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Highlights

- The 3D computerized facial approximation of the astronomer Tycho Brahe was performed
- Due to a poorly preserved facial skeleton missing data was statistically estimated
- Impact of the missing data estimation was evaluated visually from 10 test-subjects
- The procedure is an alternative to traditional methods of facial approximation

ABSTRACT: The virtual approach in physical and forensic anthropology is increasingly used to further analyze human remains, but also to propose new didactic means for visualization and dissemination of scientific results. Computerized facial approximation (FA) offers an alternative to manual methods, but usually requires a complete facial skeleton to allow for the estimation of the facial appearance of an individual. This paper presents the case of Tycho Brahe, Danish astronomer born during the XVIth century, whose remains were reanalyzed at the occasion of a short exhumation in 2010. Cranial remains of Brahe were poorly preserved, with only a partial facial skeleton, and virtual anthropology tools were used to estimate the missing parts of his skull. This 3D restoration was followed by a FA using TIVMI-AFA3D, subsequently textured with graphic tools. The result provided an interesting estimate that was compared with portraits of the astronomer. The impact of the missing data estimation was investigated by performing FAs on 10 complete test subjects and the same 10 subjects after cropping and estimating 50% of the landmarks (reproducing the preservation state of Tycho Brahe's cranial remains). The comparison between the FA based on the complete and incomplete skulls of the same subject produced a visual assessment of the estimation impact on FAs which is relatively low. This procedure is an alternative to manual methods and offers a reproducible estimate of a face based on incomplete cranial remains. Although the case report concerns a historical individual, the robust automatic estimation of missing landmarks followed by a FA has value for forensic caseworks as a support to the identification process.

Keywords: Facial reconstruction; Virtual anthropology; Missing data;Geometric Morphometrics; Computerized restoration

1. Introduction

The problem of missing data, from incomplete skulls, in facial approximation (FA) casework is relatively common. Using manual methods offers a certain freedom in reconstructing parts and interpreting the facial shape underlying missing areas, yet adequate methods to address the issue for computerized FA have not been previously presented. The aim of this paper is to provide an example for the handling of missing data using AFA3D and after presenting positive pilot validation results in a small test sample, apply the process to the historical case of Tycho Brahe.

1.1. Tycho Brahe – history and analysis of remains

Tycho Brahe (born Tyge Ottesen Brahe in 1546), was a Danish astronomer who spent the last three years of his life as Royal astronomer at the court of the *Holy Roman Emperor* Rudolf II in Prague; he died in 1601 at the age of 54 after a short illness lasting eleven days: these are the only historically assessed facts [1]. In 1566, when Brahe was twenty years old, he got into a dispute with his friend Manderuo Parsberg and lost a part of his nose in the subsequent fight. Since then and until his death, he wore a prosthetic nose allegedly made from silver and gold, which was attached to the rest of his nose by a glutinous substance [2]. He was also left with a somewhat larger scar on his forehead following the same duel. These injuries were not apparent on the facial skeletal remains, but greenish residues were found around the nasal aperture. Instrumental neutron activation analysis (INAA) of bone samples collected from around the anterior nasal spine revealed the presence of highly elevated levels copper and zinc in the approximate ratio 1:1, indicating that the presumed nose prosthesis was likely made of brass [3].

The remains of Tycho Brahe have been exhumed twice. His tomb in the *Church of Our Lady before Týn* (Prague) was first opened at the beginning of the 20th century (1901), in order to confirm the location of the astronomer´s remains. Doubts about their presence arose after historical reports about clearing the cathedral of non-Catholics remains following the Battle of White Mountain (1620) during the Catholic Reformation. Brahe was a Lutheran buried in a Utraquist church, which in 1621 came under the administration of the Catholic Church. The presence of the body of Tycho Brahe in the tomb was confirmed based on osteological research as well as the archaeological context [4-6]. The reason for the second exhumation, instigated by the archaeologist Jens Vellev in 2010 and directed by the National Heritage Institute in Prague [7], was mainly to verify the cause of death of the Danish astronomer in order to ascertain whether he was poisoned or not [8]. Several hypotheses had been formulated in relation to the astronomer´s death [9-11]. Speculations regarding his poisoning and violent death were predominantly based on the conclusions of physical and chemical analyses performed by Jan Pallon from Lund University [12] and Bent Kæmpe from the Institute of Forensic Medicine of Copenhagen University [10, 13], which allegedly demonstrated an increased content of mercury in the astronomer´s hair. Hence, samples from Tycho Brahe’s remains were collected during the second exhumation for chemical, physical, and biological analyses to investigate the cause of death. Additionally, the remains were scanned through medical computed tomography (CT) to allow further morphological analyses without tampering with the bones. As part of the identification efforts and as an opportunity to test new methods, the feasibility of a facial approximation was investigated by the research team. However, the poorly preserved facial skeleton did not allow for a straightforward approximation, so a special approach was adapted for this case.

