Influence of Alkali-Silica Reaction on the Shear Capacity of Reinforced Concrete Slabs Without Shear Reinforcement

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Influence of Alkali-Silica Reaction on the Shear Capacity of Reinforced Concrete Slabs Without Shear Reinforcement

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April 4, 2019
Preface

This thesis has been prepared as a partial fulfilment of the requirements for obtaining the PhD degree at the University of Southern Denmark. The research has been undertaken at the Section for Civil and Architectural Engineering under supervision of Professor Linh Cao Hoang.

Søren Gustenhoff Hansen
Odense, Denmark, April, 2019

Preface to published version

The thesis was defended on Wednesday 3rd of July 2019. Subsequently, the PhD degree was awarded by the University of Southern Denmark.

Søren Gustenhoff Hansen
Odense, Denmark, July, 2019

The bridge on the cover is Lindenborg Pæledæk. The photo is taken by Claus Pedersen, Ramboll. The cover is designed by Anja Kunic.
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Thanks to my friend and colleague Assistant Prof. Roberto Naboni for his transmitting enthusiasm and for making even late-night procrastinations to be about concrete and espresso.

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I gratefully acknowledge the Danish Road Directorate for making the unique experiments with beams from real ASR-damaged bridges possible.

My thanks also go out to all colleagues from the department of Civil and Architectural Engineering at University of Southern Denmark and all former bachelor and Master students whom I have had the pleasure to share my passion for ASR with.

Since concrete structures is a small research field in Denmark, my PhD colleagues from DTU and Aarhus University have been important for my PhD journey. Thank you all for the stimulating discussions, travels around the globe that I will never forget and for sharing your passions for concrete structures with me.
I will like to send a special thanks to my girlfriend and my son. To my girlfriend for always having time to provide an unsentimental constructive review of my work and for always being able to make me smile. You have been an outstanding support - you are the best. To my son for reminding me that there are after all other important things in the world than concrete structures.

Last but not the least, I would like to thank my parents for always supporting me regardless of my requests.
Abstract

Alkali-silica reaction (ASR) is a deterioration mechanism that can occur in concrete structures. It is a chemical reaction between alkalis, silica minerals in the reactive aggregates and water. The reaction causes severe cracking of the concrete, which results in significant reductions of the strength parameters. This material degradation has raised serious concerns regarding the safety of ASR-damaged structures; particularly structures, which may be sensitive to shear failure. The Danish Road Directorate has estimated that more than 600 Danish road bridges have the potential to develop ASR in the future. The majority of these bridges has been constructed as slabs without shear reinforcement, i.e. structures where the shear capacity relies entirely on the strength of the concrete. Unfortunately, there exists no satisfactory method to assess the residual shear capacity of ASR-damaged slabs without shear reinforcement - in spite of nearly 80 years of research on ASR.

The aim of this PhD project is therefore to develop an approach that can be used to determine the shear capacity of ASR-damaged slabs without shear reinforcement. The approach includes a shear model as well as recommendations and descriptions of how the relevant strength parameters should be determined by simple tests on samples taken from the structure. The works that have been undertaken to develop this approach are as follows.

In the first part of the project, a literature study on how ASR affects the parameters that are important for the shear capacity is conducted. One of the main findings here is that ASR affects slabs differently than other types of structures, e.g. the way that the ASR-induced cracks are orientated. The majority of the existing ASR research on material characteristics and/or residual capacity of reinforced members is therefore not directly applicable for this PhD project. Based on the findings as well as shortcomings in the existing literature, a number of research questions that need answers in order to develop a shear model for ASR-damaged slabs are formulated.

In the second part of the project, answers to the formulated research questions are found by means of a thorough experimental investigation, where the effects of ASR on the material properties as well as on the structural response are studied. The investigation includes a large shear testing campaign with specimens cut out from two ASR-damaged bridges. The material properties are investigated by means of standard test methods and Digital Image Correlation (DIC). By a critical examination of the results and an optical investigation of the underlying mechanisms, recommendations of testing methods to obtain the anisotropic residual compressive- and tensile strength are formulated.
In the last part of the project, a model to determine the shear capacity of ASR-damaged slabs without shear reinforcement is established. The model is based on the upper bound theorem of plasticity theory, where the specific solutions are derived with inspiration from the failure mechanisms observed in shear tests with the ASR-damaged slab bridge specimens. The calculated shear capacity correlates well with test results, both for simply supported members and for continuous members. Based on the model, some recommendations are given for how practical assessment of members subjected to arbitrary loading can be carried out.
Resumé


anisotropiske træk- og trykstyrke skal testes.


Modellen er udviklet med udgangspunkt i bjælker med en eller to punktlaster. Sidst i afhandlingen er der givet anbefalinger til, hvordan modellen kan anvendes til bjælker, der er udsat for andre belastningstyper.
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Chapter 1

Introduction

1.1 Background information on ASR

Alkali-Silica reaction (ASR) is the most frequent deterioration mechanism in concrete structures in the category of Alkali-Aggregate reactions (AAR). Ever since Stanton (1940) discovered ASR-induced cracking in concrete structures in California in the 1930s, ASR has been a subject of major concern worldwide, especially for bridges and dams (Charlwood, 1994; Wigum, 2006).

ASR is a chemical reaction between alkali hydroxides (e.g. Na$_2$O or K$_2$O) and silica minerals in reactive aggregates. The primary sources of alkalis are cement (K$^+$) and de-icing salt (Na$^+$). A secondary source of alkalis may be certain kinds of stones, or the presence of residual sea-salt from poorly washed sea-dredged stones (Godart et al., 2013).

When alkali hydroxides react with silica minerals, an inner expansion will occur. This inner expansion causes tensile stresses in the surrounding cement paste. When the tensile stresses exceed the tensile strength of the cement paste, cracks will develop around the reactive particle (Wigum, 2006). This cracking is often referred to as ASR damages.

The ASR damages have a significant effect on the durability and the load-carrying capacity of the structure. Firstly, the ASR damages result in (anisotropic) reduction of the mechanical properties of the concrete, such as the compressive and tensile strength and the stiffness (Hobbs, 1988). Secondly, the ASR-induced expansions may cause a pre-stressing in the reinforcement bars (see e.g. (McLeish, 1990)). Thirdly, the presence of ASR-induced cracks ease the access for other deteriorating mechanisms such as chloride penetration and freeze-thaw (Wigum, 2006).

The (anisotropic) material degradation rises questions regarding the residual load-carrying capacity of ASR-damaged structures. The main concern is directed towards failure mechanisms, where the load-carrying capacity relies (almost) entirely on the concrete strengths, such as shear- or punching shear failures in structures without shear reinforcement (Bakker, 2008).
1.2 ASR damages in Denmark

Even though Stanton published his discovery of ASR in concrete structures already in 1940, the knowledge about ASR was first brought to Denmark by Paul Nerenst a decade later (Nerenst, 1952). Despite a persistent effort, it was first when a new standard for concrete was published in 1987 (Byggestyrelsen, 1987) that a sufficiently low limit for the amount of reactive aggregates was introduced, and thus putting an effective stop for ASR in new structures. Concrete structures in Denmark built before 1987 may therefore contain a critical amount of reactive aggregates, and therefore have the potential to develop ASR damages in the future; this especially holds for road bridges where de-icing salt is spread in the winter. A screening of the Danish bridges shows that more than 600 bridges have the potential to develop ASR damages (Larsen, 2013). A large part of these are constructed as slabs without shear reinforcement, i.e. the type of structures which are expected to be especially vulnerable with respect to ASR damages. This type of structure will be the main concern of this PhD thesis.

1.3 Problem statement

Despite the concerns and uncertainties regarding the residual shear capacity of ASR damages structures, most of the ASR-related research has been focused on the causes and prevention of ASR rather than on the development of models and methods for strength assessment of existing structures. As a result, there exists no commonly accepted model or method to determine the shear capacity of ASR-damaged slabs without shear reinforcement. This is a problem of great importance that needs to be addressed. To the author’s knowledge, only a few approaches exist for this purpose. These are: (i) application of models developed for sound concrete employed with the ASR-affected values of the mechanical properties of the concrete; (ii) a model recommended by CUR-recommendation 102 and; (iii) a numerical model developed by Saouma et al. (2016). In the following sections, each of these approaches will be addressed and discussed briefly.

1.3.1 Models for sound concrete

The most common approach for assessment of ASR-damaged structures is probably the application of existing models for sound concrete employed with the ASR-affected values of the mechanical properties of the concrete, determined by samples taken from the structure. Due to the lack of more tailored models, this approach is explicitly mentioned by several guidelines on the assessment of ASR-damaged structures.

- The Institution of Structural Engineers (1992) states that ”the shear capacity of an ASR-damaged beam can be estimated considering the influence of the compressive stress resulting from the pre-stressing effect from the restraint. The mechanical properties of ASR-affected concrete should be used in making this estimate.”

2
• Jones and Clark (1996) state that "the assessment of structures affected by ASR can be carried out using normal design equations providing: (i) Concrete properties representing the deteriorated condition of the concrete are used. (ii) The changes in the relationship between compressive strength, tensile strength and Young’s modulus are considered. (iii) Allowance is made for the effects of prestress of reinforcement by the expansion.”

• As an option, the Dutch Guidelines on the assessment of ASR-damaged structures (CUR-Recommendation 102, 2005) recommends the application of the model for shear strength of non-prestressed structures from the Dutch standards (at the time NEN7620).

Neither the Institution of Structural Engineers (1992) nor Jones and Clark (1996) mention how the mechanical properties of the ASR-damaged concrete should be obtained. On the contrary, CUR-Recommendation 102 (2005) does provide clear guidance on how the ASR-affected tensile strength needs to be determined on the basis of tests. It is however acknowledged that their provided approach results in a conservative estimation for the shear capacity.

Even though the application of models for sound concrete employed with the measured values of the ASR-affected material properties may lead to reasonable (or at least conservative) results, it is questionable if this approach is indeed suitable for assessment of ASR-damaged structures. The (empirical or mechanical) models derived for sound concrete rely on certain assumptions and/or experimental backgrounds. Mechanical models derived for shear (e.g. the one presented by Fisker and Hagsten (2016)), typically assume a certain, predefined shape of the critical shear crack. Since ASR damages are known to influence the crack progression, it is questionable if this shape is also obtained for ASR-damaged slabs. Similarly, the empirical shear strength equations for slabs without shear reinforcement (e.g. the one presented by the Eurocode (CEN, 2004)) are derived on the basis of experiments on sound concrete only. It is questionable if these experimental results can be extrapolated to ASR-damaged structures too. Both the mechanical and empirical methods assume isotropic concrete properties, which are known to be incorrect in the case of ASR-damaged structures.

