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Economies of scale and optimal size of hospitals: Empirical results for Danish public hospitals

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Preface:

The aim of this Working Paper (WP) is to contribute to the current debate about the future configuration of the Danish hospital sector, in particular the issue of 'optimal' hospital size. From an economic perspective we find that the empirical evidence underpinning the planned hospital structure is ambiguous and partly lacking. The WP contributes to this debate with an empirical study of the question of economies of scale in the Danish hospital sector and estimates of an optimal hospital size.

Our WP is addressed to foreign and Danish economists with an interest in industrial organization and, in particular, the organisation of the (Danish) hospital sector. Furthermore, our WP serves as the back ground paper for an article submitted to a peer-reviewed health economic journal. In addition to the content of the submitted article, the WP contains details such as an appendix and a more detailed discussion of methodological issues and concepts.

Acknowledgments

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Title: **Economies of scale and optimal size of hospitals: Empirical results for Danish public hospitals**

Abstract:

Context and aim: The Danish hospital sector is facing a significant rebuilding programme, driven by a political desire to concentrate activity in fewer and larger hospitals. Our aim is to analyse whether the current configuration of Danish hospitals is subject to scale economies that may justify such plans and to estimate an optimal hospital size.

Methods: We estimate cost functions using panel data on total costs, DRG-weighted casemix, and number of beds for three years from 2004-2006. A short-run cost function is used to derive estimates of long-run scale economies by applying the envelope condition.

Results: We identify moderate to significant long-run economies of scale when applying two alternative translog cost functions. However, using a quadratic functional form we identify constant economies of scale for the medium-sized sub-groups and decreasing economies of scale for the largest sub-groups. The optimal number of beds per hospital is estimated to be 275 beds per site. Sensitivity analysis to partial changes in model parameters yields a joint 95% confidence interval in the range 130 – 585 beds per site.

Conclusions: The results indicate that it may be appropriate to consolidate the production of small hospitals (<230 beds) on fewer and larger units.

Keywords: Economies of scale, optimal size, hospitals, cost function.

JEL Classification: C13, D24, H42, I11, I12, L38, L44, L52.

1. Introduction

In Denmark the number of somatic hospitals has decreased from 117 in 1980 to 52 in 2004. Part of this decrease is due to the fact that the concept or idea of a hospital has changed. Until the early 1990's there was always a one-to-one relationship between a hospital as a management entity and its geographical location. During the past 15 years, however, hospitals at different locations have been merged so that many 'hospitals' today consist of several geographically distinct 'production units' being managed together. These new entities, consisting of several production units, are called management entities or conglomerate hospitals. This trend towards centralization is not uniquely Danish but is also found in, for instance, England, Norway and Sweden [1,2,3].

Hospital plans from the five Danish regions show that this development is expected to continue in the years to come [4]. They are planning a significant rebuilding program including green field investments at 5 new sites, significant extension and reconstruction of several existing hospitals and mergers or closures of several small hospitals[1].

Whether the increasing centralization of hospitals is to be seen as an advantage depends on a) whether there are economies of scale i.e. lower average costs and b) whether bigger hospitals lead to improved clinical outcomes [5]. Exploiting economies of scale may help to limit costs of health care outputs without compromising their quality and volume. On the other hand, hospitals may become so large that the cost of treatment will be higher due to diseconomies of scale. Furthermore, plans to concentrate further assume that the 'optimal' hospital size is bigger than in the current configuration.

From an economic perspective the evidence base underpinning centralization is weak, that is to say that there is a conspicuous absence of research and discussion of economies of scale in Danish hospital production. No econometric studies of economies of scale in hospitals have ever been undertaken in Denmark. In Europe, unlike in the U.S. where more literature exists on economies of scale, the economics of this trend towards larger hospitals have not been sufficiently analyzed. Notable exceptions are [6] and [7] along with a survey [5]. Evidence of the 'optimal' hospital size is important at a time when the hospital sector is facing major restructuring. Therefore, the aim of this study is to asses whether there are unexploited economies of scale in the current configuration and to estimate an 'optimal' hospital size. The present study

[1] The association of Danish Regions has published the rebuilding program for each region, see http://www.godtsygehusbyggeri.dk.

is limited to assessing economies of scale and 'optimal' hospital size for Danish hospitals in the period 2004-2006 from a hybrid econometric cost function perspective [8][2].

The unit of analysis is the hospital "production unit", not the hospital management entity. This approach has become increasingly relevant due to the trend towards concentration in secondary healthcare. In relation to the rebuilding programme it is the geographical hospital "production unit" which is the relevant decision unit when deciding to build new hospitals to replace one or more former hospitals. It is not the management entities with satellite production units which may be located far from each other that are the relevant analytical unit A distance of 30-50 km between units within the same management entity is quite common. Besides, using the production unit as the unit of analysis means that we can interpret the estimated economies of scale and hospital sizes in relation to the actual geographical hospital production units instead of multisited hospital management entities. In the following, consequently, the term hospital is reserved for freestanding "production units" in specific geographical sites rather than "hospital management entities" which consist of several production units at different sites.

Our presentation of earlier studies is restricted to those that use econometric cost functions that resemble the methods used in this study. However, this study differs from the majority of earlier studies in several ways – especially in its estimation of long run cost functions and 'optimal' hospital size. So far, this approach has not been used in European studies. The literature search revealed only a single Canadian study that has estimated an 'optimal' hospital size using the envelope condition [9]. All other earlier studies of 'optimal' size are based on scale estimates – excluding specific estimates of 'optimal' hospital size.

2. Earlier results

The empirical literature on economies of scale in hospitals is extensive, if all statistical techniques are included [5]. Despite the fact that the literature reflects different methods and covers many different countries the results are remarkably consistent, according to a recent survey of 103 studies by Aletras [10], i.e. these studies reveal constant economies – or even diseconomies – of scale for the average hospital with about 200-300 beds, see also Aletras et al. [11]. However, studies based on structural or hybrid cost functions only represent about one fifth of these studies. According to [11] economies of scale were evident only for small hospitals with less than 200 beds and the 'optimal' size for acute hospitals ranged form 200 to 400 beds (based on the

[2] This study is inspired by the relatively well known properties of scale and optimal scale size of parametric cost functions. The corresponding properties of the alternative non-parametric deterministic data envelopment analysis (DEA) are less explored [12].

interpretation of scale estimates). For hospitals above 400-600 beds it was concluded that the average cost increases.

Studies after 1997 based on structural or hybrid econometric cost function do not confirm the above-mentioned consistency. In North America the application of panel data has shown economies of scale in Canada [13,9]. A third study based on cross-section data also indicated economies of scale [14]. Moreover, a study of acute care hospitals in California has revealed a minor trend towards economies of scale [15].