1.2. Facial approximation and missing data estimation

According to the Scientific Working Group for Forensic Anthropology (SWGANTH), “The aims of facial approximation (sometimes referred to as facial reproduction, facial depiction, or facial reconstruction) are: (i) to estimate the antemortem facial appearance of an individual from unknown skeletal remains; (ii) to suggest the identity of persons to whom the remains might belong; and (iii) to capture public attention regarding the case” [14]. Although this definition applies to the forensic context where the recognition of an unknown individual is the goal of the process, the application of FA to historical cases should be bound by similar rules, as emphasized in a recent review by Hayes [15]. The museographic scope of FA allows for more flexibility and freedom in adding textures and details in the rendering of the face [16-19], but the scientific methodology to perform an objective facial shape is the key to any FA. Interestingly, the historical cases may include extensive information on the life history of the subject, including photographs or portraits that provide comparative elements. Therefore, in addition to providing a museum media for the public on the estimated facial appearance of known historical subjects, such FA gives an opportunity to evaluate the discrepancies of the methodology, and discuss its applicability and reliability for forensic cases [20]. Thorough scientific investigations are usually performed in historic cases, and may provide key elements to refine the resulting FA [21-24].

The recent development of computerized methods offers an alternative to the manual methods of FA [25], with more reproducibility, but also less flexibility (i.e. bound by the software's available options), although the difference in errors in both approaches has not been systematically tested. The advantage of software tools is the possibility to rapidly produce several FAs with various biological parameters (e.g. sex, age, body mass, hairstyle, etc.), and propose a range of possibilities during the post-processing phase. ReFace© is a good example of semi-automated computerized FA tools that opens new areas of investigation in the facial identification field [26]. Another computerized FA tool recently developed is the AFA3D (Anthropological Facial Approximation in 3D) module integrated in the free software TIVMI (Treatment and Increased Vision in Medical Imaging) [14, 27]. AFA3D requires a complete facial skeleton, more specifically, a full set of osseous landmarks (n=78) in order to calculate the most probable position of 100 soft tissue landmarks (from which a synthetic face will be warped to fit the target FA). A single missing landmark prevents AFA3D from estimating the facial shape.

The skull (including the cranium and the mandible) has a relatively good preservation in archaeological and forensic contexts, but cases with complete and undamaged craniofacial elements are rare [28]. Forensic anthropologists are used to dealing with fragmentary remains, whether they are affected by taphonomy or traumatic lesions. To perform a FA through AFA3D on incomplete remains, a full reconstruction of the skull is compulsory. However, the AFA3D method for quantifying the skull shape and estimating the facial shape is based on Geometric Morphometrics (GMM), which proposes a set of techniques to deal with missing coordinate data [29].

The different methods to estimate missing data in 2D or 3D have already been investigated, as well as the impact of this estimation on the results of a GMM analysis. Gunz and collaborators looked at three different methods to estimate missing data and validated the efficiency of two of them: the regression and thin plate spline (TPS) methods [30]. Couette and White tested multiple regressions and the algorithm expectation maximization on 3D data to quantify their consequences in the morphospace visualization [31]. Arbour and Brown went further on a 2D dataset, studying the impact of specimen incompleteness and sample size effects on four estimation methods [32]. These notable research projects overall confirm the applicability and limits of the estimation of missing data in GMM. However, the impact of estimating missing landmarks on FA has not been tested. Using a fully preserved facial skeleton, FA is by nature a rough estimate, and cannot represent a faithful reproduction of the individual, due to the morphological variations inherent to the human species [33]. The validation of AFA3D showed promising results in terms of standard error in the facial shape, but it is unclear to what extent it is possible to build an accurate FA based on estimated values [14]. A few missing landmarks should not alter heavily the result, but if an important amount of data is missing, performing a FA might not be pertinent. Although extensive testing should be required to investigate this aspect, only preliminary research is appended to this case report.