1.3.2 CUR-recommendation for slabs

The second approach for assessment of ASR-damaged slabs without shear reinforcement can be found in the CUR 102 recommendation (CUR-Recommendation 102, 2005), which describes an engineering model to determine the shear capacity of ASR-damaged slabs. The model applies to concrete structures in the Netherlands which are (i) plate-shaped structures (plates, walls) with a thickness of at least 400 mm; (ii) have a bidirectional, in-plane reinforcement; (iii) with cracks parallel to the surface and; (iv) with an anisotropic uniaxial tensile strength. The model is derived from an experimental study on the shear capacity of a number of (severely) ASR-damaged beams that were cut from bridge decks.
Both the experiments and the derivation of the model are described in (Den Uijl and Kaptijn, 2002).

The model assumes that failure occurs when shear-induced cracks form, i.e. when the load-induced principal stresses ($\sigma_1$) exceed the tensile strength of the concrete ($f_t$):

$$\sigma_1(\tau, \sigma_p) \leq f_t$$ (1.1)

The principle stresses are a function of the load-induced shear stresses ($\tau$) and the (potentially) ASR-induced prestressing ($\sigma_p$). Given a tensile strength $f_t$ and a prestress ($\sigma_p$), the maximum admissible shear stress can be obtained. Accounting for the anisotropic behaviour of the tensile strength, the shear capacity (expressed by the average shear stress) can be found by:

$$\tau_{1,m} = \frac{I}{dS} \sqrt{f_{ct,0,ref}(f_{ct,90} + \sigma'_{bmd})}$$ (1.2)

where $I$ is the moment of inertia for the uncracked cross section, $d$ is the depth of the cross section, $S$ is the first moment of the cross-sectional area, $\sigma'_{bmd}$ is the average prestress-induced compressive stress in the concrete (positive for compression), $f_{ct,0,ref}$ is the uniaxial tensile strength of the concrete perpendicular to the axis of the structural component (vertical for a slab) and $f_{ct,90}$ is the uniaxial tensile strength of the concrete parallel to the axis of the structural component (horizontal for a slab).

To the author’s knowledge, the model proposed in CUR 102 is the only engineering model, which is tailor-made for assessment of the shear capacity of ASR-damaged slabs. However, the model assumes a shear failure directly after the formation of shear cracks, and it should be acknowledged that it misses certain essential post-crack shear transferring mechanisms, such as aggregate interlock and shear transfer in the compressive zone (see Section 2.2). As was also stated by Bakker (2008), the CUR-model can therefore, only be considered as a good start for the assessment of ASR-damaged concrete slabs.

1.3.3 Saouma et al. (2016)

The third and probably most advanced approach for the determination of the residual shear capacity of ASR-damaged slabs is the one proposed by Saouma et al. (2016). The approach consists of two phases. The first phase regards the determination of the (time-dependent) ASR-affected concrete mechanical properties on the basis of a thermo-chemo-mechanical model proposed by Saouma and Perotti (2006). The concrete mechanical properties are assumed to be a function of the (time-dependent) environmental characteristics (humidity, temperature) and the (time-dependent) restraints present in the concrete. In the second phase, the shear capacity is determined using the concrete properties determined in the first phase and the employment of an FEM model.

The method proposed by Saouma et al. (2016) takes all facets of ASR into account; from chemical reaction to concrete expansion, which leads to crack formation and material
degradation. The method is therefore most likely capable of adequately predicting the residual shear capacity. However, the model relies on a large amount of (time-dependent) input-parameters, which are hardly available to engineers in practice. Its feasibility for practical applications is therefore questionable.

1.4 Objectives of PhD-project

At this stage, no satisfactory model exists for prediction of the residual shear capacity of ASR-damaged slabs without shear reinforcement. The models derived for sound concrete are commonly applied, but rely on assumptions and/or experimental backgrounds which are often not representative for ASR-damaged structures. The model proposed in the CUR-Recommendation 102 (2005) is derived especially for ASR-damaged slabs, however, it misses certain essential shear transferring mechanisms. The method proposed by Saouma et al. (2016) is adequate in its predictions, but its feasibility in practice is questionable due to the difficult nature of the model and the large amount of required (time-dependent) input-parameters.

The objective of this PhD thesis is therefore to develop an engineering approach for the prediction of the residual shear capacity of ASR-damaged slabs without shear reinforcement. The approach should:

- be mechanically based, such that the failure mechanisms are understandable for the engineers, and the prediction can be generalized to structures outside of the experimental background;
- adequately account for the relevant shear-transferring mechanisms, such as aggregate interlock, shear transfer in the compressive zone and dowel action (see also Section 2.2);
- be easy in application, without the need of special software or the knowledge on the history of the relevant parameters;
- adequately account for the ASR-induced cracks in the determination of critical shear crack;
- adequately account for the anisotropic mechanical properties of the ASR-damaged concrete and;
- provide guidance on the experimental determination of the required mechanical properties.

Limitations:

As the focus of this thesis lies on the mechanical properties of ASR-damaged concrete and their effects on the residual shear capacity, the chemical and physical mechanisms of ASR will not be investigated or addressed further. For this it is referred to established...
1.5 Methodology and outline

The research has been carried out in three phases, which has resulted in a threefold thesis: (Part I) a literature study and explanation of ASR-induced cracks in slab members; (Part II) an experimental investigation and; (Part III) model development. Based in the three main parts, guidelines for practical applications are also provided at the end of the thesis.

Part I - Literature study and ASR-induced cracks in slabs:

Chapter 2 presents the literature study where the state-of-the-art knowledge on topics relevant for assessment of ASR damages structures is described, and gaps in the literature are identified. Based on the findings as well as the shortcomings in the existing literature, research questions that need answers in order to develop a shear model for ASR-damaged slabs are formulated.

Section 2.1 investigates the cracking and expansion behaviour of ASR-damaged reinforced concrete structures. It is investigated how exposure conditions, restraining conditions and surface layers influence the crack-patterns and the (anisotropic) expansion behaviour, and it is explained why the cracking and expansion behaviour in slabs differ from other types of concrete structures. Based on the findings, it is discussed which results and experiments from the literature that can be considered representative for slabs without shear reinforcement.

Section 2.2 investigates which parameters should be taken into account for the determination of the shear capacity of ASR-damaged reinforced slabs without shear reinforcement. For this, a literature review is conducted on existing shear models derived for sound concrete. It is found that the concrete compressive strength, the concrete tensile strength, the prestressing, shape of the critical shear crack and the amount of longitudinal reinforcement are essential parameters. Sections 2.3 to 2.6 subsequently investigate how each of these parameters are influenced by ASR-damages.

The final subject studied in the literature is the existing experimental investigations on the shear capacity.

Besides the literature study, Part I also provides a phenomenological explanation of ASR crack growth in reinforced slabs (see Chapter 3).
Part II - Experimental investigations:

In the literature study, research questions regarding the concrete compressive strength, concrete tensile strength and the shape of the critical shear crack are raised. To answer these research questions, an experimental investigation is conducted in the second part of the thesis. The experiments are conducted on beams cut from two Danish slab bridges with severe ASR damages. To identify the influence of ASR, reference experiments are conducted on new-cast concrete without ASR damages. The experiments are evaluated on the basis measurements and Digital Image Correlation (DIC).

Chapter 4 provides an introduction to the two bridges, where the concrete beams for testing are cut from. Chapter 5 and Chapter 6, respectively, provide an experimental investigation of the compressive and the tensile strength. Chapter 7 deals with the experimental investigation of the shear capacity.

Part III - Model development:

In the third part of the thesis, a model for the shear capacity is developed on the basis of the upper bound theorem of plasticity theory. The benefit of using upper bound solutions is that they are based on failure mechanisms that are observable in experiments. In Chapter 8, the shear model is developed and compared with experimental results. Chapter 9 concludes the findings of the thesis and provides guidance to practical application of the model.

1.6 Published research and cooperations

Parts of this thesis are already published in scientific papers. Appendix A provides an overview of the connection between the thesis and the publications. Some of the experimental work presented in this thesis has been conducted in cooperation with colleagues. Appendix A provides a list of these works as well as the individual contributions.

The following research works have not yet been published:

- The analysis of the compressive strength in ASR-damaged slabs (Section 5.4).
- The investigation of the tensile strength (Chapter 6).
- The reference test series to the shear experiments for the Lindenborg programme and the investigation of the shape of the critical shear crack (Section 7.1.2 and Section 7.2).
- The Shear model (Chapter 8).
Chapter 2

Literature study

The aim of this literature study is: (i) to study the direct effects of ASR on reinforced concrete structures; (ii) to identify the parameters that are important for the shear capacity; (iii) to investigate the state-of-the-art on how ASR affects these parameters and; (iii) to find experimental studies on the shear capacity, which can be used for the development of the shear model.

The inner expansion of ASR has two direct effects on concrete structures: crack formation and expansion on a structural level (macro level). From an engineering point of view, it is important to understand these two effects, as the cracks cause the mechanical properties of the concrete to decrease whilst the expansion induces prestressing in the reinforcement. Therefore, Section 2.1 regards ASR-induced cracks and expansion.

To identify which parameters are important for the determination of the shear capacity of ASR-damaged slabs, Section 2.2 evaluates existing shear models derived for sound concrete, and identifies the governing parameters. Subsequently, Sections 2.3 to 2.6 investigate how ASR damages affect these identified parameters. Finally, Section 2.7 investigates existing experimental studies on the shear capacity of slabs of ASR-damaged concrete.

Through the literature study, research questions which are needed for the development of the shear model will be formulated. These research questions are answered in the following chapters.

2.1 Cracks and expansions

The chemical reaction of ASR induces a swelling in, or around the reactive aggregates (see Chapter 1). This inner expansion causes tensile stresses in the surrounding cement paste. When these tensile stresses exceed the tensile strength of the cement paste, cracks develop around the reactive aggregate particles. The extent of ASR, and thereby the inner expansion, depends on the availability of water, alkalis and silica. The availability of these main ingredients strongly depends on the exposure conditions, which is explained
in Section 2.1.1. Moreover, the expansion and the crack pattern are affected by the presence of restraints. Section 2.1.2 takes a closer look at outer restraints while Section 2.1.3 looks at the special restraining conditions in the surface layers.

