q = YT$ for the temperatures assuming 1D transient heat conduction. Frequency response (FR) modelling is a proven technique to design passive solar systems with weak nonlinearities where the system can be assumed to obey to the superposition principle.

![Thermal network](image)

(a) Thermal network

![Representation with a Norton equivalent](image)

(b) Representation with a Norton equivalent

Figure 2: Configuration F0

The thermal network of configuration F0 is shown in figure 2a. Here we have a very simple situation with only three main nodes: $T_{in}$ for the indoor air, $T_g$ for the glazing and $T_s$ for the storage mass on the floor. The floor
of area $A_s$ has a thermal mass layer of thickness $L$ and insulation with conductance $u_{gr}$ underneath. Infiltration and controlled ventilation exchanges are represented by $u_{vent}$ and the total conductance between the inner glazing $T_g$ and the outside air is given by $u_g$. With this notation, single or multiple layer glazings can be easily analyzed by providing the appropriate value for $u_g$ with

$$u_g = \frac{A_g}{1/U_G - 1/h_{c, glazing}}$$

where $A_g$ is the total glazed surface area (including frame), $U_G$ is the total $U$-value of the glazing material (including frame) in $W/(m^2K)$ and $h_{c, glazing}$ is the convective coefficient at the inner surface of the glazing. $U$ represents the conductance in $W/(m^2K)$ while $u$ represents the conductance multiplied by the surface area and is thus in $W/K$. Convection between the floor and the air is represented by $u_{si}$ and convection between the glazing and the air is represented by $u_{ig}$. Radiation exchanges between the floor and the glazing are represented by $u_{sg}$ and the solar radiation absorbed by the storage mass and the glazing is denoted with $S_s$ and $S_g$.

It is convenient to eliminate all exterior nodes and replace them with an equivalent source by building a Norton equivalent network, which consists of an equivalent heat source and a self admittance in parallel. In Figure 2b, the exterior node $T_o$ connected to $T_{in}$ has been transformed into $Q_{eq, vent}$, the ground node $T_{gr}$ has been transformed into $Q_{eq, gr}$ and the exterior node $T_o$ connected to $T_g$ has been transformed into $Q_{eq, g}$. For eliminating nodes connected to materials with negligible thermal mass, the equivalent source is simply equal to the conductance multiplied by the temperature of the node to be eliminated (e.g. $Q_{eq, vent} = u_{vent}T_o$). For nodes connected to massive materials, the equivalent source is equal to the wall transfer admittance multiplied by its temperature (e.g. $Q_{eq, gr} = -Y_tT_{gr}$, with a negative sign because of the convention used).

The energy balance of this system yields $YT = Q$ and is given in equation 2. The elements of the admittance matrix can by obtained by inspection: diagonal entries $Y_{ii}$ are equal to the sum of component admittances connected to node $i$; off diagonal entries $Y_{ij}$ are equal to the sum of component admittances connected between $i$ and $j$ multiplied by -1; heat source vector elements $Q_i$ are equal to the sum of the heat sources (actual and equivalent) connected at node $i$. By convention, $Q_i$ is positive if the heat source is directed to the node.
\[
\begin{pmatrix}
sC_a + u_{si} + u_{ig} + u_{\text{vent}} & -u_{si} & -u_{ig} \\
-u_{si} & Y_s + u_{si} + u_{sg} & -u_{sg} \\
-u_{ig} & -u_{sg} & sC_g + u_{ig} + u_{sg} + u_g
\end{pmatrix}
\begin{pmatrix}
T_i \\
T_s \\
T_g
\end{pmatrix}
= \begin{pmatrix}
Q_{\text{eq,vent}} \\
S_s + Q_{\text{eq,gr}} \\
S_g + Q_{\text{eq,g}}
\end{pmatrix}
\] (2)

\(sC_a\) and \(sC_g\) represents the lumped air and glazing capacitance; the capacitance of an element is given by \(C = \rho c_p V\), where \(\rho\) is the density, \(c_p\) is the specific heat capacity and \(V\) is the volume. The geometry, thermal network and admittance matrix of the five other investigated configurations are given in Appendix A.

The self and transfer admittances are complex numbers used to represent the response of a system to a specific frequency (in the form of a predetermined design period). They can be obtained by performing a Laplace transform on the 1D heat conduction equation

\[
\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\] (3)

and elaborating a Norton equivalent network. A cascade matrix for a multilayer wall can be defined by specifying boundary conditions adequate for a two-port model. For a structure with two layers (i.e. mass + insulation with negligible capacitance), the cascade matrix is given by

\[
\begin{pmatrix}
T_1 \\
Q_1
\end{pmatrix}
= \begin{pmatrix}
\cosh(\gamma_n L) & \frac{\cosh(\gamma_n L) + \sinh(\gamma_n L)}{u_{gr}} \\
\gamma_n \sinh(\gamma_n L) & \frac{\gamma_n \sinh(\gamma_n L) + \cosh(\gamma_n L)}{u_{gr}}
\end{pmatrix}
\begin{pmatrix}
T_2 \\
-Q_2
\end{pmatrix}
\] (4)

The self and transfer admittance have been defined in equations (1) and (3) in Part 1. From these definitions and the cascade matrix given above, the self and transfer admittance characterizing the floor in configuration F0 are given by

\[
Y_s = \frac{A_s (U_{gr} + \gamma_n \tanh(\gamma_n L))}{U_{gr} \tanh(\gamma_n L) + 1}
\] (5)

\[
Y_t = \frac{-A_s}{U_{gr} \cosh(\gamma_n L) + \frac{1}{\gamma_n} \sinh(\gamma_n L)}
\] (6)

where \(k\) is the storage mass thermal conductivity, \(\gamma_n\) is equal to \((s/\alpha_{th})^{1/2}\) with \(s\) being the Laplace transform variable and \(\alpha_{th}\) the storage mass thermal
diffusivity. For the frequency domain analysis conducted here, $s$ is equal to \( \omega j \) where \( \omega \) is the frequency and \( j = \sqrt{-1} \). Additional details about the procedure for elaborating the Norton equivalent network and deriving the cascade matrix for multilayered walls can be obtained in Athienitis et al. (1986) and in Athienitis and O’Brien (2015).

Since the solution is obtained in the frequency domain, all elements composing the admittance matrix must remain constant in time, including the radiative and convective coefficients. Since the convective and radiative coefficients of a surface depend on its temperature, they are determined in two iterations; estimated initial values are first provided and detailed coefficients are calculated at the second iteration based on the average temperature of the surfaces obtained from the first iteration. The convective coefficients are calculated using Khalifa and Marshall (1990) correlations. The radiative coefficient between two surfaces is calculated with (Duffie and Beckman, 2006, eq. 3.10.2)

\[
    h_{r,ij} = \frac{4\sigma T_m^3}{1 - \epsilon_i} \frac{1}{F_{ij}} + \frac{(1 - \epsilon_j)A_i}{\epsilon_j A_j}
\]  

(7)

where \( \sigma \) is the Stefan-Boltzmann constant, \( T_m \) is the mean temperature of surfaces \( i \) and \( j \) over the investigated time sequence, \( \epsilon \) represents the emissivity and \( F_{ij} \) is the view factor.

Elements composing the heat source vector must provide steady periodic conditions; therefore, the analysis with this model is focused on short design sequences. The temperature and solar radiation profiles of a specially chosen design period have to be defined for a specific location. The importance of the design sequence selection is discussed in section 3.2.5.

The temperature profile is based on a sinusoidal function with a maximum at 3pm. The average temperature is taken as the monthly average temperature for the month under consideration. The amplitude of the temperature profile is taken as the average monthly temperature variations divided by two.

The solar radiation incident on a surface for a sunny design day is computed using the Hottel clear sky method (Hottel, 1976) for the design day under consideration. A cloudy design day was constructed by setting the beam solar radiation to zero and multiplying the diffuse solar radiation by 1.5 from the Hottel model, which is representative of a completely overcast day. The solar radiation transmitted through the glazings of different ori-
presentations and absorbed by the interior surfaces is calculated separately with another routine that combines ray tracing and radiosity techniques. Detailed calculations are presented in Bastien et al. (2015).

