Upper Limit for Context-based Crop Classification in Robotic Weeding Applications

Henrik Skov Midtiby\textsuperscript{a,}, Björn Åstrand\textsuperscript{b}, Ole Jørgensen\textsuperscript{c}, Rasmus Nyholm Jørgensen\textsuperscript{d}

\textsuperscript{a}The Maersk Mc-Kinney Moller Institute, University of Southern Denmark, 5230 Odense M, Denmark
\textsuperscript{b}Intelligent Systems Laboratory, Halmstad University, SE-301 18 Halmstad, Sweden
\textsuperscript{c}Operations Management, Aarhus University, DK-8830 Tjele, Denmark
\textsuperscript{d}Signal Processing, Aarhus University, DK-8200 Aarhus N, Denmark

Abstract

Knowledge of the precise position of crop plants is a prerequisite for effective mechanical weed control in robotic weeding application such as in crops like sugar beets which are sensitive to mechanical stress. Visual detection and recognition of crop plants based on their shapes has been described many times in the literature. In this paper the potential of using knowledge about the crop seed pattern is investigated based on simulated output from a perception system. The reliability of position-based crop plant detection is shown to depend on the weed density ($\rho$, measured in weed plants per square meter) and the crop plant pattern position uncertainty ($\sigma_x$ and $\sigma_y$, measured in meters along and perpendicular to the crop row, respectively). The recognition reliability can be described with the positive predictive value (PPV), which is limited by the seeding pattern uncertainty and the weed density according to the inequality: $\text{PPV} \leq (1 + 2\pi \rho \sigma_x \sigma_y)^{-1}$. This result matches computer simulations of two novel methods for position-based crop recognition as well as earlier reported field-based trials.

Keywords: crop recognition, row structure, weeding robots

*Corresponding author.
Email address: hemi@mmmi.sdu.dk (Henrik Skov Midtiby)

## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>m</td>
<td>Coordinate along $x$-axis (direction along the crop row), $x = 0$ is the expected crop location</td>
</tr>
<tr>
<td>$y$</td>
<td>m</td>
<td>Coordinate along $y$-axis (perpendicular to the crop row), $y = 0$ is the expected crop location</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>m</td>
<td>Crop position uncertainty along the $x$-axis</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>m</td>
<td>Crop position uncertainty along the $y$-axis</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>Scaling factor</td>
</tr>
<tr>
<td>$\rho$</td>
<td>m$^{-2}$</td>
<td>Weed density</td>
</tr>
<tr>
<td>$\lambda$, NWP</td>
<td>1</td>
<td>Normalised weed pressure</td>
</tr>
<tr>
<td>$p_{\sigma_c}(c)$</td>
<td>1</td>
<td>Position probability distribution of variable $c$</td>
</tr>
<tr>
<td>$n_w(x,y)$</td>
<td>1</td>
<td>Expected number of weeds closer to the seeding location than the point $(x,y)$</td>
</tr>
<tr>
<td>$\bar{x}_k$</td>
<td>m</td>
<td>Coordinates of the $k^{th}$ plant</td>
</tr>
<tr>
<td>$\bar{x}_{\text{offset}}$</td>
<td>m</td>
<td>Coordinates of the first crop plant in the row structure</td>
</tr>
<tr>
<td>$\vec{d}$</td>
<td>m</td>
<td>Vector from one crop position to the next expected crop position</td>
</tr>
<tr>
<td>$k, i, m$</td>
<td>1</td>
<td>Index variables</td>
</tr>
<tr>
<td>$l$</td>
<td>1</td>
<td>Number of occurrences in a Poisson distribution</td>
</tr>
<tr>
<td>$c_i$</td>
<td>1</td>
<td>Position score associated to the $i^{th}$ plant</td>
</tr>
<tr>
<td>$s$</td>
<td>1</td>
<td>Scaling factor</td>
</tr>
<tr>
<td>$N$</td>
<td>1</td>
<td>Number of neighbour positions to examine</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1</td>
<td>Probability of not seeing any plants within $3\sigma$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>Crop emergence</td>
</tr>
<tr>
<td>$f$</td>
<td>1</td>
<td>Fitting parameter for classifier performance</td>
</tr>
<tr>
<td>PPV</td>
<td>1</td>
<td>Positive prediction value</td>
</tr>
<tr>
<td>ePPV</td>
<td>1</td>
<td>Expected positive prediction value</td>
</tr>
<tr>
<td>oPPV</td>
<td>1</td>
<td>Observed positive prediction value</td>
</tr>
<tr>
<td>$n_{\text{crop}}$</td>
<td>1</td>
<td>Number of crop plants in dataset</td>
</tr>
<tr>
<td>$n_{\text{weed}}$</td>
<td>1</td>
<td>Number of weed plants in dataset</td>
</tr>
<tr>
<td>$n_{\text{total}}$</td>
<td>1</td>
<td>Total number of plants in dataset</td>
</tr>
<tr>
<td>$f(l,\beta)$</td>
<td>1</td>
<td>Probability of seeing $l$ events in a Poisson process with an average number of events of $\beta$</td>
</tr>
<tr>
<td>$x_n$</td>
<td>m</td>
<td>$n^{th}$ crop location</td>
</tr>
<tr>
<td>$n$</td>
<td>1</td>
<td>Crop plant number</td>
</tr>
</tbody>
</table>
Typical work flows in agriculture are often based on crop plants placed in row structures. Cereals like barley and wheat are placed in rows with no clear structure within the row, whilst maize, sugar beets and other high value crops are placed in rows with a clear defined intra-row spacing between crop plants, see Fig. 1. Given the position of a single sugar beet plant, it is possible to predict locations of nearby crop plants, based on information about plant distances within the row. With information about crop plant locations systems such as the Garfords Robocrop (Garford, 2011) and the Robovator by F. Poulsen Engineering (Frank Poulsen Engineering, 2014) can control weeds in the crop row using mechanical means. The capacity of both the current mechanical weeding robots is around 4 ha h$^{-1}$.