2.1.1 Exposure conditions

The extent of ASR, and thereby the inner expansion, depends on the availability of water, alkalis and silica. The exposure of water and alkalis on the structure is therefore one of the most important influencing factors for expansion and crack pattern. Within a single structural element, the exposure conditions vary at the different locations of the element. In other words, the availability of the main ingredients differs within the structure, causing a dissimilarity of expansions within the structure (McLeish, 1990). This is best explained by two practical examples:

- Consider a bridge-deck with a broken water-proofing membrane. At the top face of such a structural element, there is generally easy access of water (rain) and alkalis (e.g. de-icing salt). However, in the core of the structural element the water and alkali-content may be less, and therefore also less ASR (Hobbs, 1988).

- Consider a beam immersed in alkali-free water. The inherent alkalis (e.g. cement) in the surface layers may be reduced due to an alkali diffusion where the alkali diffuses to the surrounding water with a lower alkali level. This phenomenon is called leaching, and leads to reduced alkali-content in the surface layers, while the alkali-content in the core of the beam is unaffected (McLeish, 1990).

It is found that the exposure conditions are very case-dependent, and should be considered carefully when interpreting experimental results.

2.1.2 Restraining conditions

Due to the presence of physical boundaries, applied loads or the presence of reinforcement, the ASR expansion in structures is mechanically restrained. These restraining conditions have a large impact on the way that expansions and the crack patterns develop within the structure. The influence of the restraining conditions on the expansions and crack development is threefold. Firstly, the presence of restraining reduces expansions directly due to an elastic compression from the restraining load (Jones and Clark, 1996). Secondly, the presence of restraining (most likely) opposes the ASR itself due to the possible prevention of water uptake that limits the expansion (Jones and Clark, 1996). Thirdly, the inner expansion is caused by a fluid gel pressure and will therefore develop mostly in the direction with less resistance (Morenon et al., 2017; Gautam et al., 2017).

Two different types of restraining conditions can be observed in practice (Jones and Clark, 1996). The first type is the constant restraint, which emerges due to the presence of constant compressive stresses exerted upon the concrete in a certain direction.
Thereby, an increase in concrete expansion does not lead to an increase in the restraint-induced compressive stresses. An example of such a constraint found in practice is in dams where the self-weight causes vertical restraints. The second type are the elastic restraints, which are caused by the presence of internal or external reinforcement bars in the structure. In case of elastic restraints, the ASR-expansions induce prestressing in the reinforcement bars which, in turn, places the concrete into compression (Hobbs, 1988).

Depending on the structural element under consideration, different restraining directions may apply. To classify these conditions, structural elements can be 0D-, 1D-, 2D- or 3D-restrained (see Figure 2.1). Several experimental investigations have been conducted on the effect of restraining conditions on expansion and crack formation, of which an early overview is provided in (Jones and Clark, 1996).

Figure 2.1: Typical crack pattern in specimens with different restraining conditions.

0D The vast majority of the investigations from the literature investigates expansion of unrestrained, lab manufactured test specimens. Due to the absence of restraints, the cracking will occur in the direction normal to the direction of the least restraint which occurs rather randomly. The expansions are found to be rather similar in all directions. A typical crack-pattern which could be observed for 0D-restrained specimens is shown in Figure 2.1a. It is remarked that a fully unrestrained concrete element is unlikely to be found in practice.

1D Typical 1D-restrained structural elements are for example concrete beams with
longitudinal reinforcement only (elastic restraint). A number of researchers investigated the effects of 1D restraints (constant or elastic) on the level of both the expansions and the crack pattern using accelerated ASR. Typically, these researchers conclude that the ASR expansion is anisotropic and largest in the unrestrained direction (i.e. the more restraining, the less expansion), and that the cracks tend to appear in the direction parallel to the restraints (see e.g. Rigden et al. (1992); Fan and Hanson (1998b); Giaccio et al. (2009); Morenon et al. (2017)). A typical crack-pattern for a 1D-restrained specimen is shown in Figure 2.1b. Figure 2.2 shows a crack-pattern from a real 1D-restrained specimen. The left figure shows that the cracks are parallel in the restrained direction, while the right figure shows that the cracks are formed randomly in the unrestrained directions.

![Figure 2.2: Crack-observations presented by (Fan and Hanson, 1998b) for a 1D-restrained specimen. (left) a longitudinal section and (right) a cross section.](image)

2D A typical 2D-restrained structural element is for example a slab with bi-directional, in-plane reinforcement without shear-reinforcement (which is the topic of this PhD thesis). Only a few experimental studies exist which investigate the expansion and crack formation for 2D-restrained concrete. Wald et al. (2017) investigated the effects of different (elastic) restraining conditions on the ASR expansions in laboratory cast small-scale specimens. Their results for 2D-restrained specimens showed that the expansions in the two restrained directions were approximately identical, whilst the expansion in the third direction was found to be much larger. Hayes et al. (2018) conducted a similar investigation on a large-scale slab with horizontal (plane) reinforcement in two directions (top, bottom), and concluded the same. It is remarked that neither one of the investigations address the observed crack patterns. However, observations from practice show a clear ‘laminated’ cracking pattern in 2D-reinforced structures such as shown in Figure 2.1c. Den Uijl and Kaptijn (2002) investigated a number of beams cut from a severely ASR-damaged bridge-deck in the Netherlands, and found that “[...] ASR damages result in desk-like weakened spots. Due to the absence of vertical reinforcements these ‘crack planes’ will be oriented mainly horizontally”. This crack pattern was found in the Gammelrand bridge and the Lindenborg bridge, see Chapter 4, as well.

3D A typical 3D-restrained structural element is for example a RC beam with longitudinal reinforcement and closed stirrups (i.e. reinforcement in all three directions).
In specimens that are 3D-restrained, the cracks were found to be formed with random orientation as the ASR-induced expansion has no preferred (softer) direction, see Figure 2.1d. The visual appearance of the crack-pattern is therefore similar to that of unrestrained specimens, yet the amount of expansion is less.

### 2.1.3 Surface layers

In the surface layers of ASR-damaged concrete elements a random crack pattern is often observed, which looks similar to those for unrestrained concrete elements. Such a random crack pattern on the outside may not be representative (and sometimes even misleading) for the crack pattern observed within the concrete element (McLeish, 1990; Courtier, 1990). This deviating crack-pattern is explained by the ASR-induced crack development (see Figure 2.3), which is affected by both (local) restraining and (local) exposure conditions (such as explained in the previous subsections).

In the core of the concrete element, the concrete may be considered 3D-restrained, and therefore, cracks will occur in the direction normal to the direction of the least restraint. Towards the outside edge, there is however an ever-decreasing resistance to expansion normal to the surface and therefore, cracks tend to orientate itself parallel to the surface (and to a lesser extent perpendicular to the surface). Moreover, in the surface layer (some 20 mm thick) the degree of reactivity may be reduced due to the leaching of alkalis by water or evaporation.

The overall effect of these phenomena is that the interior of the concrete expands more in the direction parallel to the surface than the surface concrete. The expansion compatibility creates tensile strains at the surface, which may eventually result in cracking visible at the surface. Generally, this results in a random crack pattern (similar to 0D- or 3D-retrained structures). The crack pattern that can be seen on the surfaces of an ASR-damaged structure may therefore not be representative for crack pattern inside the structure.

![Figure 2.3: Development of cracks due to ASR at surface of structure (McLeish, 1990)](image-url)
2.1.4 Discussion

The previous subsections showed that the exposure conditions, restraining conditions and surface layers have a large influence on the expansions and crack pattern of ASR-damaged structures. When studying experimental research from the literature, it is therefore important that the conducted experiments are representative for the situation of interest. Focus of this PhD thesis is on reinforced concrete slabs without shear reinforcement. This structural type corresponds to the 2D-restrained concrete specimen which are mainly exposed at the top.

The influence of exposure and restraining conditions on the expansions and crack patterns were found to be well explained in the literature, both qualitatively and quantitatively (see e.g. (Morenon et al., 2017)). However, these explanations are typically on laboratory specimen level. Since the cracks and their orientation are crucial for the mechanical properties of the concrete and therefore the load-carrying capacity of the structure, one of the objectives of this PhD thesis is therefore:

**RQ1: Provide a phenomenological explanation of the ASR-induced crack growth in reinforced concrete slabs without shear reinforcement.**

This research question will be addressed in Chapter 3.

2.2 Important parameters for the shear capacity of slabs of sound concrete

To identify the parameters that need to be investigated in order to assess the shear capacity of ASR-damaged slabs without shear reinforcement, the governing parameters for shear capacity of corresponding slabs of sound concrete are taken as a reference. There exists a great amount of literature on this subject, see e.g. (Zhang and Nielsen, 1997; Collins et al., 1996; Hong-Gun et al., 2006; Muttoni and Fernández Ruiz, 2008; Marí et al., 2015; Fisker and Hagsten, 2016) and many models have over the last decades been proposed for calculation of the shear capacity of members without shear reinforcement. Many of these models include the following four main effects on the shear capacity (see Figure 2.4):

- **Aggregate interlock** ($V_{agg}$): Aggregate interlock is the main contributor to the shear transfer in wide cracks. Due to the weak interface between aggregates and the cement paste, cracks tend to go around the aggregates, which results in a rough crack surface. Aggregate interlock is the resistant in a crack induced by this roughness. Since the interlock depends on the contact surfaces between the aggregates and the cement paste, it depends on the size of the aggregates and the width of the crack: Larger aggregates result in higher interlock and larger crack widths result in
lower interlock. Aggregate interlock typically depends on the concrete compressive strength ($f_c$) and the inclination of the critical crack.

- **Residual tensile strength ($V_{res}$):** The tensile strength of the concrete ($f_t$) is not zero for small crack widths. This means that a part of the critical crack has a residual tensile strength. Due to the inclination of the crack, this tensile strength will contribute to the shear capacity.

- **Compression zone ($V_{compr}$):** As shown in Figure 2.4, the critical crack does not develop to the top of the beam. The uncracked compression zone contributes to the shear capacity as well. This contribution depends on the height of the critical crack and the compressive strength.

- **Dowel action ($V_{dow}$):** The dowel action is activated by a relative translational displacement in the critical crack during failure where the longitudinal reinforcement separates the cover from the beam and horizontal cracks at the level of the reinforcement form, see Figure 2.4. This contribution depends on the concrete tensile strength and the flexural strength of the rebars.