In contrast to, for example, [13] and [14], the present writers use casemix-adjusted output measures instead of particularly constructed casemix indexes to adjust for differences in patient mix and severity. Furthermore, this study differs from [9], for example, by including costs shifters to adjust the structural model for cost drivers that are specific to hospitals.

Finally, it is apparent that the studies described do not rely on the latest data. This study applies the latest data and data adapted for managerial decision-making and efficiency-measurement in the Danish hospital sector.

3. Methods

Using econometric assessment of economies of scale and 'optimal' size of hospitals, a number of choices need to be made such as unit of analysis, model of hospital production, model for cost functions, specification of cost and output variables, and estimation technique.

As elsewhere, the number of hospitals in Denmark has declined radically over the past two decades as a result of mergers and closures. This means that many hospitals have changed from being an institution located on a single geographical site to a management conglomerate of hospitals spread across several geographical sites, often with a degree of division of labour and hence specialization. When estimating economies of scale and "optimal" hospital size this is a challenge if the cost and output data for hospitals are aggregated at the conglomerate level. This study is based on data for hospital production sites to reveal knowledge relevant for policy making about the cost and production characteristics of physical production entities in the Danish hospital sector. If the management conglomerate sites had been used as the sole unit of analysis, then estimates could have been conducted only in relation to an 'optimal management unit'.

The short-run cost function

Most studies argue that there is no evidence that hospitals operate in their long-run equilibrium, i.e. hospitals do not adjust all their inputs to their cost-minimizing levels [16]. From at theoretical point of view, therefore, it is appropriate to estimate short-run cost functions. The argument is that this approach allows hospitals to use possible non-optimal levels of the fixed inputs in the short run. Hence, hospitals are only assumed to use cost minimizing quantities of easily adjustable variable inputs, such as nurses, physicians and materials. Furthermore, cost function estimation by frontier estimators may account for deviations from the cost frontier (non-minimum cost functions).

To estimate the short-run cost function, three different functional forms have been applied to examine the sensitivity of results to the chosen functional forms, because they cannot be determined *a priori* from a theoretical model. The first two specifications are the translog model and Cobb-Douglas model. The third specification is the less common quadratic form which, like the translog model, belongs to the family of 'flexible' forms. The three functional forms can be expressed as a version of the family of 'flexible' functional forms, which are second order Taylor approximations to an unknown functional form.

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$$g(C_{it}) = \alpha_0 + \sum_{j=1}^{K} \beta_j f(x_{it}) + \sum_{j=1}^{K}\sum_{h=1}^{K} \beta_{jh} f(x_{itj}) f(x_{ith}) + u_{it} \qquad i = 1,\dots,N, t = 1,\dots,T \quad (1)$$

where g is a real valued function of the total cost for somatic treatment C_{it} for hospital i ($i = 1,..,N$) in period t ($t=1,\dots,T$), $_0$ is a constant, f a real valued function of the cost determinants, $\boldsymbol{x}_{it}$ is a matrix of outputs Q_m ($m=1,\dots M$) and cost shifters. is a vector of unknown parameters and u_{it} is a random disturbance term.

Firstly, (1) is equal to the quadratic form when f and g are chosen to be equal to $\boldsymbol{x}_{it}$. Secondly, if f and g are chosen to be the natural logarithmic function, (1) yields the translog model. Thirdly, the Cobb-Douglas version is the special nested case of the translog model where squared and interaction terms in (1) are excluded. The exact model specifications are shown in the appendix. Since this study aims to allow $\boldsymbol{x}_{it}$ to contain cost shifters for hospital i in time period t, the estimated models also belong to the family of hybrid cost functions [8].[3] This category of functional forms is now preferred to the more naïve structural functional form, see, for instance [17].

In equation (1) input prices are left out, because it is assumed that input prices do not vary significantly between the hospitals, i.e. union agreements are made on a national basis. The reason is that pay and work conditions are fairly uniform across the public hospitals in Denmark, since wages are negotiated nationally and wages mainly vary with the experience and level of education. The public regulation of the hospital's purchases of medicines and other not pay-related inputs means that it is accepted that the hospitals have more or less uniform prices in these areas. Additional arguments for the assumption about constant prices are that it is a panel data study with a relatively short time dimension and that the literature indicates the difficulty of measuring the input prices for hospitals e.g. the price of capital [6,18].

Cost and output variables

The advantages of 'flexible' cost functions are that they do not prejudge the existence or degree of economies of scale. Unfortunately, this increased flexibility is obtained at the cost of there being more parameters to be estimated than in more restricted functional forms such as the classical Cobb-Douglas cost functions. A consequence is that the estimation of 'flexible' cost functions often results in multicollinearity problems, see e.g. [6,19], especially if attempts are made to disaggregate the outputs into more subgroups. This was also the case in this study. In other

[3] Cost shifters controls for the fact that the cost of individual hospitals may be influenced by other external factors to the hospital management than the demand for output such as capital endowment, strikes, epidemics and whether conditions.

words the estimated coefficients became insignificant and unstable, and the signs changed in such a way that the results did not make sense from an economic point of view. The reason is to be found in a combination of three conditions. Firstly, there is the structure of the 'flexible' cost function with squared and interaction terms. Secondly, the aggregated output measures from hospitals tend to be correlated. Finally, the small number of observations, even in panel data, is an important issue in all small countries. These circumstances make the choice of a limited number of covariates and the aggregation of outputs two important tasks.

We have merged the multiple hospital outputs into two aggregated output measures, DRG value for inpatient and outpatient activity. This approach has the advantages that it maximizes the degrees of freedom for estimation and minimizes collinearity problems given the limited amount of observations. The disadvantage is the collapsing of the multidimensional output vector into an aggregated vector (index) [20].

The Danish DRG system adjusts each output (discharge) for casemix and to a certain extent for severity through the DRG cost weights attached to each discharge. Therefore, this study uses DRG values to measure aggregated hospital output, as do related Danish studies, e.g. [21,22] .

Outputs are defined as the total DRG value per year per hospital, i.e. 'total production value' or are divided into inpatient and outpatient production values per hospital, depending on the degrees of freedom in each of the specified models.

The number of beds per hospital is used as a proxy for the fixed cost (capital endowment) and hospital size. This approach makes it feasible to estimate the 'optimal' number of beds (optimal capital stock) by application of a 'flexible' functional form and the envelope condition [9].[4] Besides the number of beds per hospital, we have strived to include cost shifters in the model, see, for instance, [8]. The limited number of degrees of freedom in the flexible functional forms together with potential multicollinearity problems limit our ability to include many of the relevant cost shifters, e.g. patient characteristics. This study only applied a university dummy to adjust for higher cost in University hospitals due to teaching, to R&D and, possibly, to underestimation of the complexity of the patient mix at University hospitals [23,24] . Compared to earlier studies such as that by Preyra & Pink [9] which do not include cost shifters, we consider this an important issue.