In robotic weeding applications plant recognition is often based on machine vision either using spectral properties or plant morphology/shape information (Slaughter et al., 2008). Various shape descriptors (compactness,
Hu moments, skeleton features, ... were used by Weis and Gerhards (2008) to map weed infestations. Giselsson et al. (2013) used shape features derived from distance maps to distinguish between two groups of seedlings. Active shape models were used by (Søgaard, 2005) to recognise three different weed species. Plant classification based on spectral properties (Zwiggelaar, 1998) and plant morphology (Weis and Sökefeld, 2010) are vulnerable to variations in plant appearance. There can be a large variation of plant appearance within a field, between fields and during growth season. Also weed pressures and populations vary. However, the sowing pattern is more stable. Therefore, it is interesting to use classifiers that utilise the position information to discriminate between crops and weeds.

Tillett (2001) used crop position information to distinguish between crop and weed plants in a field of *brassica*. The crops were transplanted to a square pattern with side lengths of 0.48m in three adjacent rows. It was stated that it is practical to track crop plants using extended Kalman filtering, but numbers of the achieved classification rate were given. Onyango and Marchant (2003) detected grid placement of cauliflower and used this information to distinguish between crop and weed pixels. The highest obtained correct crop and weed pixel classification rates were 96% and 92%.

The two earlier examples looked at plants placed in a 2D pattern, while Åstrand and Baerveldt (2004) used crop position information in a single row to classify crop and weed plants in sugar beet fields. In a field with a weed pressure of 50 plants m⁻², they correctly recognised 96% of the crop plants by searching for a pattern consisting of five plants placed in a row structure with the inter-plant distance set to the known crop-plant distance. In Åstrand (2005) position information was combined with individual plant features for recognising crop plants. In field conditions with low weed pressure (50 plants m⁻²) they achieve a positive predictive value (PPV) of 74% for recognising crops when only using plant position information. When the weed pressure is increased to 400 plants m⁻² the PPV decreases to 47%. In both cases the crop emergence were around 70%. This decrease is explained by increase of plant occlusion/overlapping to the effect that the row structure can be difficult to recognise when the number of weed plants is large. Crop plant localisation in single crop rows were also investigated by Bontsema et al. (1998) who used frequency filtering of the amount of vegetation in the crop row to locate individual crop plants.

Recent papers by Cordill and Grift (2011) and Chen et al. (2013) also relied on recognising crop plants by knowing the distance between adjacent
plants. Cordill and Grift (2011) used four laser beams to measure maize stalk placements, the measurements were then passed through two filters (based on stalk width and distance to last located maize plant) that recognised the crop plants. Chen et al. (2013) used a stereo camera setup to get images of maize plants at the two–three leaf stage. Plants with heights lower than a given threshold were then excluded and in the remaining plants they searched for plants with a fixed distance of 250 mm ± 25 mm.

The papers cited above show that plant position information can be used for recognising crop plants sown in a known pattern when using different perception systems. In this paper the upper limit of what can be achieved by using information about sowing geometry and plant positions is investigated. The system is not limited to vision–based perception systems as it can also use input from e.g. a lidar.

One measure of how good a system that recognises crop plants performs is the probability that a crop marked as a crop plant in fact is a crop plant, this value is denoted the positive predictive value (PPV). Theoretical considerations show that the PPV is bounded upwards by the expression $\frac{1}{1 + \lambda}$ where $\lambda$ is the normalised weed pressure (NWP) defined by $\lambda = 2\pi \rho \sigma_x \sigma_y$. In this value the weed pressure $\rho$ is normalised with respect to the crop plant position uncertainty along $\sigma_x$ and across $\sigma_y$ the crop row.