Once the temperature profile and the solar radiation absorbed by the interior surfaces have been determined, complex Fourier series are required in order to calculate the time domain solution for these vectors. Five harmonics were selected for a 24 h design period. As reported in Athienitis et al. (1986), the magnitude of the self admittance $Y_{sn}$ increases with the harmonic number while the penetration depth $(\alpha_{th}/\omega)^{1/2}$ decreases. This is further exemplified by the shift in the peak $Y_{sn}$ magnitude, which occurs at a smaller thickness as the harmonic number increases (results not shown). However, by studying the effect of harmonics number on the storage temperature swings, Athienitis et al. (1986) reported that three to five harmonics are sufficient to ensure that the accuracy of the swings is about 0.1 °C.

Simulations with the FR model are executed with a time step of 0.05 h. A time step of 0.5 h would have been satisfying, but a smaller time step was selected in order to have a better resolution for the analysis of $\tau_{Q_s-T_s}$. Simulation results obtained with the FR model are presented in section 3.2 below. Unless indicated otherwise, simulations in this section are conducted for configuration N1 with dimensions of $h=3$ m, $w=2.4$ m, $l=10$ m equipped with a clear double insulated glazed unit with a concrete TES and no active air circulation for a periodic sunny day at the winter solstice for a latitude of 45°. The impacts resulting from varying these characteristics will be analyzed in the following subsections.

The analysis is focused on the main performance variables identified in Part 1. In the following discussion, the word optimal refers to the extremum of one of these variables.

3.2. Simulation results and discussion

3.2.1. Main results – all configurations

As shown on figure 3a, under the conditions described above, the minimum operative temperature becomes fairly stable for thicknesses beyond 0.20 m for all configurations. It is significantly affected by the solarium design, and positioning the mass on the wall instead of the floor (F1→N1) raises the minimum temperature by almost 2 °C.

The observation of figure 3b reveals that the maximum operative temperature is also strongly affected by the design. The maximum temperature
Figure 3: Main output performance variables – configurations F0, F1, N1 and N2
becomes relatively stable for thicknesses beyond 0.10 m for all configurations. The same observations also apply to the average temperature swing (see figure 3c).

The effect of the solarium design on the average temperature is very significant: having an opaque north wall instead of being glazed raises the average temperature by about 12 °C (F0 → F1). For the same thickness of thermal mass, locating the mass on the north wall instead of the floor will result in slightly higher average temperature (F1 → N1).

As defined in Part 1, $\tau_{[Q_a-T_s]}$ is the time lag between the peak absorbed solar radiation heat flux and the peak storage temperature. We can see in figure 3e that increasing the solar radiation collection of the space (by changing the north wall from being glazed to opaque, i.e. F0 → F1), can increase $\tau_{[Q_a-T_s]}$ by about 0.4 h. It peaks at 5-8 cm for all configurations, and becomes fairly constant for thicknesses greater than 22 cm.

Simulation results for configurations FN1 and FN2 are shown in figures 4 and 5. We can see from figure 4a that the highest minimum operative temperature is 9.2 °C and occurs for a thickness of 0.27 m for both the floor and wall storage; it remains above 9.0 °C as long as both TES are at least 0.2 m or larger. As seen on figure 4c, the minimum swing is 17 °C and occurs at a thickness of 0.18 m for both TES. It remains below 18°C as long as both TES have a minimum thickness of 0.12 m.

For FN1, the maximum wall $\tau_{[Q_a-T_s]}$ is 2.8 h and occurs when both TES are 5-6 cm thick. It is greater than 2.5 h when both TES are equal or less than 10 cm. The floor $\tau_{[Q_a-T_s]}$ is maximum at 3.3 h for a 5 cm wall and a 6-8 cm floor and is above 2.8 h when both TES are between 5 and 9 cm. Therefore, implementing a 6 cm storage on both the wall and the floor maximizes $\tau_{[Q_a-T_s]}$ for both TES.

Having storage on both the wall and the floor instead of just the wall (N1→FN1) can increase the highest minimum operative temperature by up to 7 °C. As seen in figure 5a, the highest minimum operative temperature is 11.8 °C and occurs for a thickness of 0.24-0.29 m for both TES. It remains above 11.6 °C as long as at both TES are at least 0.20 m. Having opaque east and west walls (FN1→FN2) does not significantly impact $\tau_{[Q_a-T_s]}$ for both TES.
Figure 4: Main output performance variables – configuration FN1

(a) Minimum operative temperature (°C)
(b) Maximum operative temperature (°C)
(c) Average operative temperature swing (°C)
(d) Average operative temperature (°C)
(e) Wall $\tau_{[Q_a-T_\text{s}]}$ (h)
(f) Floor $\tau_{[Q_a-T_\text{s}]}$ (h)
Figure 5: Main output performance variables – configuration FN2
3.2.2. Impact of varying floor area dimensions, aspect ratio and orientation

Although a detailed study of design parameters and their influence on the indoor climate is beyond the scope of this work, simulation results for configurations F0 and N1 with varying dimensions are presented below. The investigated dimensions are given in table 1.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>North wall height (m)</th>
<th>Width (m)</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F0-s</td>
<td>3</td>
<td>2.4</td>
<td>10</td>
</tr>
<tr>
<td>F0-m</td>
<td>4</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>F0-l</td>
<td>4</td>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>N1-s</td>
<td>3</td>
<td>2.4</td>
<td>4</td>
</tr>
<tr>
<td>N1-m</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>N1-l</td>
<td>3</td>
<td>2.4</td>
<td>10</td>
</tr>
</tbody>
</table>

As seen in figure 6, we can see that varying the floor area and aspect ratio of configurations F0 and N1 has little impact on the optimal thermal storage thickness of the main performance parameters. Varying the floor area of a fully glazed greenhouse (F0) has virtually no impact of the main performance parameters. Changes in the floor area and aspect ratio for a solarium with a massive north wall (N1) can have a small impact. From figure 6b, we can see that a solarium design aligned on a east-west axis with a higher aspect ratio experiences a higher average temperature, which is coherent with many studies that identified that it was most beneficial for greenhouses to have their longest side facing south (Harnett, 1975; Kozai, 1977; Rosa et al., 1989; Gupta and Chandra, 2002). Changing the orientation from due south reduces the absorbed solar radiation, and thus lower the average and peak operative temperatures and its swing (results not shown). It can be seen that even though the main performance parameters can be affected by variations in the floor area and aspect ratio, their response to varying thermal storage thickness is similar.
3.2.3. Impact of TES material

In this work, the reference thermal storage material is concrete as it is a widely used material that has been studied extensively, thus facilitating comparison with the existing literature. In addition, concrete is often present in buildings for structural reasons; therefore keeping it exposed and available for thermal storage could improve indoor climate with no additional cost. However, for thicknesses greater than 0.2-0.3 m, the cost and environmental impacts of this material are likely to hinder its use in bigger volumes, making the study of inexpensive and readily available materials a necessity for large TES. This section presents a comparison of the performance of concrete TES with soil and water TES.
The thermal properties of soil are highly dependent on the soil type (i.e. the proportion of sand, silt and clay) and moisture content. In particular, changes in its thermal conductivity can significantly alter heat transfer exchanges; typical values are between 0.6-2.5 W/m²-K, depending on the soil type and moisture content (ASHRAE, 2009, F25). Because of the variability of soil properties, two different types of soil are analyzed here: a sandy loam soil and clay loam soil. Properties representative of these two types of soil are indicated in Table 2.