The short-run cost functions represented by (1) were estimated by the fixed effect (FE) estimator, which is equivalent to adding a dummy variable for all but one hospital. In contrast to cross-sectional estimators, this estimator allows for unobservable systematic differences between

[4] It is not feasible to calculate a long-run cost function from a standard Cobb-Douglas cost function since it does not include squared terms.

hospitals, for instance, different managerial abilities and severity of illness. Therefore it is less likely that FE models suffer from omitted variables bias as is often the case in cross-sectional behavioural equations [25]. Besides the fixed effect model accounts for technical inefficiency among the variable inputs, even though it does not disentangle the fixed effects into an efficiency part and any unmeasured time invariant cross firm heterogeneity, [26]. The Hausman test was used to test for the choice of the FE effect estimator over the Random Effect (RE) estimator.

Calculation of the optimal hospital size

The 'optimal' size of a hospital can be calculated from the short-run cost function by application of the envelope condition [9, 27]. Given the cost function (1), this calculation can be expressed as:

$$K^* = \frac{\partial g(C)}{\partial K} = 0 \qquad (2)$$

where the 'optimal' size of a hospital K^* is a function of outputs and included cost shifters (if cost shifters are interacted with outputs). In case the estimated short-run cost function is a translog cost function, the 'optimal' number of beds K^* in (2) becomes an exponential function of hospital outputs. The calculation of (2) for the quadratic form yields a linear relationship between outputs (activity) and the 'optimal' amount of capital in terms of beds K^*, see appendix for details.

Sensitivity analysis was conducted for the 'optimal' hospital size by substituting the upper and lower bound of the 95% confidence interval for each of the beta parameters from (1), which remained in (2) after differentiation with respect to the proxy for fixed capital (K).

The long-run cost function

The long-run cost function has been estimated in two different ways. The first approach uses the estimated short-run cost function and the envelope condition to calculate the long-run cost function [9]. This means that the first order condition of the short-run cost function set equal to zero defines the 'optimal' relationship between beds and outputs as defined in (2). Substituting (2) into the short-run cost function (1) yields the long-run cost function. Since (1) is a second order Taylor approximation, this calculation yields a second order equation for the long-run translog cost function and the long run quadratic cost function respectively.[5]

[5] While it is possible to conduct the above-mentioned calculations for the flexible functional form, this is not feasible for the Cobb-Douglas model. This is due to the squared term lacking in the Cobb-Douglas model, which prohibits the calculation of an 'optimal' hospital size (K*) from (2) since it is not twice differentiable with respect to K.

The second approach applies an alternative way, where the cost function is estimated directly without use of the envelope condition [6]. This approach, the 'direct approach', has been achieved by omitting the number of beds in the estimation of the cost function. Hence, in contrast to the 'envelope condition approach', the direct approach assumes that hospitals use an optimal amount of capital in terms of beds (K^*) in the short and long run, see for instance, [16]. In other words, the direct approach assumes fixed cost to become variable in the long run, in other words as a function of, for example, outputs. Besides, it assumes that fixed cost varies with output across the data set and that the hospitals are always endowed with an optimal amount of capital. The latter is a relatively restrictive assumption to be discussed later. The degrees of freedom gained from dropping the beds variable (K) are used to include two output measures, DRG value of inpatients and outpatients instead of the total DRG value per hospital.

Derivation of economies of scale estimates

In accordance with [28], economies of scale are estimated in a way that shows the relative rise in costs when output is increased proportionally. Since the translog and Cobb-Douglas models are logged in all variables and the quadratic forms are unlogged this yields (3) and (4):

$$SE1 = \sum_m \frac{\partial g(C)}{\partial f(Q_m)} \tag{3}$$

$$SE2 = \sum_m \left(\frac{\partial C}{\partial Q_m} \Big/ \frac{C}{Q_m} \right) \tag{4}$$

SE1 in (3) expresses the sum of first order partial derivatives of the cost function (1) with respect to each output Q_m in logs. The logarithmic transformations imply that each of these derivatives is an estimate of cost elasticities for each Q_m.

SE2 in (4) measures the sum of cost elasticities with respect to output. Each of the cost elasticities in *SE2* is calculated using the standard (unlogged) approach, because the quadratic form is in cost levels. In the translog and quadratic models, in which scale estimates by definition are flexible, the sub-group median hospital was used to calculate scale estimates for each of the defined size groups. The size groups were defined by quartiles. The smallest size group (1st quartile) consists of the 25% of hospitals, which has the smallest number of beds, while the other size groups, 2nd quartile, 3rd quartile and 4th quartile, include hospitals with a size in the respective quartiles. Both *SE1* and *SE2* express the multi-product analog of marginal cost divided by average cost. The exact model specifications are shown in the Appendix.

In equations (3) and (4), SE values less than 1 indicate economies of scale corresponding to cost increases, which are smaller than the proportional output increase. SE values larger than 1 show diseconomies of scale.

Data

The data comes from a national cost database developed by the National Board of Health [29]. The cost database is based on patient activity and cost information from most public hospitals and is also used to calculate Danish DRG tariffs. Total hospital costs are actual costs incurred in respective years adjusted for costs from shared facilities with other hospitals, such as laundry.[6] They are used as the best available proxy for the total cost for somatic treatment. DRG values, or in other words the reimbursement received by hospitals, give the most appropriate picture of the value of hospital production.

There may be some inconsistencies for the DRG values for the three years 2004 to 2006, because the DRG grouper used for 2005 and 2006 was different from that used for 2004 (giving different input prices). This means that 2004 data is based on 2007 input prices while 2005 and 2006 data is based on real 2008 input prices. We assume, however, that the effect of this is negligible due to a low inflationary level. Variables used and descriptive statistics are shown in table 1.

[6] The National Board of Health calculates adjusted actual operating costs by deducting from total reported operating costs, whenever relevant. This applies, for instance, to the cost of psychiatric services, laboratory services for general practitioners, the cost of medicines provided for outpatients, adjustments for differences in accounting practice and unpaid services between hospitals. This gives the figure for 'the adjusted operational costs' which is used in the present study.