![Figure 2.4: Schematic illustration of a simply supported beam without shear reinforcement with a critical shear crack and the four tributary shear actions.](image)

From the above, it can be concluded that the following parameters play an important role for the shear capacity of beams and slabs without shear reinforcement:

- Concrete compressive strength ($f_c$)
- Concrete tensile strength ($f_t$)
- Crack width
- Shape of the critical crack
- Longitudinal reinforcement
Since the tensile strength of concrete is one of the governing parameters, it also follows that there is a specific size effect on the shear capacity of members without shear reinforcement. Furthermore, since prestressing affects the crack width; it indirectly affects the shear capacity as well. In the following subsections, it will be investigated how ASR affects each of these parameters.

2.3 Compressive strength

The concrete compressive strength was found to play an important role for the shear capacity of RC slabs without shear reinforcement. A large number of experimental studies investigated the effects of ASR on the concrete compressive strength, for an overview, see Barbosa et al. (2018a). General findings of these studies are: (i) that the concrete compressive strength is reduced compared to non-affected concrete (see e.g. (Giaccio et al., 2009; Swamy and Al-Asali, 1988; Ahmed et al., 2003)); and (ii) that the concrete compressive strength behaves anisotropic in the presence of restraints, whereby the lowest values are found for the unrestrained direction (see e.g. (Jones et al., 1994)).

The majority of the experimental studies on the concrete compressive strength is conducted on specimens where ASR was accelerated under restraining conditions which are not representative for 2D-restrained slabs such as in scope of this study. To the author’s knowledge, only two studies investigated the compressive strength of 2D-restrained ASR-damaged concrete elements:

- Gautam et al. (2017) investigated the effect of different restraining conditions on the expansion, cracking and also the compressive strength. In total, 22 cubes (254×254×254 mm) were cast; 6 of these were 2D-restrained. The compressive strength of drilled cores was measured after 2, 3, 8 and 12 months of ASR acceleration. The compressive strength was only tested in one direction (restrained). It was found that the compressive strength was reduced compared to similar specimens without ASR. Additionally, the Young’s Modulus was tested in a restrained and an unrestrained direction after 8 month of ASR acceleration. It was reduced by 16% in the restrained direction and by 25% in the unrestrained direction compared to the stiffness of the reference cube of sound concrete. They suggest that the low stiffness in the unrestrained direction may be due to crack closure.

- Hayes et al. (2018) investigated the effect of different restraining conditions for thick (2D-restrained) slabs on the expansion, the compressive strength and stiffness. Three large slabs (1000×3000×3500 mm) were cast; two with ASR and one without as a reference. The slabs with ASR were produced with reactive aggregates and alkalis were added to the mixing water. One slab with ASR was 2D-restrained by mean of a rigid steel frame and the two other slabs were 2D-restrained by means of embedded reinforcement. For each slab, 50 cylinder were made of the same concrete batches. The cylinders that represent the ASR slab restrained by the rigid
steel frame were kept in steel moulds to ensure the right restraining conditions (2D-restrained and free vertical expansion). However, the remaining cylinders were kept unrestrained despite that the associated slabs were 2D-restrained. The compressive strength was measured 7 times over 12 months. It was found that the compressive strength of the ASR-damaged concrete (restrained as well as unrestrained) was reduced up to 40% compared to the reference cylinders.

Remarkably, neither of the two studies addresses the anisotropic behaviour of the compressive strength. However, based on the findings of the 1D-restrained specimens; the fact that Gautam et al. (2017) found an anisotropic behaviour of the stiffness for 2D-restrained specimen; and the fact that the ASR-induced cracks in slabs are predominantly horizontal, it is expected that the compressive strength of 2D-restrained concrete elements is anisotropic as well. The question, however, rises how significant this anisotropy will be in practice. Since little experimental research has been conducted on large-scale, non-accelerated, 2D-restrained concrete slabs, one of the research questions in this study is:

**RQ2: To what extent is the uniaxial concrete compressive strength in 2D-restrained slabs anisotropic?**

The potentially anisotropic behaviour of the compressive strength is often attributed to the presence of ASR-damages. It is however questionable if this is indeed the case. There exist experimental investigations that show that the compressive strength of sound concrete is anisotropic as well. E.g. Hughes and Ash (1970) found as much as 50% difference in the concrete compressive strength depending on the drilling direction. This has resulted in the following research question:

**RQ3: To what extent is the anisotropic compressive strength of ASR-damaged concrete caused by ASR?**

Another obvious question that follows from this is how to address this anisotropic behaviour in the assessment of ASR-damaged slabs; should one use the uniaxial compressive strength parallel or perpendicular to the ASR-induced cracks? Most likely, the answer to this question depends on the load carrying mechanism under consideration. This is explained as follows. Figure 2.5a shows the moment transfer mechanism for a simply supported beam with a point load. Since the compressive stresses from the moment are working in the horizontal direction, the moment capacity should be found on the basis of the horizontal compressive strength. Figure 2.5b shows the shear transfer mechanism for the same beam, simplified by a strut-and-tie model. In this case, one would be interested in the compressive stresses in the direction of the compressive struts. In other words, the compression strut determines in which direction the compressive strength should be taken.
To avoid that question, the shear carrying mechanism may e.g. be investigated by using an upper bound approach. Figure 2.5c shows a failure mechanism for the considered beam. Here, the shear force is carried by the sliding resistance of a diagonal yield line/crack. The sliding resistance in this yield line/crack depends on the interaction between aggregates and cement paste. Since the interaction between aggregates and cement paste in sound concrete depends on the compressive strength as well, then the sliding resistance of the yield line in sound concrete can be found by means of the compressive strength. To be able to deduce how the sliding resistance for ASR-damaged concrete can be determined by means of measured the compressive strength, it is necessary to know which mechanisms that are governing for the compressive strength in the two directions:

\[\text{RQ4: What are the governing mechanisms for the horizontal and vertical compressive strength of ASR-damaged concrete?}\]

\[(a) \text{ Moment transfer.}\]

\[(b) \text{ Shear transfer by strut-and-tie.}\]

\[(c) \text{ Upper bound approach for shear transfer.}\]

*Figure 2.5: Schematic moment and shear transfer models in a simple supported beam.*
2.4 Tensile strength of ASR-damaged concrete

The concrete tensile strength plays an important role for the shear capacity of RC slabs without shear reinforcement. Numerous experimental studies investigated how ASR affects the tensile strength. They all show that the tensile strength is reduced by ASR, see e.g. (Fan and Hanson, 1998a). Most studies are conducted on specimens of which the restraining conditions are not representative for 2D-restrained slabs. To the authors knowledge, two studies did investigate the tensile strength of 2D-restrained ASR-damaged concrete elements:

- Siemes et al. (2002) investigated the tensile strength in existing ASR-damaged slab bridges in the Netherlands by means of uniaxial tests and Brazilian split (BS) tests. Cores from 25 different bridges were drilled vertically and tested. They found that the BS tensile strength is of the same magnitude as the tensile strength predicted by the measured compressive strength, whilst the uniaxial tensile strength was found to be substantially lower (up to 82%). For the Zaltbommel bridge, the anisotropy of the tensile strength was also investigated. Cores were drilled in three perpendicular directions and tested. The BS tests showed no anisotropy, while the uniaxial tests showed a strong anisotropy. The tensile strength perpendicular to the cracks was up to 50% lower than the tensile strength parallel to the cracks. Based on these results, they concluded: "For the basic investigation of concrete structures that showed signs of ASR it has been decided on the basis of an expert’s opinion to do the tensile testing according to the uniaxial test [...]”. It is remarked that this method is adopted in the CUR-recommendation, see Section 1.3.2.

- Barbosa (2017) published a thorough investigation on the tensile strength on the basis of three existing ASR-damaged slab bridges in Denmark. Cores and cubes were extracted from slab parts of the three bridges where there was no shear reinforcement. The tensile strength was tested by means of BS tests, wedge split (WS) tests and uniaxial tests. The BS tests were conducted for three different orientations of cracks, see Figure 2.6. The WS tests and uniaxial tests were conducted perpendicular and parallel to the cracks, see Figure 2.7.

  - For BS strength, orientation III was found to be strongest and orientation I was found to be weakest. The failure mechanism for orientation II deviated from the failure mechanism for orientation I and III. Besides the opening of a vertical splitting crack, the horizontal ASR-induced cracks opened as well, see Figure 2.8.

  - For the WS tests, the tensile strength perpendicular to the cracks was found to be up to 70% lower than the tensile strength parallel to the cracks.

  - For the uniaxial tests, the tensile strength perpendicular to the cracks was found to be approximately 0 MPa, while the tensile strength parallel to the cracks was around 0.8 MPa.
Each of the three test methods showed a clear anisotropy between the different orientations. Furthermore, there was a large difference between the results of the test methods; the BS strength was up to three times higher than the tensile strength obtained by WS tests. Since the uniaxial and BS tests were not employed on the same bridge, a direct comparison is not possible. However, the uniaxial tensile strength was up to 36% lower than the tensile strength from WS. No recommendation was provided on which test method to employ for determining the tensile strength for the use in structural calculations.

The fact that the measured reduction of the tensile strength is very dependent of the employed test method has also been investigated on specimens where ASR was accelerated without restraints, see e.g (Swamy and Al-Asali, 1988; Clayton et al., 1990; Smaoui et al., 2005; Ahmed et al., 2003). Typically, the tensile strength found by BS tests is higher than the tensile strength determined by uniaxial tests or by flexural tests. For this reason, Clayton et al. (1990) states that “[Brazilian] splitting test cannot be used to assess tensile strength loss from ASR”, while the remaining investigations do not provide a recommendation on which method should be employed for determining the tensile strength for the use in structural calculations.

Discussion

The literature review showed that the tensile strength of ASR-damages structures is reduced significantly, and that the measured reduction strongly depends on the employed test method. Nevertheless, no general consensus exists in the literature on which test method should be employed. Siemes et al. (2002) and Clayton et al. (1990) recommend not to use Brazilian split tests, however, these recommendations are based on expert knowledge, and not on a physical rationale. Therefore, the following question arises:

RQ5: Which test method shall be employed in practice to determine the tensile strength of ASR-damaged concrete for the use in structural calculations?

This question will be addressed in Chapter 6.
Figure 2.6: Orientation of ASR-induced cracks during the Brazilian split (BS) tests in the experimental investigation by Barbosa (2017). The figure is adapted from Barbosa (2017).

Figure 2.7: Crack orientation during the wedge split (WS) tests in the experimental investigation by Barbosa (2017).