Table 2: TES materials properties

<table>
<thead>
<tr>
<th></th>
<th>k</th>
<th>c_p</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>1.73 W/m²-K</td>
<td>840 J/kg-K</td>
<td>2243 kg/m³</td>
</tr>
<tr>
<td>Soil - sandy loam</td>
<td>0.8 W/m²-K</td>
<td>800 J/kg-K</td>
<td>1600 kg/m³</td>
</tr>
<tr>
<td>Soil - clay loam</td>
<td>1.2 W/m²-K</td>
<td>900 J/kg-K</td>
<td>1700 kg/m³</td>
</tr>
<tr>
<td>Water</td>
<td>0.59 W/m²-K</td>
<td>4813 J/kg-K</td>
<td>998.3 kg/m³</td>
</tr>
</tbody>
</table>

In the case of liquid TES such as water, convective heat transfer is present in addition to conduction. It is assumed here that the water temperature is uniform in a control volume. Since this is a simple case where the heat source is coming only from one side with no active heat exchangers, vertical temperature stratification due to buoyancy is expected to be small. The conductivity of water is replaced by an effective conductivity \( k_e = Nu k \)

where \( Nu \) is given by (Wright, 1996)

\[ Nu = 0.0674Ra^{1/3} \]  

(8)

For a fluid contained in a rectangular unit of width L with a temperature difference \( \Delta T \), the Rayleigh number is calculated with

\[ Ra = \frac{g \beta_{th} \Delta T L^3}{\nu \alpha_{th}} \]  

(9)

where \( \beta_{th} \) is the thermal expansion coefficient, \( \nu \) is the kinematic viscosity and \( \alpha_{th} \) is the thermal diffusivity of water. Here the effective conductivity is calculated at every thickness for a nominal temperature difference of 1 °C. Since the effective conductivity is proportional to \( \Delta T^{1/3} \), changes in \( k_e \) are
much smaller than changes in $\Delta T$; therefore the impacts of this simplification are not very significant.

As seen from figure 7a, a TES made of water experiences the smallest temperature swings from the four materials investigated. In addition, a clay loam soil is significantly more efficient than a sandy loam soil for reducing temperature swings. The average temperature is very similar for these four materials.

![Temperature swing graphs](image)

(a) Average $T_{op}$ and its swing  
(b) Minimum and maximum $T_{op}$

![Time lag graph](image)

(c) $\tau_{(Q_a-T_s)}$

Figure 7: Impact varying of storage material: --- sandy loam, --- concrete, --- clay loam, --- water

A TES made of water experiences reduced peak temperatures, followed by concrete and clay loam soil (see figure 7b). As shown on figure 7c, all investigated materials except water reach a constant time lag at 0.20 m and
beyond. Water has a significantly wider and higher peak, with a peak time lag decreasing much more slowly than for the other materials.

3.2.4. Impact of varying thermal resistance of the insulation layer

So far, all simulations have been conducted with an insulation layer having a thermal resistance of RSI 20 (R 114). Such a high value was chosen with the objective of providing near adiabatic conditions at the outside storage mass layer to focus our attention on the heat absorbed and released at the inside layer.

It is well known that increased thermal resistance reduces heat fluxes with diminishing returns. Exactly how much insulation should be selected depends on economic, space and performance constraints. The sensitivity of the five main performance variables to varying insulation levels was assessed for RSI 2, 5 and 20 insulation levels. The biggest impact was observed for the minimum and average operative temperature. The average temperature was 1.0 °C lower for all thicknesses with RSI 2 compared to RSI 20 and the minimum temperature dropped by 1.0 °C for thicknesses between 0.2-1 m. Variations of the maximum operative temperature and it’s swing were not significant and are therefore not presented.

The transfer admittance as a function of storage thickness is shown in figure 8b for the three different insulation levels. We can see that the transfer admittance approaches zero at storage thicknesses of 0.4 m and beyond for the three investigated insulation levels.

(a) Minimum and average operative temperature
(b) Magnitude of the transfer admittance

Figure 8: Impact of varying thermal resistance of the insulation layer; - - - RSI 5, - - - RSI 10, — RSI 20
Analyzing the transfer admittance in isolation could induce to conclude that there is no benefits to increase the insulation layer from RSI 2 to 5 or 20 for storage thicknesses beyond 0.4 m, but the observation of the minimum and average temperature reveals that the penalty for reduced insulation level is fairly constant for thicknesses between 0.1 and 1 m.

Although informative, it is difficult to identify design recommendations from the simulation results presented above. Therefore, the impact of varying the thermal resistance of the insulation layer will be investigated with the FD model under real weather conditions in the next section.

3.2.5. Impact of design sequence selection

Figure 9 shows the simulation results over different periodic design sequences ranging from a sunny day to seven sunny days followed by seven cloudy days. It can be seen that the optimal thickness in regards to the minimum, maximum and average operative temperatures is significantly affected by the choice of the design period.

As shown in figure 9a, the minimum temperature peak occurs at larger thicknesses with increasing consecutive cloudy days. Under a 7 sunny - 7 cloudy days design period, the minimum temperature for a TES thickness of 0.10 m is -5.4 °C while it reaches a minimum of -2.5 °C at 1.18 m. This is an important result that can justify the use of larger TES for applications where raising the minimum temperature is an important concern.

The maximum operative temperature exhibits a different behavior (see figure 9b). Under a sunny day and a 1 sunny - 1 cloudy days design periods, the extremum occurs between 0.13-0.15 m, while it occurs at thicknesses greater than 0.49 m for sequences with two consecutive cloudy days or more. As the number of consecutive cloudy days increases, two minima become visible: one around 0.10-0.15 m and a lower minima at a significantly greater thickness.

Figure 9c illustrates the swing of the daily average operative temperature for different design day periods. It can be seen that it is mainly independent of the design period. The minimum swing occurs between 0.17 and 0.23 m and then converges to a constant value at 0.40 m and beyond. The swing is 0.04-0.41 °C lower at the minima than the value for thicknesses greater than 0.40 m. At small thicknesses (< 0.10 m), the temperature swing is very sensitive to the TES thickness.

As seen from figure 9d, the maximum average temperature occurs at a greater thickness when the number of consecutive cloudy days increases, but
Figure 9: Impact of design sequence selection – configuration N1
the overall magnitude of the variations in average temperature with thickness is very small.

The impacts of solar radiation availability on $\tau_{Q_a-T_s}$ are shown in Figure 9e, where results for a sunny day and a cloudy day at the winter solstice are compared. Although the $\tau_{Q_a-T_s}$ is higher for a cloudy day, its peak occurs at the same thickness whether the design period is a cloudy day or a sunny day.

Figure 9f shows $\tau_{Q_a-T_s}$ for three different moments in the year. Here simulations were conducted under the periodic conditions of a sunny day at the winter solstice, the vernal equinox and the summer solstice. Since a solarium design cannot be optimized simultaneously for different times of the year, a designer should carefully select a day representative of the moment of the year when the time lag is most desirable. However, the choice of a specific day is not of crucial importance since the behavior of $\tau_{Q_a-T_s}$ does not exhibit very significant variations throughout the year. One could choose to optimize the time lag at the winter solstice, because then solar gains are at their lowest levels and therefore more needed.

### 3.2.6. Discussion

As we can see from the results presented so far, under periodic design sequences, there seems to be three different optimal TES thicknesses for fulfilling different objectives: the optimal thickness for reducing daily average temperature swings is between 0.15-0.25 m, the optimal thickness for maximizing $\tau_{Q_a-T_s}$ is around 0.05-0.10 m while reducing operative temperature peaks needs a greater thickness as the number of consecutive cloudy days increases. However, the magnitude of improvement of these three performance variables is relatively small; investigating how these will translate under real weather conditions is important and will be studied in the next section.

Although frequency response modelling can be used to analyze any periodic conditions, and it is possible to create period conditions from non-periodic ones by increasing the simulation period and repeating the conditions, this modelling method is more appropriate for short term analysis using design sequences. For accurate simulations, more harmonics are needed as the design period increases, which significantly impacts simulation time. For instance, for a one-day design period, 5 harmonics were sufficient, while 43 harmonics were necessary to analyze a 7 sunny - 7 cloudy days design period; this increased the computational time by a factor of 140. The use of a finite
difference (FD) model is more appropriate for yearly simulations with real weather data.

4. Finite difference model

Here the previously developed frequency response (FR) model will be compared with a model using the finite difference (FD) method. The FR model will be progressively modified towards a finite difference thermal network model that will consider non-linear radiative and convective heat transfer. The major modeling steps are presented in the first subsection where their impacts on accuracy and computational efficiency are discussed. The second subsection presents results obtained with the FD model.

4.1. Model parameters

All simulation results presented in this section are for configuration F0 subjected to a sunny day at the winter solstice unless specified otherwise.

4.1.1. Spatial discretization

The first modification introduced is the spatial discretization of the thermal mass. First, the accuracy of two different discretization schemes is analyzed. With the ”3Ne” model, half of the mass is located at the center and one quarter at the edges (see Figure 10). With the ”3Nc” model, 1/4 of the mass is in the center and 3/8 of the mass is located at 1/4 of the total thickness.