Table 1 Descriptive statistics for Danish hospitals in the years 2004-2006[a]

Variable	Year	Description	Average	Std. dev.	Min	Max
		Dependent:	in 1000 DKK (Danish currency)			
C	2004	Adjusted operational costs	530,934	648,465	21,581	3,591,319
	2005	-	616,490	784,055	19,035	4,337,614
	2006	-	439,160	449,149	16,761	1,890,084
		Independent:				
Q_1	2004	DRG value inpatient	328,514	379,452	0	2,123,694
	2005	-	361,908	477,284	0	2,516,725
	2006	-	232,824	231,748	3,265	898,218
Q_2	2004	DRG value outpatient[b]	209,637	270,439	4,909	1,180,949
	2005	-	274,984	335,966	2,514	1,572,589
	2006	-	226,344	247,555	3,555	1,000,748
Q_T	2004	Total DRG value (Q_1+ Q_2)	538,152	592,734	22,968	3,304,644
	2005	-	636,892	788,952	12,098	4,089,314
	2006	-	459,168	468,839	15,163	1,898,966
		Independent cost shifters:	in no. of beds and percentage of hospitals			
K	2004	Average number of staffed beds	281.6	250.9	25.6	1107.1
	2005	-	265.1	259.5	9	1136.7
	2006	-	176.0	152.1	9	517

[a] Unbalanced due to missing data for 2006. The numbers of observations in 2004-06 are 57, 55 & 31 respectively. [b] Including the value of grey zone DRG activity

Data in table 1 shows that hospital production units on average had operating costs in the range DKK 530 to 616 million. The DRG values are measured in local currency, DKK. The total value of DRG production for each hospital is divided into two output categories: 1) the production value of inpatients and 2) the production value of outpatients, including both so called grey zone patients and emergency patients.

Grey zone patients are patients that the hospital staff both can choose to treat as outpatient or as inpatient (in connection with hospitalization). To avoid distortion of this substitution choice, a special grey zone DRG rate is used. The grey zone DRG rate is calculated as the weighted average between what it costs to perform same-day surgery or outpatient treatment, and the corresponding price for similar inpatient treatment.

The average number of beds per hospital production unit is in the range 265 to 281, but this average covers wide variation between production units (e.g. min. 9, max. 1136 in 2005). The average number of disposable beds per hospital is used as a proxy for the size of hospitals and fixed inputs.

Table 1 also shows that the percentage of public hospitals that were university hospitals was on average approximately 21% in the period 2004 to 2005. Finally, it should be noted that the data for 2006 is generally sparser than the data for the previous year. This is due to data being missing for some of the large units in 2006, i.e. unbalanced panel data. Psychiatric hospitals are excluded from this study. Danish psychiatric hospitals do not use the DRG system. In special hospitals, e.g. Friklinikken in Braedstrup and Hammel Neurocenter, the production process is considered to be atypical. Therefore, six hospitals were excluded.

Results

Table 2 shows the results for the short-run cost models in the Cobb-Douglas, the translog and the quadratic model specification.

Table 2 Regression results – short-run cost functions

	Cobb-Douglas	Translog	Quadratic
	FE	FE	FE
Variable	2004-2006	2004-2006	2004-2006
Intercept	-0.1423*	-0.1966**	-0.0297
Inpatients (DRG value)	0.4425***	-	-
Outpatients (DRG value)	0.2736***	-	-
Total DRG value of in- and outpatients	-	0.6921***	1.4800***
Avg. number of beds	0.0511	-0.0403	-1.2360**
(Total DRG value)^2	-	-0.0262	0.1266**
(Avg. number of beds)^2	-	-0.0967	0.8131***
Total DRG value*Avg. number of beds	-	0.0938	-0.5377***
Number of observations	143	143	143
Number of hospitals	54	60	60
R^2			
Within	0.6028	0.7717	0.7657
Between	0.9837	0.9778	0.9517
Overall	0.9763	0.9663	0.9374
F-test (5,78)	28.11***	105.55***	30.10***
Hausman chi2(5)	8.49**	-[a]	-[a]

*** P < 0.01, ** P <0.05%, * P <0.10
[a] Model did not meet the assumptions of the Hausmann test

For the Cobb-Douglas and translog specification the beta estimates should be interpreted as elasticities, while in the quadratic form they indicate the absolute increase in costs due to an increase in one unit of output.

The Cobb-Douglas and the tranlog model show that elasticities for inpatients are higher than for outpatients and that the results are quite similar for cross-section and panel data specification except for the outpatient elasticity being higher in 2006 than in 2004 and 2005. This deviation is probably due to missing data in 2006 as mentioned in the data description.

The beta estimate for the average number of beds changes sign and significance across the model specifications leaving the effect ambiguous. The university hospital dummy in table 1 was eliminated in the fixed effect model 2[7].

The regression results in table 2 are used to estimate the long-run cost function based on the envelope condition, shown together with the direct approach to long-run cost function in table 3. The results in table 2 are also used to estimate the scale elasticities shown in table 4.

Table 3 Regression and calculated result – long-run cost functions

	Translog & Envelope condition	Translog, FE (without beds)	Quadratic & Envelope condition
Intercept	-0.1950	-0.1709***	1.4980
Total DRG value	0.6845	-	2.8092
Total DRG value^2	-0.0043	-	0.3524
Inpatients DRG value	-	0.5388***	-
Outpatients DRG value	-	0.4100***	-
Inpatients DRG value^2	-	0.0879**	-
Outpatients DRG value^2	-	0.1094***	-
Inpatients*Outpatients	-	-0.1483***	-
R^2			
Within	-	0.6880	-
Between	-	0.9834	-
Overall	-	0.9764	-
F-test	-	27.36***	-
Hausmann-test		12,02**	

*** P < 0.01, ** P <0.05%, * P <0.10

Table 3 shows the three long-run cost functions. The first version of the translog function and the quadratic function are calculated from the short-run cost functions in table 2 by substitution of equation (2) into equation (1).

The second version of the translog model is a directly estimated, fixed-effect, long-run cost function. In this model, the total output vector has been divided into two output measures – inpatient and outpatient DRG value – and no cost shifters have been included to avoid collinearity.

The beta estimates of the Cobb-Douglas and translog cost functions are elasticities, whereas the betas of the quadratic model show the absolute increases in costs. The difference in

[7] In an earlier cross-section analysis the university hospital dummy was positively significant for each of the years 2004-2006. This indicates, as expected, that university hospitals incur higher cost, see the method section.

signs between the translog models and quadratic form is due to the logarithmic transformations and the functional form. For example, the negative sign on the interaction term captures elements of cost complementarities between in- and outpatient activity, and the negative intercepts in the translog model will become positive after antilogarithmic transformations.

Table 4 shows estimates of economies of scale for the alternative functional forms when applying the short-run cost and long-run cost functions respectively.

Both short-run and long-run economies of scale are measured by conventional ray scale economies, which are the elasticity of cost taken along a ray that holds product mix constant. SE < 1 implies scale economies and SE >1 implies diseconomies when outputs are changed proportionately.