Figure 2.8: Photo of a failure from a Brazilian split (BS) test in Crack orientation II, see Figure 2.6. The photo is from Barbosa (2017).
2.5 ASR-induced prestressing

Section 2.2 showed that prestressing played an important role for the shear capacity of sound reinforced concrete members. Studies in the literature have found that prestressing plays an important role for the shear capacity of ASR-damaged beams as well. They give ASR-induced prestressing of the reinforcement as the explanation for the surprisingly high shear capacities of ASR-damaged beams and slabs (Fujii et al., 1986), (Institution of Structural Engineers, 1992), (Committee on AAR, 1986) published in (Clark, 1989),(The Danish Road Directorate, 1990), (Cope and Slade, 1992), (Chana and G. A. Korobokis, 1991), (Ahmed et al., 1998), (Den Uijl and Kaptijn, 2003) and (Hansen et al., 2016b).

A large number of experimental studies investigated the (magnitude of) ASR-induced prestressing. The majority of these studies are based on specimens where ASR was accelerated under restraining conditions that are not representative for 2D-restrained slabs. The results of these studies are therefore not considered further in this thesis. However, in the literature, there exists a handful well-conducted experiments that are representative for 2D-restrained slabs. For example:

- Multon and Toutlemonde (2006) investigated the effect of a number of different restraining conditions on the ASR-induced prestressing. For this, they produced eight 2D-restrained cylinders (130 × 240 mm) containing reactive aggregates and sufficient alkalis to ensure ASR. After casting, the 2D restraint was provided by 3 or 5 mm thick steel rings, such that axial expansion was free. During the ASR expansion, the axial and radial expansion was measured where the radial expansion corresponds to the prestressing. They found prestressing of 135 to 206 MPa (yielding) in the steel rings.

- Hayes et al. (2018) investigated ASR-induced prestressing in three large (1000 × 3000 × 3500 mm) 2D-restrained slabs. They found that the restraining reinforcement was prestressed to what corresponds to approximately 200 MPa.

- Wald et al. (2017) investigated the effect of different restraining conditions on the expansion as well. They produced 33 480 mm cubes; of which 6 were restrained in two directions during the ASR acceleration. They found that the reinforcement was yielding (420 MPa) due to the ASR-induced prestressing.

It can be concluded that the ASR-induced prestressing of the reinforcement is significant for 2D-restrained specimens, and should be taken into account when the load-carrying capacity is assessed.

In laboratory specimens, the ASR-induced prestressing is often measured directly on the reinforcement by means of preinstalled strain gauges. Thereby, the prestressing can be measured during the acceleration of the ASR. A similar test method may be employed for assessing the prestressing in existing structures. Strain gauges are mounted
on reinforcement bars which are cut subsequently. The prestressing can be found as the measured contraction of the reinforcement bar. Since this test method requires that the reinforcement is cut, it is undesirable in practice. Therefore, a non-destructive test method tailored for ASR-damaged structures has been developed - Crack Width Summation (Jones and Clark, 1994). As the name indicates, the test method employs accumulation of crack widths to quantify the ASR-induced prestressing. However, Jones and Clark (1996) state that "It is extremely difficult to measure accurately the widths of ASR cracks, even under laboratory conditions [...]". Consequently, for research purposes where accuracy is desired, the direct test method should be employed.

2.6 Shape of the critical shear crack

The shape of the critical shear crack was the last parameter that was found to play an important role for the shear capacity of RC slabs without shear reinforcement. The majority of studies regarding the shear capacity of ASR-damaged concrete slabs or beams without shear reinforcement focuses on the capacity, exclusively (see Section 2.7). To the author's knowledge, there exist no studies regarding the shape of the critical crack in 2D-restrained slabs. However, there are a few studies that investigated how ASR affects the shape of the critical shear crack in 1D-restrained beams:

- The Danish Road Directorate (1990) investigated how ASR affects the shear capacity of 1D-restrained beams. Figure 2.9 shows photos of the crack pattern after failure for an ASR-damaged beam (Figure 2.9a) and a reference beam (Figure 2.9b). Clearly, the shape of the critical crack is affected by ASR. It appears that the critical crack for the ASR-damaged beam consists of two parts: the flat part and an almost vertical part in the compression zone. In contrast, the critical crack in the reference beam has a smoother nature. It is remarked that the shear capacity of the ASR-damaged beam was increased compared to the reference beam.

- Also, Ahmed et al. (1998) investigated how ASR affects the shear capacity of 1D-restrained beams. Figure 2.10 shows photos of the crack pattern after failure for the ASR damaged beam (Figure 2.10a) and the reference beam (Figure 2.10b). The critical crack in the ASR-damaged beam is found to have the same shape as the critical crack in the ASR-damaged beam shown in Figure 2.9a. By comparing the crack pattern for the ASR-damaged beam and the reference beam, it is seen that the ASR has affected the shape of the critical crack. Also, Ahmed et al. (1998) found that the shear capacity was increased compared to the reference beam.
Figure 2.9: The crack pattern after failure for beams without shear reinforcement. The red arrows indicate the reaction and the loading. The photos are from (The Danish Road Directorate, 1990).

Figure 2.10: The crack pattern after failure for beams without shear reinforcement. The red arrows indicate the reaction and the loading. The photos are from (Ahmed et al., 1998).

Since these findings are based on 1D-restrained beams and the shape of the critical crack has never been investigated for 2D-restrained slabs, following question rises:

**RQ6: To what extent and how does ASR affect the shape of the critical shear crack for 2D-restrained slabs without shear reinforcement?**

Since the focus of the two studies mentioned above was on the shear capacity rather than on the shape of the critical crack, they did not provide an explanation for the change of crack shape. The obvious question therefore is:

**RQ7: Why does ASR affect the shape of the critical crack for 2D-restrained slabs without shear reinforcement?**

Both research questions are answered in Chapter 7.
2.7 Existing experimental studies on the shear capacity of ASR-damaged slabs

During the last 35 years, numerous experimental studies on the shear capacity of ASR-damaged slabs or beams without shear reinforcement have been conducted. Despite the fact that the studies had the same scope, their conclusions are not the same. Some studies find that the shear capacity is increased due to ASR (see e.g. (Bilodeau et al., 2016; Ahmed et al., 1998)) whilst other studies show that it is decreased (see e.g. (Chana and G. A. Korobakis, 1991)). Furthermore, it is found that the majority of the studies are conducted on beams where ASR is accelerated under restraining conditions that are not representative for 2D-restrained slabs. To the author’s knowledge, only two studies investigated the shear capacity of 2D-restrained ASR-damaged concrete slabs:

- Den Uijl and Kaptijn (2003) investigated the shear capacity of two ASR-damaged slab bridges in the Netherlands - Zaltbommel (ZB) and Heemraadsingel (HS). To conduct the experiments under controlled conditions, two and four beams were cut from ZB and HS and brought to a laboratory. The beams were cut from areas with only in-plane reinforcement (i.e. no shear reinforcement). The two beams from ZB failed in bending and are therefore not considered further in this study regarding shear capacity. The four beams from HS were strengthened by steel strips glued to the bottom side to ensure shear failure (and avoid bending failure). They found that the shear capacity was reduced by up to 32% compared to shear capacity predicted by a model developed for sound concrete and employing the measured compressive strength.

  It is remarked that these findings cannot be applied for other ASR-damaged slabs since the external reinforcement (i.e. the steel strips glued to the bottom side) deviates significantly from traditional embedded reinforcement: (i) the external reinforcement was installed first after the ASR expansion was induced and the ASR-induced prestressing is therefore absent; (ii) the bonding conditions which are important for the crack width and thereby the shear capacity are not the same and; (iii) the dowel action is not the same.

- Schmidt et al. (2014) investigated the shear capacity of an ASR-damaged slab bridge in Denmark. The shear capacity of the bridge slab was tested on 4 locations of the slab that only contained in-plane reinforcement (i.e. no shear reinforcement). To ensure one-way shear during the experiment, the slab was cut such that it was only supported along one edge, as a cantilevered beam. The shear capacity was tested by loading the cantilevered slab to failure. However, later inspections of the tested slabs revealed that some top-side (tensile) reinforcement bars were accidentally cut during preparation of the tests. Consequently, the anchorage condition of some reinforcement bars was impaired significantly before testing and the
test results were highly affected; the failures were mainly caused by bending and anchorage failures rather than by shear failures (Hansen et al., 2016b).

Due to external reinforcement in the study by Den Uijl and Kaptijn (2003) and due to the anchorage inadequacy in the study by Schmidt et al. (2014), the experiments cannot be used in development of a mechanical model for the shear capacity. It is therefore one of the objectives of this PhD thesis to:

**RQ8: Provide experimental evidence that can be used in the development of a mechanical shear model for ASR-damaged 2D-restrained slabs without shear reinforcement.**

Chapter 4 provides an introduction to the bridges which will be experimentally investigated in this study. Chapter 7 describes the conducted experiments and the results.
Chapter 3

Explanation of ASR-induced crack growth in 2D-restrained slabs

For the convenience in the rest of the thesis, this chapter provides a phenomenological explanation of ASR-induced crack growth in 2D-restrained slabs. Since the explanation focuses on the crack growth inside slabs, the distinctive crack formation in the surface layers is not included. Similarly, the effect of surface exposure is neither included; the reactions are assumed to occur uniformly throughout the entire slab. The explanation consists of five phases which are chronologically schematized in Figure 3.1. The five phases show:

a) The inner ASR-induced expansion occurs in or around a reactive aggregate particle (Hobbs, 1988). As the stiffness of the surrounding uncracked cement paste is the same in the $x$-, $y$- and $z$-direction, the expansion may develop uniformly in all directions.

b) The inner expansion causes tensile stresses in the surrounding cement paste. When these stresses exceed the tensile strength of the cement paste, randomly orientated cracks form. These cracks are in the rest of this thesis mentioned as ASR-induced micro-cracks.

c) The formation of the micro-cracks will lead to an expansion on a structural level. Due to this expansion, the reinforcement (elastic restraint) will be stressed to tension (prestressed), $\varepsilon_{s,ASR}$, and thus induce equilibrating compressive strains and stresses in the concrete, $\varepsilon_{c,ASR}$.

d) The $x$-, and $y$-direction will now be restrained by the reinforcement and the compressive stresses. Contrarily, the $z$-direction will be unrestrained. Since the restraining conditions are anisotropic (least restrained in the $z$-direction), further inner expansion will lead to vertical crack opening ($z$-direction).

e) The micro-cracks parallel to the direction of the reinforcement and the compressive stresses ($x$-, and $y$-direction) will grow into macro-cracks with larger crack widths.
In the rest of this thesis, the macro-cracks will be mentioned as \textit{ASR-induced cracks}.