![Figure 10: Two spatial discretization schemes](image)

Figure 11a depicts the average temperature swings obtained with the FR model with distributed mass and with the ”3Ne” and ”3Nc” discretization schemes. It can be seen that models with thermal mass at the edges are more accurate and therefore adopted.
Figure 11b shows the impact of increasing the number of control volumes from three to seven. Spatial discretization with three control volumes shows good adequacy until a thickness of about 0.2 m, but then diverges from the FR model, with the difference getting bigger with increasing thickness. As can be seen, a greater number of control volumes is needed to maintain accuracy as the thickness increases.

(a) Impact of spatial discretization scheme

(b) Impact of increasing control volumes

Figure 11: Temperature swing as a function of TES thickness; \( T_s, \ldots, T_{op} \)

Here TES with thicknesses between 0.1 and 1 m are analyzed. Analyzing smaller thicknesses would necessitate a very small time step (see next section); thus the limit was set at 0.1 m. Although TES as thick as 5.5 m have been built (as seen in the literature review), 1 m is deemed sufficient to evaluate potential benefits of large TES in solaria and greenhouses.

As seen from figure 12, for the aforementioned conditions and for thicknesses up to 1 m, introducing a spatial discretization scheme with thermal mass at the edges and lumping the capacitance in 23 control volumes provides a good adequacy with the FR model with distributed mass. Therefore these parameters are adopted.

4.1.2. Temporal discretization

Developing a lumped parameter FD model requires introducing temporal discretization. The basic Euler forward method is adopted, which has a fully
Figure 12: Operative temperature swing as a function of TES thickness; — $T_{s}$, - - - $T_{op}$, + indicates extremum, △ indicates extremum ±0.15 and ♦ indicates extremum ±0.3

**explicit scheme:**

$$T_{i,t+1} = \frac{\Delta t}{C} \left( Q_{i,t} + h_{c,i} A_{i} (T_{air,t} - T_{i,t}) + \sum_{j} (T_{j,t} - T_{i,t}) R_{ij}^{-1} + A_{i} \sum_{k} h_{r,ik} (T_{k,t} - T_{i,t}) \right) + T_{i,t}$$

(10)

where $C$ is the capacitance of node $i$, $Q_{i,t}$ is a heat source at node $i$, $R_{ij}$ is the thermal resistance between nodes $i$ and $j$ (in K/W), $j$ represents nodes experiencing conductive exchanges with $i$, $k$ represents nodes experiencing radiative exchanges with $i$ and other symbols were previously defined. For comparison purposes, the convective and radiative coefficients, initial conditions and solar radiation and temperature profiles are retrieved from the FR model and used as input for the FD model in this subsection.

The Euler forward discretization scheme was adopted since it is easy to implement, computationally fast and convenient for computing nonlinear effects. Given the large number of control volumes required for accuracy, a time step of 0.002 h (7.2 s) is used to guarantee stability. Even with such a small time step, annual simulations can be carried out at a reasonable speed (about 40 minutes). The hourly weather data input are not interpolated to fit this time step.

The temperature swings obtained with the FR model, the FR model with discretized mass and the FD model are depicted in figure 13 for comparison. It can be seen for the FD model, a small discrepancy is introduced at a
thickness of 0.4 m and beyond. Reducing the time step below 0.002 h does not reduce the discrepancy. The maximum difference of 0.13 °C between the FR and the FD 23N models is considered acceptable for the scope of this study.

4.1.3. Sensitivity to radiative and convective coefficients

With the FR model, all parameters of the admittance matrix had to be kept constant during the simulation period, including the radiative and convective coefficients. With this model, the interior long wave radiation exchanges were calculated with a constant linearized radiative heat transfer coefficient. Here these simulations are compared with the FD model using the same linearized radiative coefficient but recalculated at every time step as a function of the temperature of surfaces. A third model is also investigated with non linear radiative coefficients calculated with the Gebhart method (Gebhart, 1959; Mottard and Fissore, 2007), where the net radiative flux emitted by a surface is calculated with the Gebhart coefficients.

Here the investigated configuration is F0 and the radiative coefficient under study is $h_{r,\text{floor-glazing}}$. The simulation results of these three radiation models are very close to each other where only minimal variations can be observed in figure 14a. Mean and extreme values of the radiative coefficient obtained with different calculation methods are presented in Table 3. By observing Figure 14a, it can be seen that there is very little difference when using a variable linear radiative coefficient comparing to a constant one, while the simulation time is 4.0-5.8 times longer. Therefore, using a variable
Figure 14: Impact of radiative coefficient calculation method; $T_s$, $T_{op}$

Table 3: Comparison of mean, minimum and maximum values of radiative coefficients

<table>
<thead>
<tr>
<th>Method</th>
<th>Minimum</th>
<th>Mean</th>
<th>Maximum</th>
<th>Computation time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W/(m^2K)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Sunny/1 Cloudy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant hr</td>
<td>3.97</td>
<td>3.97</td>
<td>3.97</td>
<td>6</td>
</tr>
<tr>
<td>Variable linear hr</td>
<td>3.78</td>
<td>3.96</td>
<td>4.47</td>
<td>24</td>
</tr>
<tr>
<td>Variable non linear hr</td>
<td>3.96</td>
<td>4.15</td>
<td>4.69</td>
<td>25</td>
</tr>
<tr>
<td>Hourly non linear hr</td>
<td>3.96</td>
<td>4.15</td>
<td>4.69</td>
<td>6</td>
</tr>
<tr>
<td>5 Sunny/5 Cloudy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant hr</td>
<td>3.97</td>
<td>3.97</td>
<td>3.97</td>
<td>52</td>
</tr>
<tr>
<td>Variable linear hr</td>
<td>3.75</td>
<td>3.96</td>
<td>4.49</td>
<td>299</td>
</tr>
<tr>
<td>Variable non linear hr</td>
<td>3.93</td>
<td>4.15</td>
<td>4.71</td>
<td>318</td>
</tr>
<tr>
<td>Hourly non linear hr</td>
<td>3.93</td>
<td>4.15</td>
<td>4.71</td>
<td>56</td>
</tr>
</tbody>
</table>

The linear radiative coefficient is of little interest; if short simulation time is a priority, a well estimated fixed coefficient may yield fast and relatively accurate simulation results for a short sequence, and if accuracy is more important, then non linear radiative coefficients should be used.

In all models, the radiative coefficients were calculated at every time step (unless constant). Since surfaces temperature is not changing significantly
for the order of magnitude of the selected time step (7.2 s), computation
time can be reduced significantly without altering accuracy by updating the
radiative coefficients hourly. Using non linear radiative coefficient updated
hourly yields very similar results (as shown in Figure 14b) with significantly
less simulation time (56 s instead of 318 s); therefore this modelling approach
is adopted for the subsequent simulations.

With the FR model, convective coefficients were calculated in two steps:
they were estimated for the first iteration and their value was calculated us-
ing the Khalifa and Marshall correlations at the second iteration, based on
the average temperature of the surfaces. With a FD model, the convective
coefficients may vary during the simulation sequence. The following para-
graphs presents an assessment of the impact of employing a fixed convective
coefficient compared to a variable one.

![Figure 15: Impact of convective coefficient calculation method; T_s, T_{op}](image)

Simulation results using constant and variable convective coefficients are
compared in Figure 15. There is very little differences induced by the use
of a variable convective coefficient compared to a fixed one and virtually no
differences at all if it is calculated at every time step or hourly.

Table 4 shows the minimum, mean and maximum value of the convective
coefficient of the floor and glazing under a 1 sunny - 1 cloudy day and 5
sunny - 5 cloudy days sequences. The coefficients can reach much lower
values when they are varying at every time step; however, since they reach
such low values when the surface and air temperature are very close to each
other, the impact on the magnitude of heat transfer is fairly low.
Since there is very little difference induced by updating the convective coefficients hourly instead of at every time step while the computation time is about 4 times faster, varying convective coefficients calculated hourly are adopted for the FD model.

Table 4: Comparison of mean, minimum and maximum values of convective coefficients.