Four different strategies (position scores, path scores, random plant selector and known seeding positions) for localising crop plants based on plant locations were implemented and tested in a simulated environment and the results compared with theoretical considerations. The results from real field data originally presented in Åstrand (2005) compared with the derived theoretical upper bound.

2. Theory

2.1. Normalised weed pressure

The normalised weed pressure is the average number of weed plants closer to a seeding point than the nearest crop plant. The crop plant position probability was modelled as a Gaussian distribution with centre at (0, 0) and the uncertainties $\sigma_x$, $\sigma_y$ in the $x$ and $y$ directions, were respectively $(c \in \{x, y\})$.

$$p_{\sigma_c}(c) = \frac{1}{\sigma_c \sqrt{2\pi}} \exp \left( \frac{-c^2}{2\sigma_c^2} \right)$$  \hspace{1cm} (1)
Fig. 2: Given a crop plant at \((x, y)\), the area that should be weed free for obtaining a correct classification is marked with a grey shading. All points in the shaded area have a smaller Mahanolobis distance to the expected crop location than the observed crop at \((x, y)\). The area of the shaded region is \(\pi \sigma_x \sigma_y \left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)\). Origin is the expected crop plant position.

The position was described in terms of the distance along the crop row \((x-) coordinate) and the distance perpendicular to the crop row \((y-) coordinate). The next step is to determine the number of weed plants closer to the origin than the crop–point \((x, y)\) using the Mahanolobis distance metric. The number of weed plants is the weed density \(\rho\) multiplied with the area of the ellipse going through \((x, y)\) with semi–major axis along the \(x\)–axis and semi–major and minor axes proportional to \(\sigma_x\) and \(\sigma_y\). (Fig. 2). The boundary between the shaded and non–shaded area in Fig. 2 is a contour line of the Mahalanobis distance from a point centred on \((0, 0)\) with uncertainties \(\sigma_x\) and \(\sigma_y\) in the \(x\) and \(y\) directions, respectively. The \(\alpha\) value is a scaling factor which applies both to \(\sigma_x\) and \(\sigma_y\) when the boundary should be located, therefore the scaling factor should be the same in both directions. The expected number of weeds is then given by

\[
n_w(x, y) = \rho \pi \sigma_x \sigma_y \left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)
\]

The average number of weeds closer to the seeding point than the nearest crop plant can then be expressed with the following double integral
\[ \lambda = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{\sigma_x}(x) \cdot p_{\sigma_y}(y) \cdot n_w(x, y) \, dx \, dy \]  

(3)

The value of the double integral is \( \lambda = 2\pi \rho \sigma_x \sigma_y \) (see derivation in Appendix A.2), this value is denoted the normalised weed pressure (NWP).

2.2. Positive predictive value given normalized weed pressure

If a classifier based on the plant positions chooses the plant nearest to the estimated grid position as crop, it is interesting to look at the probability of misclassification, which happens when there is a weed plant closer to the grid location than the nearest crop plant. To calculate this probability it is assumed that weeds are uniformly distributed and the number of weed plants within an area can be modelled with a Poisson distribution

\[ f(l; \beta) = \frac{\beta^l \, e^{-\beta}}{l!} \]  

(4)

with \( l \) as the number of observed weed plants and \( \beta \) the average number of weeds seen in an area of this size, calculated as weed density \( \rho \) times the size of the area. The case where no weed plants are observed in the area corresponds to \( l = 0 \), in which the probability is given by \( \exp(-\beta) \). With a crop plant at \( (x, y) \) the probability of not seeing a weed plant closer to the grid point is \( \exp(-n_w(x, y)) \). By averaging this probability over all possible values of \( x \) and \( y \) while weighting with the probability of seeing a crop plant at these locations, the following integral appears

\[ \text{PPV} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{\sigma_x}(x) \cdot p_{\sigma_y}(y) \cdot \exp(-n_w(x, y)) \, dx \, dy \]  

(5)

The value of the double integral is \( \text{PPV} = \frac{1}{1+2\pi \rho \sigma_x \sigma_y} \) (see derivation in Appendix A.3). This value can also be expressed in terms of the normalised weed pressure, then it is \( \text{PPV} = \frac{1}{1+\lambda} \). This is an upper bound on the achievable positive predictive value that can be reached using position information alone for recognising crop plants. The upper bound can be reached when the crop sowing positions are known, but this information will usually be determined by inspection of the nearby row structure. This will increase errors to the expected crop location which will reduce the PPV.
2.3. Effect of crop emergence

In practice crop emergence is not complete. Therefore the model (PPV) needs to be adapted to handle that. The main difference between the obtained theoretical predictions and the observed classifier performance described in Astrand (2005) was that the crop emergence in the experiments were less than assumed in the derivations (100%). The article stated crop emergence $\gamma$ together with number of crops found (TP) and number of weeds misclassified as crops (FP). If the used algorithm cannot find a plant near a predicted grid location (within $3\sigma$, $\alpha = 3$), no plants are classified as crop for that location. The probability of not finding a plant within $3\sigma$ is given by

$$\phi = (1 - \gamma) \cdot \exp(-\lambda \cdot \alpha^2)$$

(6)

where $1 - \gamma$ is the probability of not having a crop plant at the grid location and $\exp(-\lambda \cdot \alpha^2)$ is the probability of not seeing any weeds within $3\sigma$ from the grid location.