All scale estimates are calculated for four size groups measured by the number of beds to obtain information on the shape of the cost curve. The policy implication of economies of scale for all size groups is an L-shaped average cost curve where average cost decreases when hospital output increases. This means the cheapest way of operating a hospital system would be to build hospitals that are as large as possible. In the extreme, we would plan one super hospital per region or a single hospital for the entire country. Diseconomies of scale imply that the average cost curve must be U-shaped because average costs increase as a function of output [30].

Table 4 Short-run and long-run scale estimates for hospital production units in Denmark.

	Short-run			Long-run		
Groups of hospitals	Cobb-Douglas	Translog	Quadratic	Translog (envelope condition)	Translog (directly estimated)	Quadratic (envelope condition)
All hospitals	0.7160	0.7086	1.0826	0.6493	0.8907	1.0545
1st quartile	0.7160	0.6235	1.3517	0.6897	0.6948	1.0138
2nd quartile	0.7160	0.6688	1.3277	0.6796	0.8462	1.0927
3rd quartile	0.7160	0.6972	1.2241	0.6716	0.9630	1.2239
4th quartile	0.7160	0.7206	0.9657	0.6674	1.0168	1.3068

The short- and long-run estimates in table 4 indicate that results are dependent on the functional forms. The Cobb-Douglas model and the translog model show significant economies of scale for all size groups and the quadratic form shows constant or decreasing economies of scale.

In the short-run, the translog model expresses decreasing economies of scale and the quadratic model expresses decreasing diseconomies of scale as the size of hospitals are increased.

Overall the models suggest that results of the flexible models depend on the functional form used, while the nested Cobb-Douglas and the translog models yield similar results. However, an important difference is that the translog and quadratic models allow us to make a non-constant scale estimate. The results indicate that scale estimates are increasing with the size of the hospitals, which should be interpreted as an indication of declining economies of scale as hospitals become larger.

However, in view of the major restructuring and centralisation that are to take place in Denmark over the next decade, it is less meaningful to base decisions on short-run scale estimates. Therefore, in the following, our focus is on the long-run scale estimates. The long-run scale estimates in table 4 show scale estimates based on (3) for the three alternative long-run cost curves, taking into account the fact that hospitals do not necessarily use the 'optimal' capital in the short run and that hospitals can change the amount of capital according to the activity level in the long run. Long-run results and short-run results are similar in the sense that the results are sensitive to the functional form. Besides, the directly estimated translog model indicates that results are sensitive to the two alternative long-run estimation approaches.

The two translog models indicate presence of scale economies, while the quadratic form indicates constant or decreasing returns to scale. However, while the findings of the translog model based on the envelope condition indicate that scale effects have a low variation between hospital size groups (0.69 for the largest hospitals and 0.67 for the smallest), the variation shows up as larger when we use the direct approach (0.70 for the smallest to 1.02 for the largest hospitals).

The translog scale estimates for the largest size groups lie around 0.67 or very close to the value 1, equivalent to constant economies of scale in the long term. Thus, there is nothing in the translog models to indicate that the hospitals experience diseconomies of scale in the long run, as the scale estimates are not significantly above the value 1.

Overall, we identify significant to moderate long-run economies of scale when applying two alternative translog cost functions. However, using a quadratic functional form we identify constant economies of scale for the medium-sized sub-groups and decreasing economies of scale for the largest sub-groups.

Figure 1 Long-run economies of scale and size of Danish hospital production units

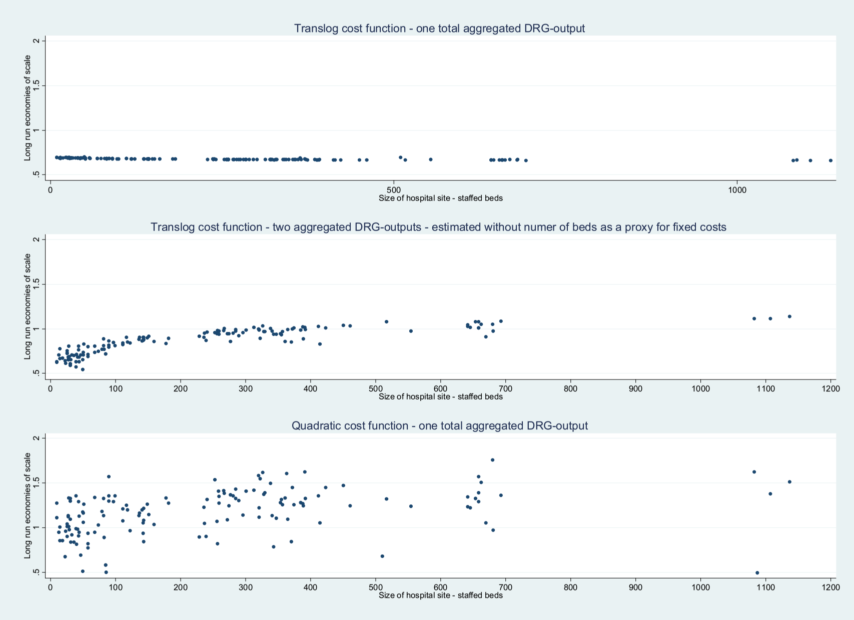


Figure 1 shows the long-run (LR) scale estimates as a function of hospital size for the three different LR model specifications. The figure shows that LR scale estimates for the translog model using the envelope condition lie below 1 for all hospital sizes, whereas they start to exceed 1 for hospitals above around 400 beds in the direct translog LR model. The estimates based on the quadratic form show less correlation between hospital size and LR scale estimates even though a positive trend can be detected with increasing size of hospitals. This increased level of noise probably stems from the lack of compression of outliers in the unlogged model.