<table>
<thead>
<tr>
<th>Method</th>
<th>Minimum</th>
<th>Mean</th>
<th>Maximum</th>
<th>Computation time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W/(m²K)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sunny/1 Cloudy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant $h_{c_{floor}}$</td>
<td>3.02</td>
<td>3.02</td>
<td>3.02</td>
<td>6</td>
</tr>
<tr>
<td>Variable $h_{c_{floor}}$</td>
<td>0.41</td>
<td>2.94</td>
<td>3.33</td>
<td>25</td>
</tr>
<tr>
<td>Hourly $h_{c_{floor}}$</td>
<td>1.77</td>
<td>2.95</td>
<td>3.33</td>
<td>6</td>
</tr>
<tr>
<td>Constant $h_{c_{glazing}}$</td>
<td>7.17</td>
<td>7.17</td>
<td>7.17</td>
<td>6</td>
</tr>
<tr>
<td>Variable $h_{c_{glazing}}$</td>
<td>4.48</td>
<td>7.13</td>
<td>7.42</td>
<td>25</td>
</tr>
<tr>
<td>Hourly $h_{c_{glazing}}$</td>
<td>6.41</td>
<td>7.15</td>
<td>7.42</td>
<td>6</td>
</tr>
<tr>
<td>Sunny/5 Cloudy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant $h_{c_{floor}}$</td>
<td>3.01</td>
<td>3.01</td>
<td>3.01</td>
<td>52</td>
</tr>
<tr>
<td>Variable $h_{c_{floor}}$</td>
<td>0.20</td>
<td>2.93</td>
<td>3.43</td>
<td>301</td>
</tr>
<tr>
<td>Hourly $h_{c_{floor}}$</td>
<td>1.59</td>
<td>2.93</td>
<td>3.43</td>
<td>58</td>
</tr>
<tr>
<td>Constant $h_{c_{glazing}}$</td>
<td>7.17</td>
<td>7.17</td>
<td>7.17</td>
<td>52</td>
</tr>
<tr>
<td>Variable $h_{c_{glazing}}$</td>
<td>3.54</td>
<td>7.11</td>
<td>7.49</td>
<td>301</td>
</tr>
<tr>
<td>Hourly $h_{c_{glazing}}$</td>
<td>5.33</td>
<td>7.11</td>
<td>7.49</td>
<td>58</td>
</tr>
</tbody>
</table>

Comparisons of the storage and operative temperature swings obtained with the FR and FD models with the parameters defined above (thermal storage divided in 23 control volumes, time step of 7.2 s, non linear $h_r$, variable $h_c$ with $h_r$ and $h_c$ updated hourly) for a sunny day design period are depicted in Figure 16 for configuration F0. We can see that the two models are in good agreement with each other. Other configurations showed a similar adequacy (results not shown).
4.2. Simulation results and discussion

Simulations are conducted for the severe 2003-2004 winter for Quebec city and for the relatively warm 2009-2010 winter for Montreal. Weather data were obtained from the SIMEB building energy simulation software website (Hydro-Québec, 2015). Figures 17a show the daily average outdoor temperature and solar radiation profiles over twelve months for the 2009-2010 year in Montreal.

Weather data for the year 2003-2004 in Quebec city are shown in figure 17b. We can see significantly lower temperatures, especially for the month of January with daily average temperatures between -19/-14 °C.

Simulations were run from April 1st to April 30th of the next year with the first month of April being used as a warm-up period. As with the FR model, the infiltration rate was constant at 0.2 ACH throughout the year. The year was divided into a cooling mode, a heating mode and a mixed mode where different ventilation rates were adopted depending on the mode. The cooling mode started in April until September, October and March were in mixed mode and the winter mode was during the months of November to February. The ventilation rules are reported in table 5.

Important simulation results for configuration N1 with Montreal weather data for the year 2009-2010 are presented in figure 18; more detailed monthly simulation results can be found in Bastien (2015). The monthly minimum operative temperature increases with thickness for all months of the year, in a small to moderate extent. The annual minimum temperature is depicted in figure 18b. Here we are looking at a weather sequence where there were
several consecutive cloudy days followed by a temperature drop, which occurred in January. With that sequence, raising the storage thickness from 0.1 m to 1 m increases the minimum temperature from -14.0 °C to -11.4 °C.
Table 5: Ventilation rates adopted during the heating, cooling and mixed modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>$T_i$ (°C)</th>
<th>Ventilation rate (ACH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heating</td>
<td>&gt;30</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>&lt;30</td>
<td>0</td>
</tr>
<tr>
<td>Cooling</td>
<td>&gt;24</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>20 &lt; $T_i$ &lt; 24</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>&lt;20</td>
<td>0</td>
</tr>
<tr>
<td>Mixed</td>
<td>&gt;24</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>20 &lt; $T_i$ &lt; 24</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>&lt;20</td>
<td>0</td>
</tr>
</tbody>
</table>

The annual maximum temperature is depicted in figure 18b, where we can see a moderate reduction of the maximum temperature until a thickness of 0.15 m and a marginal reduction beyond; regardless of the storage thickness, the maximum temperature is too high and additional measures for temperature control are needed.

The daily operative temperature swings averaged annually are shown in figure 18c, where a reduced swing is observed at 0.18-0.20 m of storage thickness. However, the penalty for having a larger thickness is about 0.2 °C and is therefore hardly significant.

The observation of figure 18a reveals that December experiences an increased average operative temperature with thickness while January is relatively constant and February experiences a reduction of its average temperature. This could be explained by the seasonal character of large TES. When the average outdoor temperature is on the fall, a thicker TES can slightly increase the indoor average temperature, while with increased solar radiation availability in February, a thicker storage would stock more heat and thus the average indoor temperature would take longer to increase.

Annual simulation results under the weather of the year 2003-2004 in Quebec city are presented in figure 19. As seen in figure 19b, at the coldest indoor conditions, which occurred in January, the minimum temperature at 0.1 m is -23.1 °C and is raised to -18.9 °C for a 1 m storage. An inflection
point with a changing slope of the minimum operative temperature is visible
at 0.2 m for the year 2009-2010 and at 0.3 m for the year 2003-2004.

Figure 19a reveals a similar behavior than in figure 18a: the average operative temperature for December increases with thickness, is mostly constant for January and is decreasing with thickness for February. This confirms that thick passive TES walls exhibit a seasonal behavior and are thus most advantageous in early winter than late winter for raising the average temperature.

The influence of heating was investigated and annual simulation results for configuration N1 with a heating set point of 5 °C are reported in figure 20. The minimum temperature is still increasing with thickness, but less markedly than in the unheated case and with an inflection point at 0.25 m. As shown in figure 20a, increasing the storage thickness from 0.25 m to 1 m increases the minimum temperature only by 0.1 °C. The heating requirements for keeping the minimum air temperature at 5 °C are shown in figure 20b. A significant reduction of the heating requirements with increasing storage thickness is observed between 0.10 to 0.20 m and a more moderate reduction is observed at greater thicknesses.

Simulations of a solarium with RSI 2 instead of RSI 20 behind the storage wall have been carried out and the most important results are reported here. The minimum operative temperature dropped from -14 °C to -15°C at a 0.1 m thickness and from -11.4 °C to -11.8 °C at a 1 m thickness. The monthly average operative temperature of the three coldest months was about 0.5 °C lower at all thicknesses. The maximum operative temperature and average daily swing were not significantly affected. These observations are similar to those derived with the FR model. Since there are significant impacts on some of the main performance variables even at thicknesses where the transfer admittance was almost zero, we conclude that the analysis of the transfer transmittance is of little practical use and suggest to focus the attention on the main performance variables identified in Part 1.

The annual minimum operative temperature is shown in figure 21 for configuration FN2. The minimum temperature is -11.5 °C when both TES have a thickness of 0.1 m. It reaches -8.7 °C for a 0.1 m floor storage and a 1 m the wall storage while it reaches -9.0 °C for a 0.1 m wall storage and a 1 m floor storage. This shows that locating a thick thermal storage on the north wall instead of the floor is slightly more efficient for raising the minimum temperature. The minimum temperature is raised to -6.3 °C when both storages are 1 m thick.
(a) Average $T_{op}$

(b) Annual minimum and maximum $T_{op}$

(c) Annual daily average $T_{op}$ swing

Figure 18: Montreal, year 2009-2010

Figure 19: Quebec, year 2003-2004
5. Methodology

The simulation results presented above can be used to provide insight when planning the design of solaria and greenhouses in cold climates. However, the design of a high performance solarium or greenhouse would benefit from carrying tailored simulations for analyzing potential design improvements. To this aim, we suggest to follow the methodology presented in this section.