If a crop plant is present at the investigated grid location, the expected PPV is given by $\frac{1}{1+\lambda}$. If no plants are within $3\sigma$, no action is taken and the observed PPV is not affected. The expected PPV value in a field with crop emergence $\gamma$ is then

$$ePPV = \frac{1}{1+\lambda} \cdot \frac{\gamma}{1 - \phi}$$

(7)

where $(1 - \phi)$ is the proportion of grid locations where at least a single plant is within $3\sigma$ from the location.

3. Materials and methods

In this section an experimental model for evaluating context–based crop recognition is described. The model was based on simulating plant positions of both crop and weed plants, in an artificial field. A list of all the plant positions was then used as input to a context–based plant recognizer, which localised crop plants based on the known sowing geometry. This process is shown in Fig. 3. A measure of the ability of context–based recognition to locate crop plants is the positive predictive value (PPV). It was found that the PPV was bound by the expression $\frac{1}{1+\lambda}$, where $\lambda$ is the normalised weed pressure and it had the value $\lambda = 2\pi \rho \sigma_x \sigma_y$. 

January 25, 2016
3.1. Test suite

For evaluation of the implemented crop recognition methods, a simulation environment was implemented. Given a set of parameters (weed density and crop position uncertainty), the simulation environment produced a set of crop plant positions and a set of weed plant positions. The generation of these position sets is described in 3.1.1. The crop plant positions were used as ground-truth values in the evaluation. These two sets of positions were combined to one set which was then handed over to the method that should be tested. The performance of the method was then evaluated and a resulting PPV value was obtained. The evaluation procedure is described in 3.1.2.

3.1.1. Generation of plant positions

The $n^{th}$ grid location had the position

$$\vec{x}_n = \vec{x}_{\text{first}} + n \cdot \vec{d}$$

where $\vec{x}_{\text{first}}$ is the position of the first plant and $\vec{d}$ is the distance from one crop plant to the adjacent crop plant. For each grid location a crop plant was placed on that point and adjusted with $x$ and $y$ displacements drawn from normal distributions with zero mean and $\sigma_x$ and $\sigma_y$ standard deviations.
Weed plant positions were drawn from a uniform distribution of the simulated area. The number of weed plants, $n_{\text{weed}}$, was adjusted to match the desired weed pressure. Crop and weed plant positions were then placed in a list and sorted by their $x$-coordinates.

### 3.1.2. Interpretation of results

The tested method then generated a list of the plants recognised as crops and combined with the knowledge of the true crop plant positions a confusion matrix was built. To avoid boundary effects on the result, the simulated field was split into three parts: beginning, middle and end. The parts at the beginning and end of the simulated field were defined as the space required for 20 adjacent crop plants. Only plants in the middle part contributed to the confusion matrix. The confusion matrix kept track of the number of true positives TP (crops classified as crops), false positives FP (weeds classified as crops), false negatives FN (crops classified as weeds) and true negatives TN (weeds classified as weeds). PPV was then calculated as

$$PPV = \frac{TP}{TP + FP}$$

### 3.2. Crop localisation based on plant locations

The following subsection shows the different methods used for recognising crop plants given plant positions as input. Two novel context-based methods are described, *Position score* and *Path score*, along with two reference methods for establishing upper and lower bounds on classifier performance. All methods took a list of all observed plant positions ($\vec{x}_k$, $k \in 1 \ldots n_{\text{total}}$) as input together with the number of crop plants $n_{\text{crop}}$ present in the dataset and values describing the sowing pattern (distance between crop plants and position uncertainty).