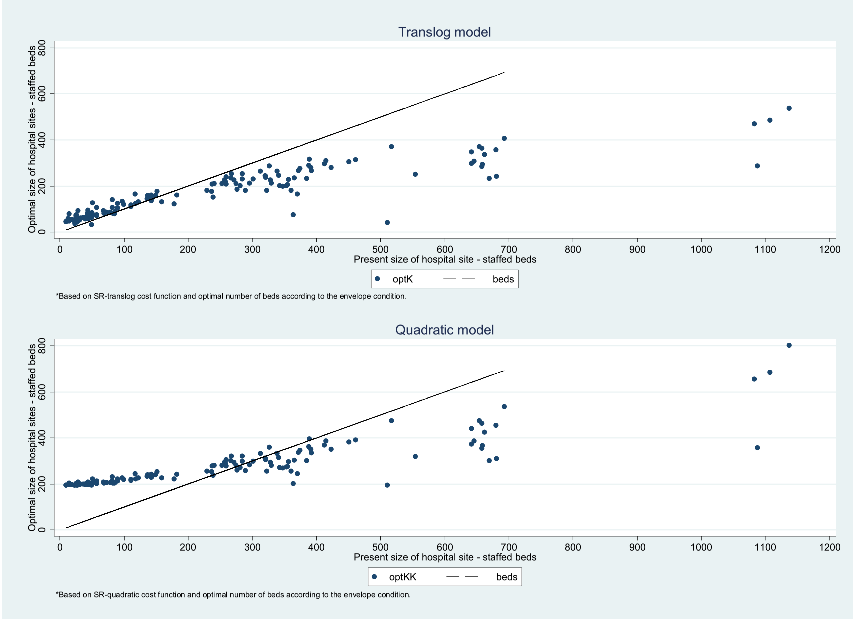
In contrast to table 3, which showed scale estimates for the median hospitals data in each size groups (representative units), figure 1 shows the short-run scale characteristics for all observed hospitals. The smallest quartile has estimates in the range of 9-50.9 beds, while the following three size groups (2nd quartile, 3rd quartile and 4th quartile) have observations in the intervals 50.9-229.0, 229.0-356.6 and 356.6-1136.7 beds.

Finally, figure 1 shows that there are three outlier observations that we did not find any arguments to exclude and that there are relatively few observations among the largest hospital production sites.

Optimal hospital size

The estimation based on (1) and the calculation of an 'optimal' hospital size based on (2) yields 204.9 beds for the median Danish hospital in the translog model and 275.2 beds for the quadratic functional form.

Fig.2 Optimal sizes of a Danish hospital production unit as a function of size – short-run*



***The number of beds has been used to illustrate the size of hospitals instead of the DRG values**

In figure 2 the estimated optimal hospital size is shown as a function of present size. Using the 45 degree line as point of departure, the figure illustrates how the 'optimal' size of each hospital deviates from the present size ('45º' line). The results of both models indicate that small and medium-sized hospitals with less than 204 or 275 beds are too small, while the larger hospitals are too large. However, it is not evident whether, for example, 'small is too small' in the translog model, since optimal and actual sizes are not different, at least not statistically. Both results are in line with the above-mentioned literature review by Aletras & Jones, which points to optimal sizes of hospitals as being between 200 and 400 beds. Another example is a recent Canadian study, which estimated an 'optimal' value of 179. 5 beds [9].

Applying the 'directly' estimated long-run cost function, it is not possible to derive an optimal hospital size through (2). However, the long-run scale estimates in table 4 can be analyzed to reveal whether they indicate an 'optimal' hospital size (*SE1*=1). The results indicate that economies of scale have been exploited for the largest quartile (357-1137 beds), and that these hospitals are facing constant economies of scale where no economic gains can be obtained by centralization.

From an econometric perspective, the quadratic form for estimates of 'optimal' hospital size is preferred to the translog model. This is due to transformation problems in the translog model, which may result in biased cost estimates as well as biased estimates of 'optimal' hospital sizes [31,32]. To avoid these transformation issues (i.e. the introduction of approximate correction factors), we choose to use the quadratic form to construct confidence intervals for 'optimal' hospital sizes.

Table 5 Sensitivity analysis for estimate of optimal hospital size

Model parameters	95% confidence intervals Quadratic model (275 beds)
Average number of beds	130.0 – 419.9
(Average number of beds)^2	179.9 – 585.3
Total DRG value * Average number of beds	235.2 – 315.2

Table 5 shows how the estimated 'optimal' size (275 beds) is sensitive to partial changes in each of the estimated parameters included in the envelop condition (2). According to the estimated model, changes in the parameter estimate of the average number of beds within the 95% confidence interval can result in 'optimal' size estimates in the range 130.0 to 419.2 beds per hospital. Finally, including the intersection of possible partial changes in each of the model parameters indicates that the 'optimal' number of beds is estimated to be in the interval 130 to 585.3 beds per hospital.

4. Discussion

This study shows that parametric estimates of economies of scale and of the optimal size of public hospitals in Denmark in line with other studies are sensitive to the chosen model specification.

Theory makes assumptions about first and second order effects of cost functions, i.e. marginal cost, scale elasticity and the envelope condition. This study is based on the relatively well-known properties of parametric flexible cost functions guided by neoclassical production

theory [33]. Still, due to the fact that input prices were assumed to be constant, this study was limited to adopting a second-best practice where it was impossible to test all cost function regularity conditions, see for instance, [6].The translog cost function, which has become a standard approach, has been applied in many studies of economies of scale, for instance, [15,18,34] and in a number of studies of hospital efficiency [17]. In the same way, the Cobb-Douglas function is a standard functional form, but it has been less used to estimate economies of scale since the development of the more flexible functional form, such as the translog model [35].

The only functional form that is not very frequently used is the quadratic form, even though it is a flexible functional form like the translog model [9]. This might be due to the fact that the coefficients are more difficult to interpret in the quadratic form. On the other hand, the quadratic form has the advantages that it does not require logarithmic transformation, which can result in a transformation problem regarding the dependent variable if heteroscedasticity is present in the model [32].

From one perspective, the translog and quadratic functional form have the great advantage that they are flexible forms, which minimizes the risk of misspecification. From another perspective, however, increased flexibility requires an increasing number of parameters to be estimated. The last argument is a real issue in small countries where the number of observations always will be limited. The small number of observations means that only a small number of aggregate output measures can be applied in flexible functional form, despite the fact that hospitals have multi-dimensional outputs. Secondly, flexible functional forms have a latent multicollinearity problem, which is likely to be more severe in a 'small country' context, where the number of hospitals is small, [6,19,36].

The structural model requires all parts of the flexible functional model, including squared and interactions terms, to be included, so that the model can capture varying economies of scale. Therefore, as mentioned above and found in [36], it is not possible to include more than one or two different output measures without getting multicollinearity, which, of course, leaves open critical issues regarding collapsing the multidimensional output vector, see, for instance, [20]. In order to minimize these obstacles, the approach used in this study has been to aggregate the output vector to one aggregated output and to use only two cost shifters in order to have sufficient degrees of freedom left for model estimation without getting more collinearity than the nature of the flexible forms imposes through squared and interactions terms. Besides, the unit of analysis has been defined as hospital production units instead of conglomerate hospitals. This is done both to define the unit of analysis relevant for policy and to increase the number of observations that are higher for hospital production units than conglomerate hospitals.

The above-mentioned approach, which uses only one output index, is debatable. From one point of view, the multi-dimensional output can only be aggregated if the original dimensions are broad isoresource categories [9]. From another point of view, it can be argued that this is exactly what the DRG system allows.