Especially for a thick mass, a FR model is significantly easier to implement than a FD model because of the avoidance of spatial discretization and is thus selected for this methodology. Although annual simulations cannot be performed with this type of model, most design decisions can be made from an appropriate sequence of clear and cloudy days. Such a model requires a
constant admittance matrix and therefore constant radiative, convective and conductive values. As shown in section 4.1.3, the use of constant convective and radiative coefficients does not significantly impact the average daily operative temperature swings. Average convective and radiative heat transfer coefficients are presented in table 6 for the different configurations. The main drawback of FR models would be in the case where shading devices are used, where the varying thermal resistance of the glazing will surely impact the indoor climate. In this case, an FR model could still be employed, but the sensitivity to varying glazing thermal resistance should be assessed. To assist in the design of glazing and shading systems, a methodology for selecting high performance fenestration systems can be found in Bastien and Athienitis (2015) and a control strategy for improving the operation of shades is presented in Bastien et al. (2015).

Table 6: Average convective and radiative coefficients, [W/m²-K]

<table>
<thead>
<tr>
<th>Configuration</th>
<th>h_c,floor</th>
<th>h_c,glazing</th>
<th>h_r,floor–glazing</th>
</tr>
</thead>
<tbody>
<tr>
<td>F0</td>
<td>2.9</td>
<td>7.1</td>
<td>4.1</td>
</tr>
<tr>
<td>F1, N1, FN1</td>
<td>2.9</td>
<td>7.5</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>1.3</td>
<td>3.1</td>
</tr>
<tr>
<td>F2, FN2</td>
<td>2.8</td>
<td>7.7</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>2.8</td>
<td>1.1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

It is suggested to carry out simulations with a FR model over two design periods: over a period of a sunny day for analyzing τ[Q_a–T_s] and over a five sunny - five cloudy days period for analyzing the other main performance variables. The analysis of τ[Q_a–T_s] could be carried out at the winter solstice where solar gains are at their lowest levels or at another moment where the time lag effect would be the most desirable. We have seen that τ[Q_a–T_s] is not too significantly affected by the choice of the design day.

In order to provide conditions that will be representative of the most challenging weather conditions for the five sunny - five cloudy days design
period, it is recommended to model the solar radiation at the winter solstice with the Hottel model. Estimation of the average absorbed beam fractions, $f_x$, are provided for different conditions for avoiding tedious ray-tracing calculations; they are given in table 8 below for a latitude of 45° and in Bastien (2015) for a latitude of 55°. The roof tilt angle does not affect much the solar radiation distribution, so $f_x$ can be estimated to be identical to those showed in the tables even if the roof angle is different. Values can be interpolated for different width to north wall ratio, floor aspect ratio, orientation or latitude. For a glazed surface, the sum of its $f_x$ must be equal to 1. Then, the portion of a window area illuminating directly surface $i$ can be calculated as $f_{w,i} = f_{x,i}A_w$ where $A_w$ is the area of the window. Finally, the beam and diffuse radiation absorbed by an interior surface is calculated following the procedure defined in Bastien et al. (2015, section 2.1.2).

For the average outdoor temperature, it is recommended to select the minimum monthly air temperature over the last 22 years for the location under consideration; this information can be readily obtained from the NASA surface meteorology and solar energy web site (NASA, 2015). The average daily temperature range can also be obtained from the same source, where the latitude and longitude of the location of interest have to be provided. The parameters Air temperature at 10 m and Daily temperature range at 10 m have to be selected in order to visualize only the variables of interest.

The FR model should be used to explore design variations for improving the performance. As identified in this study, the parameters that most significantly impact the performance are the presence of a TES, its thickness and material, the glazing type and the aspect ratio of the space. The positioning of glazed and opaque surfaces also has a significant impact; however, it is very time consuming to analyze many variants with numerical simulations. It is therefore suggested to follow recommendations presented in table 7 to select an energy efficient design. When the positions of glazed and opaque surfaces have been selected, the energy balance equation for the desired configuration can be found in this work with which a FR model can be readily implemented in a programming software.

A high value for the thermal resistance of the insulation layers is strongly recommended during the first design exploration phase. When most of the design parameters have been decided, the impact of varying the thermal resistance should be investigated at last.

The main steps of the proposed methodology are presented in table 9...
Table 7: Design recommendations for increasing the average temperature in solaria and greenhouses

- A floor area with a high aspect ratio should be selected and oriented with the longest side facing south.
- South-facing surfaces should be fully glazed.
- North facing surfaces should be opaque, insulated and have a thermally massive inner layer.
- Opaque east and west walls increase the average temperature, but also reduce the solar radiation homogeneity on the floor compared to glazed east and west walls.
- A glazing with a high solar transmittance and thermal resistance should be selected.
- Shading devices should be installed and controlled efficiently.
- The floor should have a thermally massive inner layer with insulation underneath.
- The use of water as a TES material should be considered.
Table 8: Average absorbed beam radiation fraction at the winter solstice – $\lambda = 45^\circ$

<table>
<thead>
<tr>
<th>Floor aspect ratio</th>
<th>Indoor surface</th>
<th>Glazed surface</th>
<th>Glazed surface</th>
<th>Glazed surface</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>south</td>
<td>south</td>
<td>east</td>
</tr>
<tr>
<td></td>
<td></td>
<td>wall</td>
<td>roof</td>
<td>wall</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Longest side facing south</td>
<td>Longest side facing 30° west of south</td>
<td></td>
</tr>
<tr>
<td>Longest side facing south</td>
<td></td>
<td>4:1</td>
<td>floor</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>north wall</td>
<td>0.12</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>east and west walls</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>south wall and roof</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2:1</td>
<td>floor</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td></td>
<td>north wall</td>
<td>0.07</td>
<td>0.81</td>
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<td></td>
<td></td>
<td>east and west walls</td>
<td>0.20</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>south wall and roof</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td></td>
<td></td>
<td>1:1</td>
<td>floor</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>north wall</td>
<td>0.04</td>
<td>0.67</td>
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<tr>
<td></td>
<td></td>
<td>east and west walls</td>
<td>0.37</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>south wall and roof</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1:2</td>
<td>floor</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>north wall</td>
<td>0.00</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>east and west walls</td>
<td>0.51</td>
<td>0.50</td>
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<td></td>
<td></td>
<td>south wall and roof</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>roof angle of 35°, width=2×north wall height</td>
<td></td>
<td>4:1</td>
<td>floor</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>north wall</td>
<td>0.08</td>
<td>0.80</td>
</tr>
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<td>east and west walls</td>
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<td></td>
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<td>2:1</td>
<td>floor</td>
<td>0.78</td>
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<td>north wall</td>
<td>0.04</td>
<td>0.73</td>
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<td>east and west walls</td>
<td>0.18</td>
<td>0.16</td>
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<td>south wall and roof</td>
<td>0.00</td>
<td>0.00</td>
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<td></td>
<td></td>
<td>1:1</td>
<td>floor</td>
<td>0.68</td>
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<td>north wall</td>
<td>0.00</td>
<td>0.60</td>
</tr>
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<td></td>
<td></td>
<td>east and west walls</td>
<td>0.32</td>
<td>0.30</td>
</tr>
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<td>south wall and roof</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1:2</td>
<td>floor</td>
<td>0.54</td>
</tr>
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<td></td>
<td></td>
<td>north wall</td>
<td>0.00</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>east and west walls</td>
<td>0.46</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>south wall and roof</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table 9: Methodology for thermal mass design in solaria and greenhouses