#### 3.2.1. Position scores

In this method a position score was calculated for all of the observed plant positions. Given a plant location, the position score depends on location of nearby plants, this is shown in Fig. 4. If the nearby plants follow the crop plant structure, the calculated score is high. The score function investigates the $N$ adjacent expected crop locations and for each of them it finds the plant closest to this position (Euclidean distance). If there are plants nearby
Fig. 4: Two examples of how position scores are calculated, when looking at five neighbour sites \((N = 5)\). Grey circles represent plant locations, the black scale marks expected crop plant locations and the red lines are the distance from an expected crop plant location to the nearest observed plant. The used plant positions are shown in a), green circles represent crop plants and red circles weed plants. In b) the position score of a plant, which belong to the crop–row structure, is visualised. c) is similar to b) but now with a plant outside the crop–row structure.

the expected crop locations, the score for that position is high, otherwise it is reduced.

\[
c_i = \sum_{m=1}^{N} \left[ \max_k \exp \left( \frac{-||\vec{x}_k - \vec{x}_i - m \cdot \vec{d}||^2}{2s^2\sigma_x\sigma_y} \right) \right]^{10}
\]

The scaling factor \(s\) was set to 5, tests showed that values outside the interval \([2, 8]\) reduced the performance of the method, inside the interval the performance was unaltered. After calculating the position score for all observed plant locations the \(n_{\text{crop}}\) plant positions with the highest score were marked as being crop plants.

3.2.2. Path scores

This method calculates plant position scores by looking at the position score of neighbouring plants. All plant position scores are divided in two parts, a base value of 1 and a contribution of a percentage of a neighbouring plant’s position score. The contribution percentage was determined by the relative positions of the neighbouring plants; if the plant positions were likely neighbours, the percentage was increased but otherwise it was reduced. This
percentage was modelled as an exponential function with the distance from
expected to observed plant positions squared and divided by the crop position
uncertainties. In this description it is assumed that the plant which position
score should be calculated is positioned exactly at a grid location. Plants near
the prior grid location were all investigated and it was determined which plant
(and corresponding position score) could best increase the current plant’s
position score. Such a measure can be computed efficiently by calculating the
position scores from left to right using dynamic programming. The position
score of the \( i \)th plant could then be expressed as

\[
\begin{align*}
c_i &= 1 + \max_k \left[ c_k \cdot \exp \left( \frac{-1}{2 \cdot (2\sigma_x) \cdot (2\sigma_y)} \right) \right]
\end{align*}
\]  (11)

3.2.3. Upper and lower bounds on context-based crop recognition random
plant selector (lower bound)

From the set of plant positions \( \bar{x}_k \) are \( n_{\text{crop}} \) plant positions drawn ran-
domly with no replacements. This method uses no information about the
row structure and can therefore be used as a lower bound on the achievable
recognition rate.
3.2.4. Known seeding positions (upper bound)

In addition to the plant positions, this method also has access to information about all grid locations on which there is placed crop plants. For each of these grid locations the nearest plant is identified and marked as crop. This method was implemented for confirming the predicted upper bound on the achievable PPV given a normalised weed pressure $\lambda$.

4. Experiments and results

Using the test framework described in 3.1, the implemented classification methods were evaluated. 0.5m by 72m parcels containing 360 crop plants were used. PPV values were calculated for weed densities ($\rho$) in the interval $[0.03 m^{-2}; 180 m^{-2}]$ and crop positions uncertainties ($\sigma_x, \sigma_y$) in the interval $[0.015 m; 0.200 m]$. For each set of simulation variables, the simulation was repeated 36 times.

By plotting the obtained PPV as a function of the normalised weed pressure, it is seen that the normalised weed pressure was a suitable combination of the weed pressure and the crop position uncertainty as the simulation results (black dots in Fig. 6) lie in a thin band. The visualised simulation data in Fig. 6 was from the known seeding positions method. It is also seen that the simulation results closely resemble the predicted upper bound on PPV given a specified normalised weed pressure.

4.1. Influence of reduced crop emergence

The effect of reduced crop emergence is visualised in Fig. 6, based on the assumption that plants with a distance larger than $3\sigma$ from the nearest expected grid location were always classified as weeds. At a NWP above $\lambda \simeq 0.1$ the effect of reduced crop emergence was a direct reduction in the achievable PPV value. At lower NWP values the dependency on crop emergence was reduced, as the probability of finding a weed within $3\sigma$ was low.

4.2. Lower limits for crop position uncertainty

Low NWP values are required for context-based crop recognition to perform well, but there are limitations on how low the NWP value can get, as there will be weeds in the field and the crop plant position uncertainty will always be greater than zero. Crop plant position uncertainty depends on several factors such as seed bouncing, displacement during sowing and the difference between seed and seedling locations (Nørremark et al., 2007).
Fig. 6: Visualisation of the predicted PPV as a function of the normalised weed pressure and the crop emergence; emergence percentages are indicated. The black dots are simulation results of the known seeding positions method for recognising crop plants, it is seen to follow the prediction PPV. The two coloured dots are PPV from experiments described in Astrand (2005).

Position uncertainty will in general be larger in the direction of the crop row due to the seeding mechanism. Distances between seed and seedling location were quantified for light and heavy soil types by (Griepentrog et al., 2005). For light soils with fine seed beds the uncertainty was $\sigma_{\text{seed-plant}} = 12.4 \text{mm}$ (Nørremark et al., 2007).