All in all, the slab consists of randomly orientated micro-cracks, which are evenly distributed and horizontal macro-cracks which are discrete distributed whilst the reinforcement is prestressed.

![Conceptual explanation of ASR progression in a slab element (2D-restrained). The sections are also representative for sections in the x-z plane.](image)

The described cracking behaviour is seen in the experiments by Hayes et al. (2018). They investigated the ASR-induced expansions in a 2D-restrain slab, see Figure 3.2. Figure 3.3 shows the expansions in the restrained direction (horizontal axis) vs. the expansions in the unrestrained direction (vertical axis). It is seen that initially, the expansions are approximate equal in the restrained and unrestrained directions. This corresponds to the micro-cracking phase in the explanation (phase b and c). Subsequently, the expansions occur mainly in the unrestrained direction, which corresponds to the macro-cracking phase (phase d and e).
Figure 3.2: The tested concrete 2D-restrained slab. Adapted from (Hayes et al., 2018).

Figure 3.3: Expansion in the restrained direction vs. expansion in the unrestrained direction. Adapted from (Hayes et al., 2018).
Chapter 4

Introduction to the ASR-damaged bridges

The experiments with ASR-damaged concrete in this thesis are conducted with concrete from two bridges, namely Gammelrand and Lindenborg. This chapter provides a short introduction to the two bridges, while a detailed description of the experiments and the results is given in the respective chapters.

4.1 Gammelrand bridge

Gammelrand was a three-span concrete bridge built in 1976. The entire bridge was constructed as a flat slab with wings, see Figure 4.1. The slab was approximately 720 mm thick at the centre. The slab had only reinforcement parallel to the surfaces; there was no shear reinforcement.

![Figure 4.1: Schematic cross section of Gammelrand bridge.](image)

Inspections of the bridge showed that the entire slab was cracked due to ASR. Reactive aggregates were found in the sand fraction. Figure 4.2 shows a UV-photo of a Ø110 mm core from the final inspections (Niras, 2011). The core was drilled vertically and impregnated with fluorescence epoxy such that, cracks, voids and porosities fluoresce under UV light. It is seen that the concrete was severely cracked with horizontal cracks in the entire height. That the cracks are mainly horizontal (vertical on Figure 4.2) is consistent with the phenomenological explanation of crack formation in Chapter 3.
Due to uncertainties regarding the residual load-carrying capacity, the bridge was demolished in 2010. In connection with the demolition, four beams were cut and brought to laboratory facilities where the experiments were conducted, see Figure 4.3. The four beams were cut from the same area of the bridge, and there were no visual differences of the extent of the ASR-cracks between the four beams.

**Figure 4.3: The four beams cut from Gammelrand at the laboratory facilities.**

### 4.2 Lindenborg bridge

*Lindenborg* is a pile-supported multi-span concrete bridge with a total length of 312 m. The superstructure consists of a 300 mm flat slab without shear reinforcement, see Figure 4.4.
The bridge was built in 1966-1967. Inspections of the bridge showed extensive ASR-induced cracks, leading to concerns for the residual shear capacity. Consequently, it was decided to test the shear capacity. To minimise the uncertainties during testing, such as the boundary conditions, supports and loading; and to enable detailed measurements, six slab segments were cut from the bridge and brought to laboratory facilities for testing, see Figure 4.5.

Since the slab segments were cut over a zone of approximately 180 m, variations of the extent of the ASR-induced cracks cannot be neglected. Figure 4.6 shows UV-photos of ø100 mm epoxy-impregnated cores drilled vertically in Slab Segment 2, 4 and 6, respectively. As can be seen, the three cores have horizontal ASR-induced cracks in the entire height. As for Gammelrand, this is consistent with the phenomenological explanation of crack formation in Chapter 3. It is seen that there are more cracks in the cores from Slab Segment 4 and 6 than in the core from Slab Segment 2.
Figure 4.6: ASR-induced cracks shown on fluorescence-impregnated cores drilled vertical from Slab Segment 2, 4 and 6 from Lindenborg bridge.

Each slab segment was cut into three beams and four strips, see Figure 4.7. While the beams were used to test the shear capacity, the strips were used to test the mechanical properties of the concrete. The reinforcement bars in the strips were used for testing of the mechanical properties of the steel and for testing of the level of prestressing.
Figure 4.7: Slab segment cut into test beams, Lindenborg bridge.
Chapter 5

Concrete compressive strength

This chapter investigates the compressive strength of ASR-damaged concrete. In the literature study (Chapter 2), three research questions (RQ) regarding compressive strength were asked:

RQ2 To what extent is the uniaxial concrete compressive strength in 2D-restrained slabs anisotropic?

RQ3 To what extent is the anisotropic compressive strength of ASR-damaged concrete caused by ASR?

RQ4 What are the governing mechanisms for the horizontal and vertical compressive strength of ASR-damaged concrete?

To answer to what extent the uniaxial concrete compressive strength in 2D-restrained slabs is anisotropic (RQ2), the compressive strength is quantified in two directions in two existing ASR-damaged bridge decks, see Section 5.1.

It is not only in ASR-damaged concrete that the compressive strength is anisotropic. Several experimental studies have found the compressive strength of sound concrete to be anisotropic as well (Hughes and Ash, 1970; Johnston, 1973; Leshchinsky, 1990; Bartlett and MacGregor, 1995; Ergun and Kurklu, 2012; Den Uijl and Yang, 2009). The studies however, strongly disagree on the magnitude of the anisotropy; absolute as well as relative. E.g. Hughes and Ash (1970) found as much as 50% difference in the concrete compressive strength depending on the drilling direction, whereas Van Mier (1984) found that the compressive strength in sound concrete was isotropic. However, due to serious shortcomings in the conducted experimental studies it is difficult to conclude which findings represent reality most; the ones ‘in favor’ of the strong anisotropic behaviour of sound concrete, or the ones ‘in favor’ of the traditional isotropic behaviour. The shortcomings are:

A) The experimental results are generally analysed without proper statistical techniques, and it is therefore uncertain if the obtained results can be attributed to
coincidents. E.g. Hughes and Ash (1970) drew their conclusion based on a comparison of two samples only while Leshchinsky (1990) and Ergun and Kurklu (2012) drew their conclusions based on a comparison of mean-values without regards to the variances.

B) The geometry and origins of the employed test specimens are generally not representative for real structures. E.g. Johnston (1973) and Van Mier (1984) studied the anisotropy by testing cubes that were sawn from unreinforced concrete prisms and Leshchinsky (1990) drilled cores from unreinforced concrete cubes (200 x 200 x 200 mm).

It is therefore still debatable how significant the anisotropy in sound concrete actually is.

To be able to determine whether the possible anisotropy in sound concrete may play a role for the anisotropy found in ASR-damaged slabs, a better understanding of the anisotropy in sound concrete is needed. Therefore, an experimental investigation of the anisotropy in sound concrete has been conducted, see Section 5.2. Here it is investigated to what extent there is anisotropy in sound concrete and what the reason behind is. By comparing the outcome of this investigation with the anisotropy in ASR-damaged slabs (RQ2), it is possible to determine to what extent the anisotropic strength of ASR-damaged concrete specimens is caused by ASR (RQ3), see Section 5.3.

To answer what the governing mechanisms for the horizontal and vertical compressive strength in ASR-damaged slabs are (RQ4), a thorough analysis of the compressive strength in ASR-damaged slabs is conducted, see Section 5.4.

The chapter closes with some recommendations on practical applications of Young’s modulus for ASR-damaged concrete in slabs.

5.1 Quantification of the anisotropy of the compressive strength in ASR-damaged bridge decks

This section provides a quantification of the anisotropy of the compressive strength of two ASR-damaged slabs from two existing Danish bridges: Gammelrand and Lindenborg. A general description of the two bridges is given in Chapter 4.

5.1.1 Cores

To quantify the anisotropy of the concrete compressive strength in 2D-restrained slabs, ø99 mm cores were drilled from beam segments from Gammelrand and slab segments from Lindenborg parallel (core\(\parallel\)) and perpendicular (core\(\perp\)) to the ASR-induced cracks, see Figure 5.1. For convenience, the compressive strength of (core\(\parallel\)) and (core\(\perp\)) is called \(f_{c\parallel}\) and \(f_{c\perp}\), respectively.
For Gammelrand bridge, 20 cores (10 core∥ and 10 core⊥) were drilled from the ends of the beams (anchorage zones) after the shear tests; which are presented in Chapter 7. For Lindenborg, the number of drilled cores is shown in Table 5.1. The cores were drilled from the remaining strips after cutting the beams from the slab segments, see Figure 4.7. Since the vertically drilled cores (core⊥) were extracted by drilling through the slab segments, the cores often contained reinforcement. Consequently, approximately 50 mm of the upper and lower parts of the cores were removed. Since the horizontally drilled cores (core∥) were extracted in the mid-height of the slab segment, they did not contain concrete from the upper and lower part of the slab segments. Figure 4.6 shows that the ASR-induced cracks are approximately evenly distributed over the height of the slab segments. Therefore, it is concluded that core∥ and core⊥ represent the entire cross section.

<table>
<thead>
<tr>
<th>Slab segment</th>
<th>Core⊥</th>
<th>Core∥</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6 + 6*</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

*Deformation-controlled tests

Table 5.1: The number of tested cores from Lindenborg bridge.

The cores from both bridges were drilled, stored and prepared in the same way. Drilling took place by use of a water-cooled diamond drill. The cores were thereafter wiped off and wrapped in cling-film to avoid desiccation and stored in sealed plastic bags. The cores were stored at a temperature of approximately 20°C for approximately 3 weeks. Before testing, the cores were sawn and grinded to ensure a plane loading surface. The cores from Gammelrand had a height of 200 mm. Due to the removal of the upper and lower concrete, this was not possible for all core⊥ from Lindenborg, where the height varies between 165 and 200 mm, see Appendix C.
5.1.2 Tests

The compressive tests were conducted in a machine with steel loading platens where one loading platen was fixed and the other hinged to allow a potential uneven deformation. To compensate small unevennesses on the loading surfaces 3 mm wood fibre boards were placed between the loading platens and the core during testing. The cores from Gammelrand were tested deformation-controlled with a constant rate of 0.5 mm/min. Generally, the cores from Lindenborg were tested load-controlled with a constant rate of 0.6 MPa/sek. To assess whether the loading method affects the measured strength, a reference test series was conducted where additional 6 cores were tested deformation-controlled. Figure 5.2 shows the test results with the $2 \times 6$ cores from Slab segment 3. The results are converted to standard cylinder compressive strength, see description below. The test results show that the mean values of the two test series are close to be identical (27.6 MPa vs. 28.4 MPa). Therefore, no distinction is made between the compressive strength obtained by load- and deformation-controlled tests.