The goal of this paper is to propose a virtual restoration of the partial facial skeleton of Tycho Brahe, and perform a computerized FA of this famous Danish astronomer. The AFA3D module in TIVMI will be used, and the impact of missing data on FA will be evaluated for application in forensic casework.

2. Material and Methods

2.1. Material

The partial facial skeleton of Tycho Brahe was CT-scanned in Prague in November 2010 with a Siemens Somatom 16 at an optimal resolution (matrix 512*512, voxel size = 0.35 mm, voxel thickness = 0.75 mm, slice thickness = 0.4 mm) and reconstructed in DICOM with a hard convolution filter (to improve the rendering of the bony tissues). The 3D volume of the facial skeleton was computed using the HMH3D algorithm implemented in TIVMI [34], which allows for an accurate reconstruction with lowered measurement uncertainty [35]. The obtained mesh (Figure 1) was then cleaned from surrounding artefacts in MeshLab (v.1.3, Visual Computing Lab – ISTI – SNR, http://meshlab.sourceforge.net).

2.2. Missing data estimation

In order to create a pool of reference individuals covering a good morphological variability, 145 individuals (out of the 500 reference subjects of AFA3D) were selected. This sample includes 87 full skulls (for which the 78 landmarks are known), and an additional 58 subset of skulls for which at least 80% of the 78 landmarks were available. A first estimation of these few missing landmarks in the 58 subjects allowed creating a reference pool of 145 complete individuals, and ensuring a wide range of different cranial shapes. The estimation was performed with the R package *geomorph* [36] that proposes the TPS and regression methods in the *estimate.missing* function. The TPS method was used in this research, since it is more robust than regressions with this sample size [31].

The available landmarks on Tycho Brahe's facial skeleton were positioned in TIVMI (see [14] for methodology). Only 38 landmarks could be recorded (48% of the required face information); the remaining 40 missing landmarks were estimated with geomorph TPS method using the 145 abovementioned individuals as a reference. At this stage, the 78 resulting landmarks could be imported in AFA3D to perform a FA using the biological attributes male, older than 40 years, and (arbitrarily) with an overweight build, and with a global accuracy of 3 and a local accuracy of 4 (which defines the strength of the warping algorithm). To obtain a visual rendering of the reconstructed skull, a complete skull was necessary to be used as a basis for a warping using the *Morpho* package [37]. In order to limit the distortion, a Principal Component Analysis (PCA) was performed on the 87 reference individuals and Tycho Brahe's available points (using only the 38 landmarks on the complete individuals), and the closest subject (in terms of Procrustes distance) was preferred for the warping.

2.3. Facial approximation

The product of AFA3D is a synthetic face warped to fit the closest statistical shape appearance of the target individual [14]. The addition of textures is necessary to further individualize the face. A basic texture was created in FaceGen Modeller (v. 3.14, https://facegen.com/modeller.htm), and its corresponding UV map was applied to the FA in Blender (v. 2.78, Blender Foundation, https://www.blender.org). Additional details (texture, accessories), were manually added in 2D using Adobe Photoshop CS6 (v.13, Adobe Systems Incorporated).

To visually evaluate the impact of performing a FA with 52% of missing data, 10 full test subjects from the AFA3D reference pool were extracted and virtually truncated from the same missing landmarks on Tycho Brahe's facial skeleton. After the estimation of these missing data (as explained in 3.2), FAs were performed on both estimated and true skulls. The 10 subjects were excluded from the 145 reference database before the estimation process. The FAs of the estimated and true shapes are each computed with the same biological factors (sex, age, and body mass) to avoid bias, and observe only the impact of the estimation of missing data. Distance mapping between each couple of faces was automatically calculated in Avizo (v.7.0.0, Visualization Sciences Group, SAS) using the *SurfaceDistance* tool.

3. Results

3.1. Tycho Brahe's skull restoration

The 40 missing landmarks in Tycho Brahe's skull were estimated using the 145 reference individuals with the TPS method, and the missing bone surfaces were virtually created by warping a complete skull template on the 78 landmarks (40 estimated and 38 available landmarks). The result is displayed in Figure 2.

The vault shape is extrapolated from the complete template, and was not bound by any landmark in the warping (since this study focuses on 78 facial landmarks). Additional work on the model was required since Brahe's maxillary teeth are visibly worn and the teeth of the complete individual did not present any attrition. The teeth in the template model were thus virtually cropped to allow a proper occlusion of the mandible, avoiding an extreme overestimation of the facial height. The cropping was performed subjectively.