4.3. Best case PPV estimation from NWP

This is an example how NWP can be used to predict the performance of a system. Considering a case with a high weed pressure of $\rho = 400m^{-2}$ and crop position uncertainties of $\sigma_x = \sigma_y = 30mm$. What is the effect of using a better sowing machine to reduce the crop position uncertainty by a factor of two in both $x$ and $y$ directions? The normalised weed pressures and expected PPV values in these two cases are
Fig. 7: Images from sugar beet fields, showing four–five sugar beet plants and some weed plants. Image from DS1 and DS2 in Åstrand (2005).

\[ \lambda_1 = 2\pi \cdot 400m^{-2} \cdot 0.03m \cdot 0.03m = 2.26 \quad \text{PPV} = \frac{1}{1 + \lambda_1} = 0.31 \] (12)

\[ \lambda_2 = 2\pi \cdot 400m^{-2} \cdot 0.015m \cdot 0.015m = 0.57 \quad \text{PPV} = \frac{1}{1 + \lambda_2} = 0.64 \] (13)

This increase in PPV value indicates that for context–based methods the precise placement of crop plants is vital for good performance.

4.4. Comparison with reported classification rates

Åstrand (2005) provides information on weed pressure and crop plant position uncertainties for two datasets; these numbers are provided in Table 2. Sample images from the two datasets are shown in Fig. 7. Given this information the upper bounds of PPV can be estimated using the relationship derived in section 2.3. Table 3 contains information about classifier PPV in
Table 2: Weed pressure, crop emergence and crop position uncertainties for two example datasets and the derived normalised weed pressure. \( \rho \): weed density, \( \sigma \): crop position uncertainty, \( \lambda \): normalized weed pressure, \( \gamma \): emergence.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>( \rho [m^{-2}] )</th>
<th>( \sigma_x [m] )</th>
<th>( \sigma_y [m] )</th>
<th>( \lambda )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS1</td>
<td>50</td>
<td>0.0240</td>
<td>0.0136</td>
<td>0.1025</td>
<td>0.71</td>
</tr>
<tr>
<td>DS2</td>
<td>400</td>
<td>0.0148</td>
<td>0.0108</td>
<td>0.4017</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Table 3: Predictions based on Åstrand (2005). # loc: number of crop locations, # TP: number of correctly classified crop plants, oPPV: observed positive prediction rate, CI: credible interval of the true PPV given the observations.

<table>
<thead>
<tr>
<th>Dataset</th>
<th># Loc.</th>
<th># TP</th>
<th># FP</th>
<th>oPPV</th>
<th>CI</th>
<th>( \frac{1}{1+\lambda} \cdot \frac{\gamma}{1-\phi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS1</td>
<td>643</td>
<td>424</td>
<td>148</td>
<td>0.741</td>
<td>[0.70; 0.78]</td>
<td>0.728</td>
</tr>
<tr>
<td>DS2</td>
<td>273</td>
<td>120</td>
<td>135</td>
<td>0.471</td>
<td>[0.41; 0.53]</td>
<td>0.525</td>
</tr>
</tbody>
</table>

The observed PPV can be calculated as

\[
oPPV = \frac{TP}{TP + FP}
\]  

The 95% credible interval (CI) in Table 3 was calculated using the minimal length method described in Ross (2003). In dataset DS1 the classifier performed slightly better than the predicted upper bound, but was not significantly greater. In DS2 the classifier did not perform as well as the predicted upper bound, but the difference was not significant. For both datasets the theoretical predictions were close to the observed classifier performance, which supports the predictions and indicates that the classifier implemented in Åstrand (2005) performed close to optimum.

4.5. Performance of implemented methods

Two novel methods for context–based crop recognition were presented in section 3.2. One method was based on position scores and the other on path scores. To compare the performance of the different methods, PPV values for different circumstances are shown in Fig. 8. The position score classifier were used in four different configurations where the number of examined neighbour positions was 2, 5, 10 and 20. From Fig. 8 part b-f, the simulation results appear very similar to the data in Fig. 6, except that they are shifted...
Fig. 8: Comparison of performance of different context–based crop recognition methods. The position score method is shown with results from four different numbers of neighbours. Simulation results based on the same weed pressure (but different crop position uncertainties) have identical colours. For each method the model \( \frac{1}{1 + f \cdot \lambda} \) is fitted and the \( f \) value is indicated in the figure.

horizontal towards lower NWP values. To quantify this shift, the following model was fitted to the simulation results for each classifier.

\[
PPV = \frac{1}{1 + f \cdot \lambda}
\]  

In the model \( f \) is a measure of the shift were low \( f \) values indicate a small shift. The theoretical upper bound (i.e. optimal value), derived in section 2.2, corresponds to \( f = 1 \) which means no shift. \( f \) is a measure of the ability of the classifier to locate the true crop locations compared to the expected performance of a classifier using true crop grid locations. A value of 3 means that the classifier performs as a classifier using true crop grid locations in a field with a three times higher NWP.