![Figure 5.2: Concrete compressive strength parallel with the ASR-induced cracks for Slab segment 3 from Lindenborg, tested load- and deformation-controlled.](image)

5.1.3 Conversion to standard cylinder strength

The compressive strength of a drilled core may not be directly representative for the compressive strength in the structure of origin. Therefore, the tested compressive strength is subsequently converted to a standard cylinder compressive strength that can be adopted in models for the load-carrying capacity. In this project, the conversion formula provided by the Danish Road Directorate (Vejdirektoratet, 2017) is used:
\[ f_{c,\text{std}} = k_1 \cdot k_2 \cdot k_3 \cdot f_{c,\text{measured}} \] (5.1)

where \( f_{c,\text{std}} \) is the standard cylinder compressive strength, \( k_1 \) corrects for a height/diameter ratio different from 2.0 (see Equation 5.2), \( k_2 \) corrects for a diameter different from 150 mm (see Equation 5.3), \( k_3 \) corrects for the fact that the cores are drilled, not cast individually (see Equation 5.4) and \( f_{c,\text{measured}} \) is the measured compressive strength.

\[ k_1 = 0.2 \frac{h_{\text{core}}}{d_{\text{core}}} + 0.6 \] (5.2)

\[
k_2 = \begin{cases} 
\frac{0.05}{50 \text{ mm}} (d_{\text{core}} - 70 \text{ mm}) + 0.9 & \text{for } 70 \text{ mm} \leq d_{\text{core}} \leq 100 \text{ mm} \\
\frac{0.05}{50 \text{ mm}} (d_{\text{core}} - 100 \text{ mm}) + 0.95 & \text{for } 100 \text{ mm} \leq d_{\text{core}} \leq 150 \text{ mm}
\end{cases} \] (5.3)

\[
k_3 = \begin{cases} 
\frac{-0.05}{50 \text{ mm}} (d_{\text{core}} - 70 \text{ mm}) + 1.2 & \text{for } 70 \text{ mm} \leq d_{\text{core}} \leq 100 \text{ mm} \\
\frac{-0.05}{50 \text{ mm}} (d_{\text{core}} - 100 \text{ mm}) + 1.15 & \text{for } 100 \text{ mm} \leq d_{\text{core}} \leq 150 \text{ mm}
\end{cases} \] (5.4)

where \( d_{\text{core}} \) and \( h_{\text{core}} \) are the diameter and height of the core, respectively. All compressive strength results of drilled cores shown in this thesis are converted to standard cylinder compressive strength.

**5.1.4 Estimation of the undamaged compressive strength**

To estimate how ASR affects \( f_{c,||} \) and \( f_{c,\perp} \), the **undamaged concrete compressive strength**, \( f_{c,\text{und}} \) is needed. The strength written on the as-built drawings is not a good estimator for \( f_{c,\text{und}} \); it is a lower limit for \( f_c \) that the contractor shall ensure. In this research project, \( f_{c,\text{und}} \) is estimated by means of microscopic examinations (Jensen et al., 1985). This estimation is based on Feret’s formula (Neville, 2011):

\[ f_{c,\text{und}} = 280\text{MPa} \left( \frac{V_c}{V_c + V_w + V_a} \right)^2 \] (5.5)

where \( V_c \), \( V_w \) and \( V_a \) are the absolute volumetric proportions of cement, water and air, respectively. The three proportions are quantified by microscopy on epoxy-impregnated thin section samples (Jensen et al., 1985). The formula holds for ordinary Portland cement. \( f_{c,\text{und}} \), determined by this method, correlates well with \( f_c \) determined by cores drilled from real undamaged concrete structures - new and old (Thaulow et al., 1982).

**5.1.5 Results**

Figure 5.3 and 5.4 show the test results from Gammelrand and Lindenborg, respectively. The results of individual compressive tests are shown as black crosses, and the mean value thereof as red dots. For comparison, the undamaged compressive strength (from (Barbosa et al., 2018a)) is shown as well, where the mean value is shown as a blue dot and the minimum and maximum value are shown as grey whiskers.
5.1.5.1 Gammelrand

Figure 5.3 shows that both $f_{c\parallel}$ and $f_{c\perp}$ are reduced significantly compared to $f_{c,\text{und}}$; up to 50%. Furthermore, it can be seen that $f_{c\parallel}$ is significantly larger than $f_{c\perp}$; in average 4.3 MPa.

![Figure 5.3: Concrete compressive strength parallel with and perpendicular to the ASR-induced cracks for Gammelrand.](image)

Each test result and the conversion are provided in Appendix B.

5.1.5.2 Lindenborg

Figure 5.4 shows the same tendency as Figure 5.3; both $f_{c\parallel}$ and $f_{c\perp}$ are reduced significantly for the most of the slab segments; by up to 50%. However, the reduction of $f_{c\parallel}$ for Slab segment 2 is negligible. Furthermore, it can be seen that $f_{c\parallel}$ is significantly larger than $f_{c\perp}$. The average difference varies from approximately 3 to 11 MPa.
Each test result and the conversion are provided in Appendix C.

5.1.6 Conclusion - Answer to RQ2

It can be concluded that there is a significant anisotropy where $f_{c\perp}$ is lower than $f_{c\parallel}$. For Gammelrand the difference was 4.3 MPa whilst it was up to 11.0 MPa for Lindenborg.

5.2 Anisotropy of sound concrete

This section provides an experimental investigation of the anisotropy of sound concrete. The investigation aims: (i) to determine to what extent the compressive strength in sound concrete is anisotropic and; (ii) to determine what the reason behind this anisotropy is. To enable a systematic investigation of the underlying mechanisms of anisotropy, the influence of a number of parameters on the anisotropy is tested. The investigated parameters are: a) the reference cylinder strength, b) the presence of reinforcement, c) the curing time and d) the type of structural members (slabs versus beams). For the statistical analysis of the experimental results, multiple regression models with interactions of explanatory variables are employed. These models enable a detailed analysis of the significance and the magnitude of the anisotropy and the influences of the studied parameters.

Based on the findings in the investigation, a plausible physical explanation of strength anisotropy of sound concrete is provided.
5.2.1 Definition of anisotropy

The existence of strength anisotropy in sound concrete is often explained by segregation or water migration in the fresh concrete, which causes weak interfaces or initial micro-cracks between the cement paste and the undersurface of the large aggregate particles (Hughes and Ash, 1970; Johnston, 1973; Leshchinsky, 1990; Bartlett and MacGregor, 1995; Ergun and Kurklu, 2012; Van Mier, 1984, 1996). The compressive strength is therefore often indexed by its direction relative to the casting direction: the core compressive strength parallel to the casting direction \((f_{c,0})\) and the core compressive strength perpendicular to the casting direction \((f_{c,90})\), see also Figure 5.5. The most commonly used measure for anisotropy is the ratio between \(f_{c,0}\) and \(f_{c,90}\). Contrary, in this thesis, the anisotropy will mainly be discussed on the basis of an absolute strength difference, i.e.:

\[
\Delta f_c = f_{c,0} - f_{c,90}
\]  

(5.6)

The reason for measuring the anisotropy as a strength difference rather than a strength ratio is that the statistical analyses to be presented below show that the reference cylinder strength (i.e. \(f_{c,\text{cylin}}\)) has no significant influence on \(\Delta f_c\). A more detailed discussion is provided in Section 5.2.6.1.

![Figure 5.5: Illustration of a slab with notation of casting and drilling direction.](image)

5.2.2 Tests

The experimental programme to investigate the anisotropy consists of two test series. Each test series consists of a large number of cores drilled from beam- or slab members produced at a local manufacturer of precast concrete elements.
5.2.2.1 Test series 1 - Slabs

The goal of Test series 1 is to investigate the anisotropy in slabs. The parameters varied in this series are the reference cylinder strength, $f_{c,\text{cylin}}$, (i.e. basically the w/c-ratio) and the presence of reinforcement. The influence of the reinforcement is interesting to investigate because, the reinforcement mesh in flat slabs (without shear reinforcement) may induce unidirectional micro-cracks due to anisotropic shrinkage conditions. These micro-cracks may influence the strength anisotropy (Van Mier, 1996).

The drilled cores in this series were obtained from four slabs with the dimensions 1200 x 1200 x 200 mm. To study the influence of $f_{c,\text{cylin}}$, two slabs were cast with a relatively low $f_{c,\text{cylin}}$ (Mix A) and two slabs were conducted with a relatively high $f_{c,\text{cylin}}$ (Mix B). Details on Mix A and Mix B can be seen in Table 5.2. To study the influence of the presence of reinforcement for both Mix A and Mix B, one slab contained top and bottom mesh reinforcement and one slab contained no reinforcement. The reinforcement meshes consisted of \( \varnothing 6 \) mm rebars per 150 mm in both directions. Each pair of slabs (Mix A and Mix B) was cast from the same batch of concrete. After casting, the slabs cured for 24 hours covered in plastic before they were demoulded, wrapped in plastic and stored indoor until core drilling.

Cores with a diameter of 100 mm were drilled with a water-cooled diamond drill according to the drilling plan displayed in Figure 5.6. The drilling plan ensures that all cores were taken from positions not intersected by rebars. 116 drilled cores were used for compressive tests and 110 were used for split tests (the split tests are not a part of this investigation, see (Aabjerg and Jørgensen, 2016)). The cores, used for compressive tests, were grinded in both ends to ensure plane loading surfaces. The height of the cores after grinding is shown in Appendix D. The cores were tested after 83 (Mix A) and 91 (Mix B) days, respectively.

<table>
<thead>
<tr>
<th>Test series</th>
<th>Max aggregate size [mm]</th>
<th>Aggregate type</th>
<th>Cement</th>
<th>$w/c$</th>
<th>Air-entraining admixture</th>
<th>Super-plasticizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - Mix A</td>
<td>8</td>
<td>Round</td>
<td>Basis Portland (CEM 52.5)</td>
<td>0.60</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>1 - Mix B</td>
<td>8</td>
<td>Round</td>
<td>Basis Portland (CEM 52.5)</td>
<td>0.43</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>Crushed</td>
<td>Rapid Portland (CEM 52.5)</td>
<td>0.40</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

*Table 5.2: Concrete mix composition for Test Series 1 and 2.*

5.2.2.2 Test series 2 - Beams

The goal of Test series 2 is to investigate the strength anisotropy in beams and the influence of curing time. The primary motivation to focus on the curing time in this test series is that a large part of the studies in the literature are based on test specimens with very different curing times. Hence, it is relevant to investigate the correlation and possibly provide a partial explanation for the published results.