<table>
<thead>
<tr>
<th>Step</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1. Determine the position of glazed and opaque surfaces.</td>
<td>Table 7</td>
</tr>
<tr>
<td>Step 2. Identify the corresponding configuration.</td>
<td>Section 2</td>
</tr>
<tr>
<td>Step 3. Find the heat balance equation associated to the chosen configuration.</td>
<td>Equation 2 and A.1-A.5</td>
</tr>
<tr>
<td>Step 4. Determine the absorbed solar radiation.</td>
<td>see Hottel (1976) or Duffie and Beckman (2006, section 2.8)</td>
</tr>
<tr>
<td>Step 4.1 Define the solar radiation incident on the glazed surfaces for a sunny and a cloudy day at the winter solstice with the Hottel model; for a cloudy day, ( I_{\text{beam}} = 0 ) and ( I_{\text{diffuse, cloudy}} = 1.5 \cdot I_{\text{diffuse, sunny}} )</td>
<td></td>
</tr>
<tr>
<td>Step 4.2 Calculate the solar radiation transmitted through the glazings and absorbed by interior surfaces.</td>
<td>see Bastien et al. (2015, sec. 2.1.2) with ( f_x ) from table 8</td>
</tr>
<tr>
<td>Step 5. Define the exterior temperature profile by setting ( T_o = T_{av} + dT_{av}/2 \cos(\omega t + 3\pi/4) ) where ( T_{av} = \text{Air temperature at 10 m} ), ( dT_{av} = \text{Daily temperature range at 10 m} ) and ( \omega = 2\pi/86400 ).</td>
<td>(NASA, 2015), where ( T_{av} ) is the coldest month of the 22-year minimum</td>
</tr>
<tr>
<td>Step 6. Determine the conductances in the admittance matrix.</td>
<td>Table 6</td>
</tr>
<tr>
<td>Step 6.1 Define the convective and radiative heat transfer coefficients.</td>
<td></td>
</tr>
<tr>
<td>Step 6.2 Set ( U_{\text{vent}} = ACHV \rho_{\text{air}}C_{\text{air}}/3600 ) where ACH is the infiltration + ventilation rate, in air changes per hour.</td>
<td></td>
</tr>
<tr>
<td>Step 6.3 Define ( U_{gr} = A_f/R_{gr} ) and ( U_o = A_s/R_o ) where ( R_{gr} = R_o = 20 , \text{m}^2\text{K}/\text{W} ) at first.</td>
<td>Equation 1 for ( U_g )</td>
</tr>
<tr>
<td>Step 6.4 Calculate ( U_g ) and ( U_w ) for the glazing and wall materials of interest, where ( U_w = A_w/(R_w + 1/h_o) ), ( R_w ) is the total wall thermal resistance and ( h_o = 20 , \text{W/m}^2\text{K} ).</td>
<td></td>
</tr>
<tr>
<td>Step 7. Build the equivalent sources.</td>
<td>Section 3.1 and Appendix A</td>
</tr>
<tr>
<td>Step 8. Represent the heat sources (real and equivalent) with complex Fourier series. Use ( N=5 ) for a sunny day and ( N=31 ) for a five sunny - five cloudy days sequence.</td>
<td>Section 3.1 and Appendix A</td>
</tr>
<tr>
<td>Step 9. Define the admittances and solve the heat balance equation identified in step 3 for the temperature vector.</td>
<td></td>
</tr>
<tr>
<td>Step 10. Run simulations for a sunny day and observe ( \tau[Q_o - T_s] ); then for a five sunny - five cloudy days design period and observe the other five performance variables.</td>
<td></td>
</tr>
<tr>
<td>Step 11. Use the model to explore design variation for improving performance. When most design variables are identified, explore varying the R-value of the insulation layers at last.</td>
<td></td>
</tr>
</tbody>
</table>
For exemplifying the use of this methodology, simulation results for a five sunny - five cloudy days period are presented in figure 22 for configuration N1 located in Montreal. For this location (45.5°N, 73.6°W), the minimum monthly air temperature is -13.7 °C and occurs for the month of January, which has an average daily temperature range of 6.5 °C. Here we can see that the minimum operative temperature is -17.5 °C at 0.01 m, -13.0 °C at 0.1 m, -10.2 °C at 0.5 m and -9.5 °C at 1 m. The average operative temperature is 0.2 °C for all thicknesses. The daily average swing drops significantly from 0.01 m to 0.1 m, reaches a minimum of 20.5 °C at 0.17 m and increases marginally for greater thicknesses. The maximum operative temperature drops significantly from 0.01 m to 0.1 m and reduces slightly for greater thicknesses. All these results are representative of those obtained with the FD model for the coldest month, presented in figure 18. Therefore, we can conclude that the five sunny - five cloudy design period with the conditions defined above can provide a good estimation of the behavior of a solarium during the harshest conditions in a year and thus provides a good basis for the comparative assessment of design improvements.

<table>
<thead>
<tr>
<th>Thickness (m)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top av</td>
<td>Top min</td>
</tr>
<tr>
<td>Top max</td>
<td>Daily av Top swing</td>
</tr>
<tr>
<td>0.01</td>
<td>-15</td>
</tr>
<tr>
<td>0.1</td>
<td>-10</td>
</tr>
<tr>
<td>0.2</td>
<td>-5</td>
</tr>
<tr>
<td>0.3</td>
<td>0</td>
</tr>
<tr>
<td>0.4</td>
<td>5</td>
</tr>
<tr>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>0.6</td>
<td>15</td>
</tr>
<tr>
<td>0.7</td>
<td>20</td>
</tr>
<tr>
<td>0.8</td>
<td>25</td>
</tr>
<tr>
<td>0.9</td>
<td>30</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
</tr>
</tbody>
</table>

Figure 22: Proposed methodology – example for configuration N1 located in Montreal

6. Summary, design recommendations and conclusions

The key design targets of solaria/greenhouses equipped with various passive TES systems have been analyzed with frequency response (FR) and finite difference thermal network (FD) models. Upon analysis of the simulation results obtained with these models, within the main performance parameters identified above, we can conclude that passive TES in solaria and greenhouses can most significantly impact the timing of the heat delivery (as captured by \( \tau_{[Q_a-T_s]} \)), the daily average operative temperature swing and the minimum
operative temperature. Therefore we suggest focusing on these three design
targets when designing TES in solaria and greenhouses.

As seen from figure 3c, the presence of a storage mass on the north wall
or the floor significantly reduces the daily operative temperature swing up
to a thickness about 0.10-0.20 m and becomes mostly constant beyond –
under the periodic conditions of a cold sunny day. This behavior is mostly
independent of the design period under consideration. Results obtained with
the FD model with real weather data confirmed this observation and revealed
an annual daily swing minimum around 0.20 m with minimal increase for
greater thickness. Therefore, it is recommended to adopt a TES with a
thickness of at least 0.15 m for reducing temperature swings.

Under periodic conditions, the minimum operative temperature is strongly
affected by the design period under consideration (see figure 9a). Simulations
with the FD model with real weather data revealed an ever increasing min-
imum temperature with storage thickness – at least up to 1 m. However,
the slope of the minimum operative temperature changes with an inflection
point at about 0.20-0.30 m; therefore the biggest contribution for raising
the minimum operative temperature are made up to that point, where the
minimum temperature keeps increasing beyond but less markedly. Result
obtained with the FD model revealed that increasing the thickness of TES
from 0.10 m to 1 m can raise the minimum temperature in unheated solaria
and greenhouses by 3 to 5 °C, depending on the configuration and weather
conditions. For raising the minimum operative temperature in heated solaria
and greenhouses, it is recommended to select a TES thickness of about 0.25
m. For unheated solaria and greenhouses, it is recommended to select a TES
with a minimum thickness of 0.3 m and even thicker if allowable by space
constraints. For a fixed thickness of thermal mass, it is recommended to
place about 0.10 m on the floor and the remaining on the north wall.

As seen in figure 3e, it is not possible to design the thickness of a pas-
sive TES system to provide a time lag appropriate for night cooling. It is
however possible to improve thermal comfort in the evening by selecting a
TES thickness with a high $\tau_{Q_a - T_s}$. If providing comfort during the evening
is an important design goal, it is recommended to implement a TES on the
north wall or the floor with a thickness between 0.05-0.10 m and to select a
configuration with opaque north, east and west walls. Implementing a 6 cm
TES on both the north wall and the floor maximizes $\tau_{Q_a - T_s}$ for both TES
and is thus the best option for improving evening thermal comfort.
If providing evening warmth and higher minimum temperatures are both important design goals, it is suggested to select a configuration with a massive floor and north wall and insulated east and west walls (i.e. configuration FN2) and to locate a 0.06-0.08 m TES on the floor and a TES on the north wall as thick as possible while meeting practical constraints. The use of water as TES material should be considered.