Furthermore, the Cobb-Douglas model has been applied to try to avoid multicollinearity. The lack of flexibility in the Cobb-Douglas model implies that fewer parameters have to be estimated than in the 'flexible' cost functions. From one point of view, this may be preferred in situations where only a relatively small number of degrees of freedom are available as in the 'small country' case. From another angle, use of the Cobb-Douglas model implies constant scale elasticities, which decrease flexibility and increase the risk of misspecification. The inflexibility of the Cobb-Douglas model also shows that it cannot be used to calculate 'optimal' scale and size as a function of size or activity, as can be seen from table 4 where the short-run scale estimates become constant. The implication is that the Cobb-Douglas only can be used as a supplementary measure, as was the case in this study.

A time wise bigger panel would give more degrees of freedom and minimize multicollinearity through more variation in data, although it would not solve the whole problem if the output vector was broken down into homogeneous resource groups. The amount of extra parameters would increase relatively more than the degrees of freedom gained from extra years of panel data. Besides, as mentioned in the data section, missing data for 2006 may have influenced the results compared to an ideally balanced panel.

In this paper we used only three years of panel data. The reason is that changes in DRG groupers, DRG values, and allocation of costs and other structural changes mean that it is difficult to compare more years than the three used here, Danish Regions, the Ministry of Finance [37].

Another consequence of the above mentioned "small country" problem is that the number of observations is too small to conduct specifications tests to determine a "correct" functional form. Outliers will simply influence too much. Instead the results of several functional forms are presented to "measure" sensitivity to different functional forms.

While the present study assumes that the quality of hospital output is homogenous across the hospitals, it is realized that this undoubtedly is a crude generalisation. The reasoning behind this assumption is a combination of a lack of recognized and objective approaches to adjust for differences in structural, process and outcome related parts of quality [38,20,39]. Furthermore, the explanation is that it is difficult to secure a sufficient amount of degrees of freedom to include more variables to adjust for quality.

The literature indicates that researchers should include variables to describe hospital products in terms of, for example, quality, patient characteristics and institutional conditions in order to avoid bias due to omitted variables. On the other hand, empirical studies show that quality data, for example, can be insignificant in hospital cost functions, despite the belief in some quarters that better quality comes at a cost [17].

From an economic point of view, the ideal definition of total cost is based on an opportunity costs approach. However, this approach was not feasible since the Danish National Board of Health has adopted an accounting approach to costing. . There are no available data based on the opportunity cost approach. The more correct economic approach requires calculation of both amortizations, interests on debt capital and interest on equity capital. The cost of debt capital can be observed, but the cost of equity and the market based value of assets needs to be estimated from "market" data and managerially determined discount rates, which were unavailable in this study. As a consequence of the lack of a monetary measure of the capital stock this study uses the number of beds as a proxy for fixed cost. This is due to the fact that fixed costs are assumed to be driven by the number of beds in each hospital.

As mentioned earlier, in the literature the number of beds per hospital has become a standard method to measure hospital size. However, it could be claimed that increasingly the number of beds captures size imperfectly as outpatient treatment and same-day surgery dominates hospital activity. In a long-run perspective, this is due to a trend towards substitution of hospital outpatient care for inpatient care as a measure initiated in the beginning of the 1970's to reduce rising hospital costs, see, for instance, [40]. The number of non-critical care medicine beds in particular has been declining. Despite the criticism, we did not find a more appropriate and recognized approach to measuring hospital size.

According to a trend, both Danish and international, towards relatively more outpatient activity, the number of beds as a proxy for fixed cost and hospital size is debatable, [41-43]. The relative decrease in number of beds is primarily due to new technologies which allow more outpatient activity. Therefore, fewer beds do not necessarily mean less activity, seen from a marginal perspective. In the future, it may be more appropriate to use different measures for fixed costs and hospital size.

So far, we have used the standard method to be able to compare our results with the literature. Furthermore, since we do not know the monetary capital costs, the meaning for the results in this study of the choice between the applied physical measure and the monetary measure of total cost is ambiguous.

It is not realistic to assume that hospitals can adjust all their inputs quickly as was the case in the 'direct approach' [44]. Most studies indicate that hospitals cannot adjust all inputs quickly when output levels or factor prices change

Consequently, it is probably more reasonable to assume that hospitals only apply optimal quantities of the most easily adjustable variable inputs such as manpower and medical supplies, given likely non-optimal levels of fixed inputs (measured in terms of beds). Therefore, it is more appropriate to estimate short-run cost functions and exploit the envelope condition to derive the long-run cost function. A test for long-run equilibrium exists [16]. To use this test, however, input prices should be available. In this study prices are assumed to be constant.

For the short run Cobb-Douglas model and the directly estimated long-run translog model, the Hausman test confirmed that the fixed effect model is preferred to the random effect model. This indicates that there is correlation between the individual effects and the covariates. As shown in table 1, it was not feasible to conduct the test for the short-run translog and quadratic models. Both models failed to meet the assumptions of the Hausman test. Hence, the fewer assumptions of the FE model led us to choose to use the FE rather than the RE model. Even though the fixed effect model was preferred, it still has some disadvantages. One example is that time invariant effects cannot be estimated *per se*. This prohibits relevant covariates being included directly, such as, teaching or university status, which are believed to be structural cost driver in hospitals. However, FE models indirectly correct for individual differences through the fixed effects. In addition, optimally consistent estimators require that both time series and cross-sectional dimensions go to infinity [25].

It could be argued that the old technology is not representative for the future technology in new hospitals. The reason is that Danish hospital planners expect a significant one time productivity gain from the rebuilding programme, e.g. better logistics and efficient patient pathways in the new hospitals. Planners expect that length of stay will continue to decrease [45].

It is feasible to test whether the short-run scale estimates in table 4 are significantly different from one using the delta method or bootstrapping, see for instance, [46,47]. Since we are focusing on the long-run scale estimates we have omitted these tests.

Since the FE approach is a version of regression analysis, cost function estimation usually requires that it is assumed that hospitals apply variable input in a cost-minimizing way, or in other words that hospitals are on the cost frontier (and that they cannot alter the number of beds in the short run). The present approach which assumes constant prices does not take into consideration the possibility of allocative inefficiency in the hospital's production. Besides, it can be questioned whether the FE approach correctly accounts for technical efficiency since, for instance, the unmeasured time invariant cross firm heterogeneity must be assumed away [48]. In

the present study, individual effects were not used to calculate technical inefficiencies. The estimated scale estimates and optimal hospital size are solely based on the interpretation of slopes in (1). On the other hand, Mester [49] among others has concluded that whether or not a firm is on its efficient frontier does not appear to exert much influence on the quality of the estimation of economies of scale.