3.2. Facial approximation of Tycho Brahe

The facial approximation performed with AFA3D, the generic texture arbitrarily created in FaceGen Modeller, and the resulting face (after fitting the texture to the approximation) are shown in Figure 3.

3.3. Impact of missing data estimation on facial approximations

The impact of the missing landmarks estimation is calculated by the mean difference (in terms of surface distance) between the facial approximation of 10 test-subjects using their true landmarks, and the approximation of the same subjects after estimating the same 40 landmarks missing in Tycho Brahe's skull. The addition of the 58 subjects with at least 80% of available landmarks has a negligible impact (results not presented). The errors are mapped on a generic face after registration on all landmarks through partial Procrustes superimposition (Figure 4).

Errors in the FA after estimation of the missing landmarks located on the scalp and neck (superior to 3 mm) are not relevant since these regions of the model are not bound by any landmark in the process. The error in the facial region with estimated landmarks is almost as minimal as the error in the region with the known landmarks, the lips, ears and chin being the areas with a higher uncertainty. The further away one gets from the known landmarks, the greater the error seems to be. Overall, the shape of the face and the position of the different features are consistent between FAs before and after estimation of the missing landmarks. This result indicates that errors are more likely to occur in the approximation of Tycho Brahe in the lower face and ears region, but the overall rendering of the estimated shape can be validated.

3.4. Final rendering and comparison with portraits

Using the generic texture obtained with FaceGen Modeller, wrinkles, hair features and a copper-colored patch were added to the FA with Adobe Photoshop for a refined result (Figure 5).

The resulting FA is directly influenced by the multiple portraits of Tycho Brahe, recognizable by a specific hairstyle and the prosthetic nose. A relatively large number of portraits (approximately 90) of Brahe portrayed in refined Renaissance attiring have been produced [2]. Several watercolors demonstrate only little dissimilarities between them, and must have been based on a single model when Brahe was 52 [38]. According to the Czech historian Mikovec, the best portrait of the astronomer was painted by Tobias Gemperlin on the wall of the Uranienburg Museum [39]. Additional historical remarks about the authenticity of Brahe's portraits were published at the end of the 19th century in the Nature Magazine [40, 41]. The refined oil portraits by Tobias Gemperlin (Figure 6) depict Brahe in his forties with the same long flowing mustache and pointed beard [2]. Another credible portrait was painted by Michiel Jannsz van Mierveld, but his dating is not certain.

Fine texture details of the face cannot be estimated from the cranial morphology, but the shape of the facial features and their positions relative to one another are consistent with Brahe's iconography (e.g. height of the nose, eye width, and relative position of the mouth).

4. Discussion

The recent research trends in the FA community have focused on efforts towards the standardization of the methods, exploring the tools of virtual anthropology to improve the estimation accuracy and the rendering of the faces [14, 25-26]. However, very few initiatives have addressed the issue of missing data and FA. The protocol of AFA3D is robust compared to other methods, and provides an objective and repeatable synthetic face [14]. The experimental part of this study confirmed that the estimation of the position of missing landmarks has a minimal impact on the FA, although the preservation of a complete (or mostly complete) facial skeleton is still advised, especially for forensic cases. Since the further the estimate is from the known area, the greater the error, extensive missing parts in a skull should prevent from performing a FA. Additional testing of the missing landmarks' impact will be necessary, and should be performed on a bigger sample, in order to investigate both the quantitative changes of the estimation according to the missing area (e.g., superior *versus* inferior part of the face missing, sparse *versus* grouped missing landmarks, etc.), and the qualitative changes on the general appearance of the resulting FA.

The addition of textures is arbitrary in several cases, but the availability of portraits of Tycho Brahe guided the selection of features with FaceGen Modeller, which is a cost-efficient method (single license at $299) already applied to FA [42]. Any other computer-assisted 3D graphics solution may lead to similar results depending on the proficiency of the user.