The random classifier (a) \( (f = 19.18) \) performed really badly, which was to be expected as it did not utilise knowledge about the row structure. The classifier based on position scores (b, c, d and e) was tested in four different
configurations where the number of examined neighbour sites, \(N\), was varied. Using two neighbouring sites, the performance was much better than the random classifier, but the performance gap up to the ideal classifier was large. When the number of neighbours was increased from 2 to 5, 10 and 20 the performance of the classifier gradually increased. The \(f\) values can be seen to have the approximate dependency on \(N\):

\[
f = 1.94 + \frac{6.55}{N} \tag{16}
\]

The path score classifier \((f)\) was seen to perform similarly to the position score when examining 20 neighbouring positions.

A different aspect to look into is how available the different plant positions are in practice. If a crop recognition system has to look at the previous 20 crop plant locations before it can make a decision, then it is vulnerable to the small deviations in crop plant spacing that can be caused by slipping wheels on the seeder. This effect can be reduced by looking at a smaller number of adjacent crop plants.

5. Discussion

In this section three aspects of the paper are discussed: 1) the sole use of context information for crop recognition, 2) different error types that appear when using context–based methods and 3) the assumptions behind the simulations and their validity.

5.1. Is context information enough?

Is it possible to rely on context–based classification only? Assuming that a PPV of 95\% is precise enough that farmers will find that the decrease in yield due to incorrectly removing crop plants will outweight the decrease in cost of manual weed control. To reach a PPV = 0.95 the NWP should be \(\frac{1}{0.95} - 1 = 0.0526\) or lower. If the crop positioning uncertainty is \(\sigma_x = \sigma_y = 1\) cm, the weed pressure must not exceed \(\sim 84\) m\(^{-2}\). Higher weed pressures are often observed and therefore a context–based classification is not sufficient to reach the required classification accuracy. The classification accuracy can be improved by taking advantage of other kinds of information like plant shape and spectral signatures.
5.2. Error types

The observed errors can be divided into three groups. The first group contains all the cases where there is a weed plant closer to the expected crop location than the nearest crop. These errors cannot be avoided, but the error rate can be estimated from the NWP. Errors caused by missing crop plants, e.g. due to low emergence, belong to a second group. The third error group occurs when the row-structure recogniser fails to locate the row pattern. As a result, the search for the nearest plant is unlikely to find a crop. The performance differences observed from Figs. 6 to 8 can all be explained by this third group of errors.

5.3. Assumptions and their validity

To derive the central equations in this paper, a set of assumptions were used. During the analysis it was assumed that the weed density was uniform. Research by Nørremark (2009) showed that weed pressure is lower close to sugar beet seedlings. This effect is only present within a few centimetres from the crop seedling so the effect on the obtained results should be negligible. If weed pressure close to the crop plants is lower, the PPV of the context-based classifiers will slightly increase.

Under field conditions occlusion of leaf parts is often seen at high weed densities. Occlusion of leafs can disturb estimation of the plant centres, but in the simulations it was assumed that the plant centres could be located under all conditions. The plant-centre-detection method determined how fragile the system will be to excessive occlusion. The method described in Midtiby et al. (2012) can predict plant centres of partial occluded plants.

5.4. Comparison with existing methods

To compare performance of the position score method with the two datasets obtained from (Åstrand, 2005) equation (15) can be rewritten as

\[ f = \frac{1}{\lambda} \cdot \left( \frac{1}{\text{PPV}} - 1 \right) \]  

\[ (17) \]

By inserting values from DS1 and DS2, the follow values of the \( f \) parameter were found
These values are similar to the $f = 3.37$ value found in Fig. 8, indicating that the method in (Åstrand, 2005; Åstrand and Baerveldt, 2004) and the position scores using 5 neighbours perform similarly. The limited emergence in DS1 and DS2 have not been taken into account in this comparison.

6. Conclusion

An upper bound on PPV for a given normalised weed pressure can be determined using the relation $PPV = \frac{1}{1+\lambda}$. The classifier performance of the context–based crop recogniser described in the literature was compared with the estimated upper bound. The observed performances were similar to or lower than the predicted upper bounds. The predicted relationship between the normalised weed pressure and the achievable PPV was supported by both simulations and the reported classifier performance in the literature. The direct relationship between $\lambda$ and PPV indicates that for context–based methods the precise placement of crop plants is vital for good performance.

Two novel context–based crop–recognition methods were implemented and evaluated in a simulated environment. The methods based on position scores performed better when the number of examined neighbour crop positions was increased. When 20 neighbour positions were examined the method performed similarly to the path–score method. All tested methods had PPV values below the theoretical upper bound. The performance of the two best methods could be predicted by the model $PPV = \frac{1}{1+2\lambda}$.