5. Conclusion

Overall this study shows that, when applying two alternative specifications of translog cost functions, there are significant to moderate long-run economies of scale for all size groups of Danish hospitals in 2004-2006. These results indicate an L-shaped unit cost curve. However, using a quadratic form, this study identifies constant economies of scale for the medium-sized sub-groups and decreasing economies of scale for the largest sub-groups. This illustrates a U-shaped unit cost curve.

According to the scale results it may be appropriate to consolidate the production of small hospitals on fewer units from a cost point of view. This may imply merging the small hospitals in the smallest groups with hospitals in the medium-sized groups. In both cases, it is necessary to take into account the other determinants that may be important for whether it is appropriate to concentrate hospital production, such as the need for a local emergency facility, transport costs and opportunity costs through increased travel time.

Furthermore, the 'optimal' number of beds per hospital is estimated to be 275 beds per site within a 95% confidence interval between 130 to 585 beds per hospital. This is roughly in line with international results.

The analysis precludes drawing conclusion about the consolidation of hospitals leading to hospital sizes exceeding 1200 beds, because it is outside the range of data used here. In other words, it is not known whether the unit cost will decline (be L-shaped) or it will be U-shaped when hospital size increases above 1200 beds. Overall, this study supports the hypothesis that there may be cost advantages (or no disadvantages) for the smallest sub-group in producing hospital services in larger hospitals in the Danish hospital system than was the case in 2004-2006. However, policy conclusions should not be drawn solely on the basis of this study, which is solely based on panel data for the years 2004-2006. The findings reported here should be investigated further, for example, through the use of other/supplementary data, alternative model specifications, alternative estimation methods such as data envelopment analysis (DEA), and estimation of potential efficiency gains from consolidation.

Appendix I

(1) Short run Quadratic (1a), Translog (1b) and Cobb Douglas (1c) cost functions:

$$C(Q_T,K) = \beta_0 + \beta_1 Q_T + \tfrac{1}{2}\beta_2 Q_T^2 + \beta_3 K + \tfrac{1}{2}\beta_4 K^2 + \beta_5 Q_T K \qquad (1a)$$

$$\ln C(Q_T,K) = \beta_0 + \beta_1 \ln Q_T + \tfrac{1}{2}\beta_2 \ln Q_T^2 + \beta_3 \ln K + \tfrac{1}{2}\beta_4 \ln K^2 + \beta_5 \ln Q_T \ln K \qquad (1b)$$

$$\ln C(Q_T,K) = \beta_0 + \beta_1 \ln Q_1 + \beta_1 \ln Q_2 + \beta_3 \ln K \qquad (1c)$$

(2) The optimal hospital production unit size measured in terms of beds is calculated from the short run cost functions (1a-c) by application of the envelope condition:

$$\frac{\partial C(Q_T,K)}{\partial K} = \beta_3 + \beta_4 K + \beta_5 Q_T = 0 \quad \Rightarrow K = \frac{-\beta_3 - \beta_5 Q_T}{\beta_4} \qquad (2a)$$

$$\frac{\partial \ln C(Q_T,K)}{\partial K} = \beta_3 + \beta_4 \ln K + \beta_5 \ln Q_T = 0 \quad \Rightarrow \ln K = \frac{-\beta_3 - \beta_5 \ln Q_T}{\beta_4} \Rightarrow K = e^{\left(\frac{-\beta_3 - \beta_5 \ln Q_T}{\beta_4}\right)} \qquad (2b)$$

$$\frac{\partial \ln C(Q_T,K)}{\partial K} = \beta_3 = 0 \quad \Rightarrow \quad unfeasible \qquad (2c)$$

Calculation of long run cost function

The long run cost function is calculated from the short run cost function (1a,b) by substitution of the optimal number of beds (2a, 2b respectively) derived by the envelope condition (2). For the long run Quadratic cost function this yields:

$$C(Q_T) = \beta_0 + \beta_1 Q_T + \tfrac{1}{2}\beta_2 Q_T^2 + \beta_3\left(\frac{-\beta_3 - \beta_5 Q_T}{\beta_4}\right) + \tfrac{1}{2}\beta_4\left(\frac{-\beta_3 - \beta_5 Q_T}{\beta_4}\right)^2 + \beta_5 Q_T\left(\frac{-\beta_3 - \beta_5 Q_T}{\beta_4}\right)$$

After mathematical reduction the long run cost function can be reduced to:

$$C(Q_T) = \beta_0 + \beta_1 Q_T + \frac{\beta_2 Q_T^2}{2} - \frac{(\beta_3 + \beta_5 Q_T)^2}{2\beta_4}$$

We omitted the long run Translog cost function since the only differences from the above mentioned quadratic cost function is that the total DRG-production value Q_T is replaced by logged levels.

Appendix II

Calculation of long run economies of scale

The expression for long run economies of scale (3a, 4a) is calculated from the long run cost function (2a) and (3, 4 respectively).

(3) Long run economies of scale - Translog cost function

$$SE1 = \frac{\partial \ln C(Q_T)}{\partial \ln Q_T} = \underbrace{\beta_1 - \frac{2\beta_3\beta_5}{\beta_4} + \frac{\beta_3\beta_4\beta_5}{\beta_4^2}}_{Intercept} + \underbrace{\left(\beta_2 + \frac{\beta_4\beta_5^2}{\beta_4^2} - \frac{2\beta_5^2}{\beta_4}\right)}_{slope}\ln Q_T \qquad (3a)$$

By mathematical reduction expression (3a) can be reduced to the following:

$$SE1 = \frac{\beta_1\beta_4 - \beta_3\beta_5 + \left(\beta_2\beta_4 - \beta_5^2\right)\ln Q_T}{\beta_4} \qquad (3a)$$

(4) Long run economies of scale - Quadratic cost function

$$SE2 = \frac{\partial C(Q_T)}{\partial Q_T} \Big/ \frac{C}{Q_T} = \left(\underbrace{\beta_1 - \frac{2\beta_3\beta_5}{\beta_4} + \frac{\beta_3\beta_4\beta_5}{\beta_4^2}}_{Intercept} + \underbrace{\left(\beta_2 + \frac{\beta_4\beta_5^2}{\beta_4^2} - \frac{2\beta_5^2}{\beta_4}\right)}_{slope} Q_T\right) \Big/ \frac{C}{\underbrace{Q_T}_{point}} \qquad (4a)$$

By mathematical reduction expression (4a) can be reduced to the following:

$$SE2 = \left(\frac{\beta_1\beta_4 - \beta_3\beta_5 + \left(\beta_2\beta_4 - \beta_5^2\right)\ln Q_T}{\beta_4}\right) \Big/ \frac{C}{Q_T} \qquad (4a)$$

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