Artistic depictions are known to be dependent from the artist's interpretation and style, and computerized FA proposes an objective method to estimate a realistic facial appearance, at least for the shape component [17]. The superimposition of Brahe's FA with his portraits was attempted but not presented in this study since it was judged too limited by numerous factors: differences in orientation and perspective effects between the two sources are difficult to control, and ultimately the artistic interpretation in the portraits makes such analysis debatable. Although this type of analysis has already been performed [23, 42], the distortion of the face in the portraits is impossible to evaluate. In a forensic context, facial superimposition can be made with photographs, and the calculation of the distortion is essential to ensure a good comparability [43]. However, even if these factors are controlled, facial superimposition is not considered as a positive identification technique, but a tool used for exclusion [44]. The superimposition of a skull or a FA with Renaissance portraits is moreover complex due to the artistic influence on the facial shape [45, 46]. Specific artists, whose technique and style are well documented, may have produced portraits using optical instruments allowing to infer to some extent the resulting distortion, but these cases are scarce [47]. Consequently, the distortion of the computerized FA could be modified (between an orthographic and a perspective view) along with its orientation, until it more or less fits the targeted portrait, without a strict objectivity in the comparison. Nevertheless, the proportions of Brahe's FA and his depictions on Gemperlin's portraits are relatively similar and do not exclude the identity.

The standardization of the virtual approach to FA in anthropology allows for new possibilities of treatment of human remains [48]. In the case of individuals of interest to history and heritage or forensic relevance, it becomes essential to have non-invasive techniques available for use. The CT acquisition of Tycho Brahe's facial skeleton was performed in a rapid manner (less than 1h use of the scanner for high definition acquisitions), and its virtual restoration and facial approximation could be performed without time restraints after the inhumation of his remains, avoiding ethical concerns [49]. The protocol described in this paper allows an objective virtual restoration of fragmented cranial remains that can be used for further anthropological analyses (e.g. craniometrics). The multi-disciplinary integration of previous observations with computerized techniques and anthropological knowledge carried out in this study allowed us to propose a plausible representation of the appearance of Tycho Brahe. Although the estimated missing parts provided an overall consistent face, the approximation of a face based on statistically inferred data may be debatable, especially for forensic applications. However, FA is not a positive identification method, but a help in the identification process. Disseminating a FA based on estimated data may still provide additional information leading to the positive identification of the unknown.

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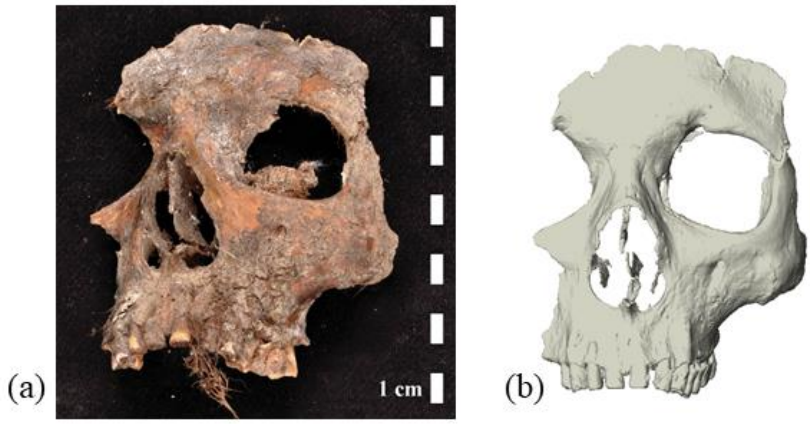

Figure 1 – (a) Tycho Brahe's preserved facial skeleton; (b) CT-based surface reconstruction.

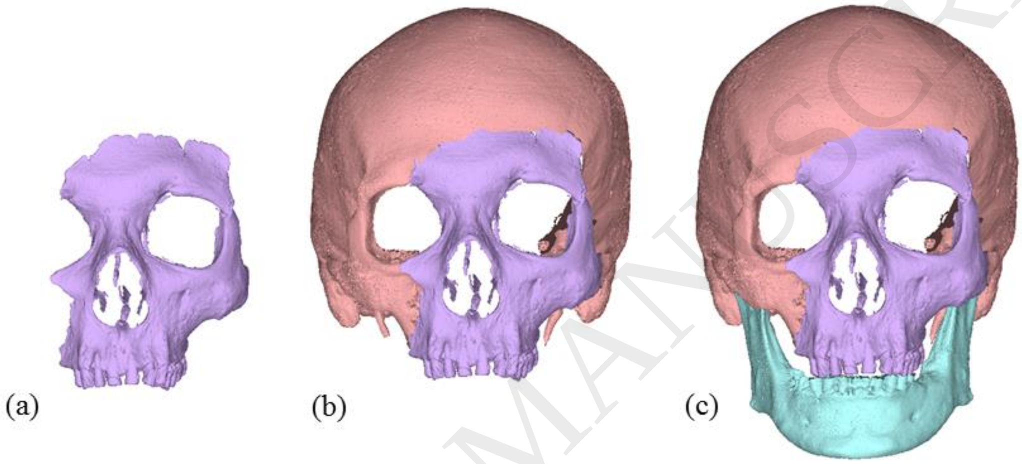

Figure 2 – Tycho Brahe's skull reconstructed after estimation of the missing facial landmarks, warping of a complete individual, and correction of the occlusion line. (a) preserved facial skeleton of Tycho Brahe; (b) fusion of a complete calvaria warped to fit the estimated missing landmarks; (c) addition of the mandible after cropping the teeth height to reproduce teeth attrition.

