Review of Incurvati "On the Concept of Finitism"

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In this paper, the author argues that the concept of finitism may be explicated in several ways since it involves two different ideas, namely: (1) that the finitary is epistemologically distinguished from the infinitary by being graspable by indubitable intuition, and (2) that it is ontologically or semantically more secure than the infinitary the existence of which is doubtful. Three explications of the concept of finitism are investigated, one due to Charles Parsons, who elaborates the epistemological tenet of finitism. The other two develop ideas inherent in the work of William Tait. One of them works out the ontological tenet of finitism whereas the other—called "Cartesian finitism" by Incurvati—is based on epistemological considerations. Incurvati argues that the parts of mathematics respectively justifiable by the three explications of the notion of finitism are by no means co-extensive.

The variety of finitism which has been developed by Charles Parsons since 1980 (cf., e.g., the collection MR2381345) relies upon a certain interpretation of Hilbert’s views concerning the distinguished role of mathematical intuition. This interpretation develops the Kantian traits of Hilbert’s view by declaring mathematical statements as justified from a finitary point of view if they can be known intuitively, where intuition is understood as the grasp of abstract entities by means of concrete objects presented by perception or imagination. Thus, for instance, perception or imagination of the token $\exists$ may provide the intuition of the abstract type of strings consisting of four items. Here the abstract type is intuited because the concrete token is seen as such a string. Intuitions of this kind may provide us with mathematical knowledge since they concern items which belong to a structure isomorphic to that of the natural numbers (p. 2415). If, then, we follow Parsons by restricting the finitary part of mathematics to statements justifiable by intuitions of the kind described, finitary mathematics becomes a proper part of PRA (primitive recursive arithmetic) since the definitional procedure of introducing functions by means of primitive recursion cannot, as Parsons admits, be seen to be legitimate by means of such intuitions only. Incurvati puts forward two objections against this conception of finitism. First he argues that making the intuition of abstract objects dependent upon the perception of concrete tokens leads up to a dilemma. Either the abstract mathematical objects inherit the vagueness of their concrete counterparts, which is intolerable for Parsons (p. 2417). Or grasping abstract objects by identifying concrete tokens as their realizations may become conceptually so involved that one hardly can speak of an “immediate” act of intuition any more (p. 2418), whereas—according to both Hilbert and Parsons—such an immediacy is constitutive for acts of intuition (p. 2415). The second objection raised by Incurvati is that the realm of the finitary is even narrower than assumed by Parsons since the successor function, as characterized within PRA, cannot be seen intuitively well-defined. Parsons attempts to justify its well-definedness by showing that we have intuitive knowledge of the “string analogue” of the statement saying that
every number has a successor which is a number again. Incurvati, however, argues (pp. 2419–2422) that there is no legitimate way in the foundational framework admitted by Parsons to prove such a general statement. He concludes that Parsons’ conception of finitism will not sanction any substantial part of mathematics unless the intimate connection between intuition of abstract types and perception of concrete tokens is weakened (p. 2422).

The second variant of finitism dealt with by Incurvati is characterized by the tenet that finitistic theories do not appeal to the actual infinite, i.e., to the existence of completed infinite totalities. The views advocated by William Tait (cf., e.g., the collection MR2119996) are taken as paradigmatic for this understanding of finitism. Incurvati points out (p. 2425) that different things can be understood by a theory’s “appeal to the existence of infinite collection”: (a) the theory entails a statement asserting the existence of such a collection; (b) the variables of the theory can only be interpreted as ranging over such a collection; or (c) the theory can only be understood by means of infinitary concepts. According to “Tait’s Thesis” (p. 2424) precisely the primitive recursive functions are finitistic. This provides PRA with the distinguished status of being that system which formalizes the finitist understanding of number. On the other hand, Tait does not consider the notion of a finitistic function itself to be finitistic. From this, Incurvati concludes (p. 2427) that Tait’s finitism is of variety (c). The question, then, arises how — on such presuppositions — one can distinguish between PRA and PA (Peano arithmetic) on the basis of the distinction between the finite and the infinite. As PRA, PA too does not entail a statement asserting the existence of some infinite object. Furthermore, it is controversial whether the understanding of PA really requires the grasp of infinitary concepts. Generality in PRA is expressed in a purely schematic way by means of free variables; hence it does not require the existence of any infinite totality (of numerals or number terms). Given the derivability of statements of the form \( \forall x. F(x) \lor \exists x. \neg F(x) \), the interpretation of the quantifiers of PA, in contrast, seems to require the existence of such a totality. But this is a distinction which — though it can drawn within a (b)-variety of finitism — is not available in Tait’s finitism of the (c)-kind, at least not without further assumptions.

The last variety of finitism, Cartesian finitism (p. 2428ff), transfers Descartes’ epistemological strategy for finding a secure basis for all knowledge by eliminating everything doubtful to mathematics. The finitary legitimate, then, is the absolute minimum necessary for numerical reasoning: doubting this last foundation would mean to doubt numerical reasoning in general and would thus lead up to a radical skepticism which “the finitist need not address” (p. 2430). Some remarks made by Tait point into the direction of Cartesian finitism. Since Tait, furthermore, conceives of PRA as the finitary legitimate part of mathematics, he has to defend the thesis that this system of arithmetic is the minimum necessary for a satisfactory body of numerical reasoning. Consequently, Tait rejects more restrictive subsystems of PRA such as EA, which characterizes the so-called “(Kalmar) elementary functions”, as ad hoc. In EA, definition by primitive recursion is restricted by the requirement that the function to be defined should not grow faster than previously defined functions; cf. MR0020529, MR0021918. According to Tait, such requirements restricts iteration in an arbitrary way. Iteration, however, is constitutive for
our notion of a number. Tait’s idea seems to be, as Incurvati (p. 2431) points out, that there can be no genuine numerical reasoning without unrestricted iteration. Incurvati criticizes that an argument supporting this position is wanting in the relevant writings of Tait. However, a *prima facie* argument for it may be constructed from some explanations provided by Tait. But, as is shown by Incurvati, that argument suffers from an ambiguity of the expression *finite sequence* entering into it in a crucial way. Two different meanings of that expression are compatible with Tait’s explanation but each of them renders one of the reconstructed argument’s premises false and thus the entire argument non-conclusive (p. 2432).

In the conclusion of his article, Incurvati quotes a passage from the second volume of Hilbert and Bernays’ *Grundlagen der Mathematik* ([MR0272596](https://doi.org/10.1007/BF01443767)) where the authors explain that their term *finitistic* is not sharply defined but rather denotes a methodical guideline for distinguishing between certain kinds of concept-formations. He declares this to be in accordance with his article which shows the concept of finitism to be informal and amenable to different explications. However, Incurvati’s article leaves us with a much darker view of finitism — at least as regards the three variants of it which he considers — than this conciliatory remark suggests. Parson’s finitism is found to be not even able to establish the well-definedness of the successor function. Tait’s first explication of finitism is criticized for failing to distinguish between PRA and PA as regards the avoidance of infinite totalities. Finally, Cartesian finitism — according to Incurvati — is not able to explain why PRA should be preferred to the more restrictive EA.

Reviewed by Klaus Robering