Under real weather conditions, the monthly average temperature exhibited a seasonal behavior where the average temperature of December increased with thickness, remained mostly constant in January and reduced with thickness in February. Thus we may conclude that large passive TES tend to increase the average temperature in early winter and to decrease it in late winter. With increased solar availability in February compared to December, this could be an acceptable drawback.

As seen in figure 9a, increasing the storage thickness to about 0.10 m significantly reduces the maximum operative temperature under a periodic sunny winter day. The analysis under real weather conditions revealed a further reduction until a thickness of about 0.20 m for most months and varying results beyond. Regardless of the TES thickness, additional measures should be implemented to prevent overheating such as ventilation and the use of shading devices.

A reduction of the thermal resistance of the insulation layer behind the storage mass from RSI 20 to RSI 2 was found to reduce the average operative temperature of winter months by about 0.5 °C independently of the TES thickness, while a greater reduction of the minimum temperature was observed for lower thicknesses. It is recommended to analyze the effects of varying the insulation level at the end of the design process, when most of the key design variables have been selected.

It was found the use of water as TES material instead of concrete or soil can significantly reduce the daily average operative temperature swing and peak temperatures as well as increase $\tau_{Q_a-T_s}$. Therefore, including water in solaria and greenhouses is recommended.

Although most simulation results presented here were obtained for a 24 m² solarium with specific dimensions, it was shown that spaces with different dimensions have similar optimal TES thicknesses for the main output variables; thus, results presented here can be valuable for designing solaria and greenhouses with different dimensions.

This paper presented a methodology for sizing passive TES in solaria and greenhouses that can be used for reaching different design objectives. Future
studies should focus on improving the efficiency of large TES in isolated-gain spaces, where the use of active ventilation and the development of efficient strategies for controlling the airflow is of particular interest.

Acknowledgments

Financial support from the ecoENERGY Innovation Initiative and NSERC/Hydro Quebec IRC is gratefully acknowledged. In addition, José Candanedo is warmly thanked for his involvement in fruitful discussions and for providing valuable recommendations about this work.

Appendix A.

Thermal networks and energy balance equations for configurations F1, N1, N2, FN1 and FN2

Figure A.23: Configuration F1 - Thermal network and energy balance equation

\[
\begin{pmatrix}
  sC_a + u_{si} + u_{ig} + u_{wi} + u_{vent} & -u_{si} & -u_{ig} & -u_{wi} \\
  -u_{si} & Y_{s,s} + u_{si} & -u_{sg} & -u_{ws} \\
  -u_{ig} & -u_{sg} & sC_g + u_{ig} + u_{sg} + u_{wg} & -u_{wg} \\
  -u_{wi} & -u_{ws} & -u_{wg} & sC_w + u_{wi} + u_{ws} + u_{wo}
\end{pmatrix}
\begin{pmatrix}
  T_{in} \\
  T_s \\
  T_g \\
  T_w
\end{pmatrix}
= \begin{pmatrix}
  Q_{eq,vent} \\
  S_s + Q_{eq,gr} \\
  S_g + Q_{eq,g} \\
  S_w + Q_{eq,wo}
\end{pmatrix}
\tag{A.1}
\]

with \( Q_{eq,wo} = u_{wo}T_o \), the other equivalent sources provided in section 3.1 and \( Y_s \) and \( Y_f \) given in equations 5 and 6.

\[
\begin{pmatrix}
  sC_u + u_{si} + u_{ig} + u_{fi} + u_{vent} & -u_{si} & -u_{ig} & -u_{fi} \\
  -u_{si} & Y_{s,s} + u_{si} + u_{sg} + u_{fs} & -u_{sg} & -u_{fs} \\
  -u_{ig} & -u_{sg} & sC_g + u_{ig} + u_{sg} + u_{fg} + u_f & -u_{fg} \\
  -u_{fi} & -u_{fs} & -u_{fg} & sC_f + u_{fi} + u_{fs} + u_{fg} + u_{gr}
\end{pmatrix}
\]
In equations A.2 and A.3, \( Q_{eq,o} = -Y_t T_o \), \( Q_{eq,gr} = u_{gr} T_{gr} \), \( Q_{eq,w} = u_w T_o \), the other equivalent sources are provided in section 3.1 and \( Y_s \) and \( Y_t \) are given in equations 5 and 6 where \( u_{gr} \) has to be replaced with \( u_o \).
Figure A.25: Configuration N2 - Thermal network and energy balance equation

\[
\begin{bmatrix}
 sC_u + u_{si} + u_{ig} + u_{fi} + u_{iw} + u_{vent} \\
 -u_{si} \\
 -u_{ig} \\
 -u_{fi} \\
 -u_{iw}
\end{bmatrix} \begin{bmatrix}
 T_{in} \\
 T_s \\
 T_g \\
 T_f \\
 T_w
\end{bmatrix} = \begin{bmatrix}
 Q_{eq,vent} \\
 S_s + Q_{eq,o} \\
 S_g + Q_{eq,g} \\
 S_f + Q_{eq,gr} \\
 S_w + Q_{eq,w}
\end{bmatrix}
\]

(A.3)
Figure A.26: Configuration FN1 - Thermal network and energy balance equation

\[
\begin{pmatrix}
  sC_u + u_{Swi} + u_{ig} \\
  -u_{Swi} \\
  -u_{ig} \\
  -u_{Sfi}
\end{pmatrix}
\begin{pmatrix}
  T_{ln} \\
  T_{Sw} \\
  T_g \\
  T_{Sf}
\end{pmatrix}
=
\begin{pmatrix}
  Q_{eq,vent} \\
  S_{Sw} + Q_{eq,o} \\
  S_g + Q_{eq,g} \\
  S_{Sf} + Q_{eq,gr}
\end{pmatrix}
\]
Figure A.27: Configuration FN2 - Thermal network and energy balance equation

\[
\begin{bmatrix}
 u_{C} + u_{Sw} + u_{ig} + u_{Sf} + u_{iw} + u_{vent} \\
 -u_{Sw} \\
 -u_{ig} \\
 -u_{Sf} \\
 -u_{iw}
\end{bmatrix}
\begin{bmatrix}
 T_i \\
 T_Sw \\
 T_g \\
 T_{Sf} \\
 T_w
\end{bmatrix}
= \begin{bmatrix}
 -u_{Sw} \\
 -u_{Sw} \\
 -u_{Sw} \\
 -u_{Sw} \\
 -u_{Sw}
\end{bmatrix}
\begin{bmatrix}
 Q_{eq,vent} \\
 S_{Sw} + Q_{eq,o} \\
 S_{g} + Q_{eq,g} \\
 S_{Sf} + Q_{eq,gr} \\
 S_{w} + Q_{eq,w}
\end{bmatrix}
\]

(A.5)

with \( Q_{eq,o} = -Y_{t,Sw}T_o \), \( Q_{eq,gr} = -Y_{t,Sf}T_{gr} \), \( Q_{eq,w} = u_wT_o \), the other equivalent sources are provided in section 3.1 and self and transfer admittances calculated with:

\[
Y_{s,Sw} = \frac{A_{Sw}}{u_o} \left( L_{Sw} + k_{Sw}\gamma_{Sw} \tanh(\gamma_{Sw}L_{Sw}) \right)
\]

\[
Y_{s,Sf} = \frac{A_{Sf}}{u_{gr}} \left( L_{Sf} + k_{Sf}\gamma_{Sf} \tanh(\gamma_{Sf}L_{Sf}) \right)
\]

\[
Y_{t,Sw} = -\frac{A_{Sw}}{u_o} \cosh(\gamma_{Sw}L_{Sw}) + \frac{1}{k_{Sw}\gamma_{Sw}} \sinh(\gamma_{Sw}L_{Sw})
\]

\[
Y_{t,Sf} = -\frac{A_{Sf}}{u_{gr}} \cosh(\gamma_{Sf}L_{Sf}) + \frac{1}{k_{Sf}\gamma_{Sf}} \sinh(\gamma_{Sf}L_{Sf})
\]
References