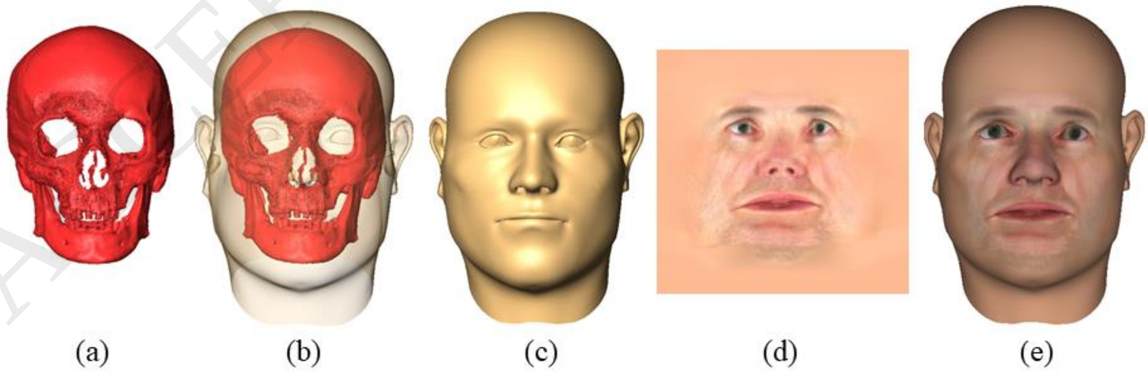

Figure 3 – (a) Virtual estimation of the skull of Tycho Brahe; (b) Facial approximation of Tycho Brahe superimposed with the restored skull; (c) Texture-free facial approximation of Tycho Brahe; (d) UV layout of the texture created in FaceGen Modeller; (e) 3D rendering of Tycho Brahe's face after texture application.

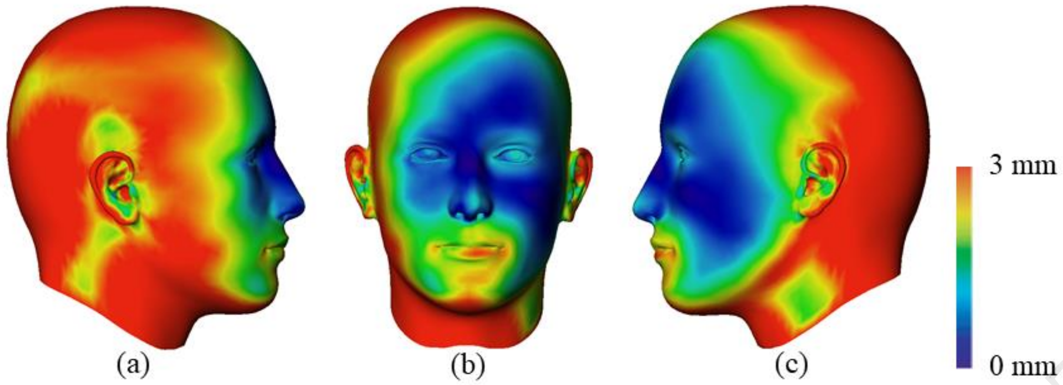

Figure 4 – Mapping of the impact of estimating 40 missing landmarks for a facial approximation (based on the average of 10 test subjects after). (a) right lateral view; (b) anterior view; (c) left lateral view.

Figure 5 – Artistic rendering of Tycho Brahe's virtual facial approximation including a partial nose prosthesis and using the facial hair type as depicted in historic portraits.

Figure 6 – Oil portraits of Tycho Brahe, (a) by Tobias Gemperlin, Karen Brahe Cloister, Odense, Denmark. (b) by Tobias Gemperlin, Royal Observatory, Edinburgh, Scotland. (c) by Michiel Jannsz van Mierveld , Royal Society, London, England.