If positive predictive values above 95% are required, classification based on plant position information alone will not be enough for typical conditions and additional information such as plant morphology, spectral characteristics or similar will be needed.
Appendix A. Derivations

Appendix A.1. Definitions

\[ p_{\sigma_c}(c) = \frac{1}{\sigma_c \sqrt{2\pi}} \exp \left( \frac{-c^2}{2\sigma_c^2} \right) \]  
\[ (A.1) \]

\[ n_w(x, y) = \rho \pi \sigma_x \sigma_y \left( \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right) \]  
\[ (A.2) \]

Appendix A.2. The average number of weed plants closer to the grid location than the nearest crop plant

\[ \lambda = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{\sigma_x}(x) \cdot p_{\sigma_y}(y) \cdot n_w(x, y) \, dx \, dy \]  
\[ (A.3) \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sigma_x \sqrt{2\pi}} \exp \left( \frac{-x^2}{2\sigma_x^2} \right) \cdot \frac{1}{\sigma_y \sqrt{2\pi}} \exp \left( \frac{-y^2}{2\sigma_y^2} \right) \cdot \rho \pi \sigma_x \sigma_y \left( \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right) \, dx \, dy \]  
\[ (A.4) \]

Moving constants out of the double integral.

\[ = \frac{\rho}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left( \frac{-x^2}{2\sigma_x^2} \right) \cdot \exp \left( \frac{-y^2}{2\sigma_y^2} \right) \cdot \left( \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right) \, dx \, dy \]  
\[ (A.5) \]

Changing integration variable to get rid of \( \sigma_x \) and \( \sigma_y \) inside the double integral. \([x' = x/\sigma_x \rightarrow dx = \sigma_x dx']\) and \([y' = y/\sigma_y \rightarrow dy = \sigma_y dy']\)

\[ = \frac{\rho}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left( \frac{-x'^2 - y'^2}{2} \right) \cdot \left( x'^2 + y'^2 \right) \sigma_x \, dx' \sigma_y \, dy' \]  
\[ (A.6) \]

Taking constants out of the integral and changing to polar coordinates.

\[ = \frac{\rho \sigma_x \sigma_y}{2} \int_{0}^{2\pi} \int_{0}^{\infty} \exp \left( \frac{-r^2}{2} \right) \cdot r^2 \, r \, dr \, d\theta \]  
\[ (A.7) \]

This can be solved using \( \theta \) integral and thereafter the \( r \) integral.

\[ = \pi \rho \sigma_x \sigma_y \int_{0}^{\infty} \exp \left( \frac{-r^2}{2} \right) \cdot r^3 \, dr = 2\pi \rho \sigma_x \sigma_y \]  
\[ (A.8) \]
Appendix A.3. Probability of not finding any weeds

The positive predictive value, given crop position uncertainty $\sigma_{x,y}$ and weed density $\rho$ can be given by

$$PPV = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{\sigma_x}(x) \cdot p_{\sigma_y}(y) \cdot \exp(-n_w(x,y)) \, dx \, dy$$  \quad (A.9)

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma_x^2}\right) \cdot \frac{1}{\sigma_y \sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)$$
$$\cdot \exp\left(-\rho \pi \sigma_x \sigma_y \left[\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right]\right) \, dx \, dy$$  \quad (A.10)

Separating the two integrals and collecting common factors

$$= \frac{1}{2\pi \sigma_x \sigma_y} \int_{-\infty}^{\infty} \exp\left(-x^2 \cdot \left[\frac{1}{2\sigma_x^2} + \frac{\rho \pi \sigma_y}{\sigma_x}\right]\right) \, dx$$
$$\cdot \int_{-\infty}^{\infty} \exp\left(-y^2 \cdot \left[\frac{1}{2\sigma_y^2} + \frac{\rho \pi \sigma_x}{\sigma_y}\right]\right) \, dy$$  \quad (A.11)

Solving the integrals

$$= \frac{1}{2\pi \sigma_x \sigma_y} \cdot \frac{\sqrt{\pi}}{\sqrt{\frac{1}{2\sigma_x^2} + \frac{\rho \pi \sigma_y}{\sigma_x}}} \cdot \frac{\sqrt{\pi}}{\sqrt{\frac{1}{2\sigma_y^2} + \frac{\rho \pi \sigma_x}{\sigma_y}}}$$  \quad (A.12)

Simplifying gives

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{1}{2} + \frac{\rho \pi \sigma_x \sigma_y}} \cdot \sqrt{\frac{1}{2} + \frac{\rho \pi \sigma_x \sigma_y}}} = \frac{1}{1 + 2\rho \pi \sigma_x \sigma_y} = \frac{1}{1 + \lambda}$$  \quad (A.13)

References