The drilled cores were obtained from four unreinforced beams with dimensions 204 x 204 x 1200 mm. Details on the concrete mix composition may be found in Table 5.2.
The beams were cast from the same batch of concrete and cured for 18 hours covered in plastic before they were demoulded. The beams were subsequently wrapped in plastic and stored indoor for further curing until it was time for core drilling. Drilling of the $\varnothing 99$ mm cores took place according to the plan displayed in Figure 5.7. The curing time until core drilling and testing was different for the four beams (2, 4, 8 and 12 weeks respectively). When necessary, the cores were ground before testing to ensure a plane loading surface. The height of the cores after grinding was $200 \pm 1$ mm.

5.2.2.3 Test

All compression tests were carried out in an electro-mechanical compression machine with a capacity of 1200 kN. The load was applied in a displacement-controlled manner with a constant velocity of 1.0 mm/min in test series 1 and 0.42 mm/min in Test series 2. The tests were conducted without interlayers between the steel loading plates and the specimens. The upper loading plate was free to rotate during the entire test, while the lower load plate was in a fixed position.

5.2.3 Multiple linear regression models

For the statistical analysis, multiple linear regression models are employed on the data (Goos and Meintrup, 2016; Bowerman et al., 2003). An introduction to the theoretical background and the adopted regression model, will together with the actual statistical calculations be provided in Appendix D. In this thesis, a significance level of 5% is used.
5.2.4 Results, Test series 1

This section provides a graphical presentation of the test results of Test series 1 and shows the outcome of the statistical calculations. Tabulated results and the statistical analysis are provided in Appendix D.

Figure 5.8 shows a box plot of the test results from Test series 1. The following can be observed:

• The median of $f_c$ is higher for Mix B than for Mix A. This is expected, as $f_{c,\text{cylin}}$ of Mix B is higher than for Mix A.

• The median of $f_c$ seems to be slightly higher for the slabs with reinforcement than for the slabs without reinforcement.

• $f_{c,90}$ seems systematically lower than $f_{c,0}$.
The statistical calculations show that:

- The influence of the reference cylinder strength is statistical insignificant ($p = 0.38$).
- The influence of presence of reinforcement is statistical insignificant ($p = 0.065$).
- There is extremely strong evidence that the compressive strength is anisotropic ($p < 0.0001$).
- The mean anisotropy ($\Delta f_c = f_{c,0} - f_{c,90}$) is 4.5 MPa.

### 5.2.5 Results, Test series 2

This section provides a graphical presentation of the test results of Test series 2 and shows the outcome of the statistical calculations. Tabulated results and the statistical analysis are provided in Appendix D.

Figure 5.9 shows a box plot of the test results from Test series 2. The following can be observed:

- $f_c$ increases with increasing curing time, as expected.
• For 2 weeks, 4 weeks as well as 12 weeks curing, $f_{c,90}$ is slightly higher than $f_{c,0}$. This is opposite to the results from Test series 1, where $f_{c,90}$ was systematically lower than $f_{c,0}$. A detailed discussion on this is provided in Section 5.2.6.3.

![Figure 5.9: Box plot of test results ($f_c$) for Test series 2.](image)

The statical calculations show that:

• The influence of the curing time is statistical insignificant ($p = 0.652$).
• There is strong evidence that the compressive strength is anisotropic ($p = 0.0033$).
• The mean anisotropy ($\Delta f_c = f_{c,0} - f_{c,90}$) is -2.5 MPa.

### 5.2.6 Discussion

The experimental findings are discussed in this section.

#### 5.2.6.1 Reference cylinder strength

The statistical analysis of the results from Test series 1 shows that the reference cylinder strength, $f_{c,cylin}$, has no significant influence on the anisotropy in slabs if the anisotropy is measured as a strength difference, $\Delta f_c$. In the literature, the anisotropy is often measured as a strength ratio, $f_{c,0}/f_{c,90}$. In that case, Test series 1 shows that the anisotropy
is decreasing for increasing $f_{c,\text{cylin}}$. This finding contrasts with some findings in the existing literature (Johnston, 1973; Ergun and Kurklu, 2012). Possible explanations to the diverging findings are provided below.

Johnston (1973) concluded that for practical purposes $f_{c,0}/f_{c,90}$ should be taken as a constant (approximately 1.08) independent of $f_{c,\text{cylin}}$ (he tested concrete with w/c-ratios from 0.35 to 0.65). However, it should be noted that there was a variation in the mean value of $f_{c,0}/f_{c,90}$ for different w/c-ratios and the suggested constant of 1.08 was simply the average of these mean values.

Similarly, Ergun and Kurklu (2012) concluded that $f_{c,0}/f_{c,90}$ is constant (approximately 1.04) for concrete class C16 to C22 and $\varnothing$100 x 200 mm cores. However, the conclusion was drawn by comparing the mean anisotropy without distinguishing between the concrete classes. Thus, like the case of Johnston, the significance of the influence of $f_{c,\text{cylin}}$ was not analysed.

Since Test series 1 shows that $f_{c,\text{cylin}}$ has no significant influence on $\Delta f_c$ and there exists no counterevidence based on statistical analysis in the literature, it is more correct to present the anisotropy as a strength difference, see Equation 5.6, rather than the currently applied strength ratio. Adoption of the strength difference to describe the anisotropy would in fact shed new light on the apparently drastic finding of Hughes and Ash (1970), who reported 50% anisotropy. By converting the results of Hughes and Ash into a strength difference, it is found that their results correspond to only 6 MPa difference between $f_{c,0}$ and $f_{c,90}$. In this light, the finding by Hughes and Ash (1970) seems less dramatic, and is actually similar to the finding in this thesis.

### 5.2.6.2 Presence of reinforcement

The statistical analysis of the experimental results from Test series 1 shows that the presence of reinforcement has no significant influence on the anisotropy in slabs.

### 5.2.6.3 Curing time

The statistical analysis of the experimental results from Test series 2 shows that, the curing time has no significant influence on the anisotropy in beams. Since there is a direct relation between curing time and reference cylinder strength, the result here actually just confirms the conclusion above; namely that $f_{c,\text{cylin}}$ has no significant influence on the anisotropy (in terms of $\Delta f_c$). Hence, the curing time cannot be used to provide additional explanations to the large variation of the strength anisotropy (in terms of $f_{c,0}/f_{c,90}$) published in the literature.

### 5.2.6.4 Type of structural member

Test series 1 shows that the anisotropy is positive (4.5 MPa) for slabs and Test series 2 shows that the anisotropy is negative (-2.5 MPa) for beams. These results show that anisotropy is governed by more than just the weak interfaces or micro-cracks between the cement paste and the undersurface of the large aggregate particles (which is the usual
explanation). Otherwise, the anisotropy would have the same sign. It is well known that $f_c$ has a spatial variation, see e.g. (Bartlett and MacGregor, 1995). In the slabs of Test Series 1, the drilling locations for core90 and core0 are sufficiently interspersed to neglect the influence on the measured anisotropy. For the beams of Test Series 2, on the other hand, the drilling locations for core90 and core0 were at each end of the beams. Therefore, spatial variation might have influenced the measured anisotropy in Test Series 2. This may be the reason why the anisotropy varies for the four beams. However, it is unlikely that it has an influence on the overall tendency, namely the anisotropy was positive for the slab members and negative for the beams. The reason for the influence of the type of structural member is further discussed in Section 5.2.7

5.2.7 Physical explanation of anisotropy in sound concrete

Based on the findings in Test series 1 and 2, a physical explanation of anisotropy in sound concrete is made. Van Mier (1996) states that "... behaviour at one level can be explained in term of the material structure observed at a lower level". The anisotropy of the compressive strength is a macro-level behaviour. At this level, the concrete is considered as one material (continuum) where the internal structure is not considered. Therefore, anisotropy must be explained on a meso-level, where the concrete is considered as a composition of individual sand and aggregate particles in a paste of hydrated cement (Wittmann, 1983). The plausible explanation is based on a meso-level approach for both fresh and hardened concrete. The explanation consists of two parts. The first part explains the formation of weak interfaces between the aggregate particles and the cement paste. The second part explains how these weak interfaces influence the compressive strength and induce strength anisotropy. Finally, the proposed explanation is qualitatively evaluated against the experimental tendencies observed in the presented tests as well as tests from the literature.

5.2.7.1 Formation of weak interfaces

The strength anisotropy originates from weak interfaces (or micro-cracks) between the larger aggregate particles and the surrounding cement paste. There are many plausible reasons for formation of these weak interfaces, e.g. segregation in the fresh concrete, shrinkage (Van Mier, 1996), thermal effects, external loads etc. (Nielsen and Hoang, 2010). A discussion of the weak interfaces is therefore needed to explain and understand the anisotropy found in Test series 1 and 2 and in the literature. The main reason for the existence of weak interfaces in the considered test series is probably segregation. The proposed explanation is therefore based hereon.

Segregation in the fresh concrete may lead to formation of weak interfaces when larger aggregate particles separate from the cement paste. The separation occurs when the aggregate particle moves through the more or less liquid cement paste. Figure 5.10 shows an aggregate particle moving (relatively) through the cement paste, where $\mathbf{u}$ describes the direction of the relative motion. Due to this relative movement, a weak interface
forms between the cement paste and the surface of aggregate particle facing away from the direction of movement. 

![Diagram of cement paste, weak interface, aggregate particle, and direction of relative motion]

**Figure 5.10:** Weak interface formed when an aggregate particle moves through the cement paste.

In some literature, segregation is described in terms of *dynamic segregation* when the concrete is flowing, e.g. during casting or vibration, and *static segregation* when the concrete is at rest, see e.g. (Shen et al., 2009). For static segregation, it is well known that the aggregate particles move downward through the cement paste due to their higher density. Therefore, the static segregation leads to horizontal weak interfaces (Kovler and Roussel, 2011). It is also well known that flowing concrete can lead to (dynamic) segregation as well. For example, if the concrete flows or is worked along the form, or in case of improper use of vibrator, see e.g. (Neville, 2011). However, dynamic segregation is often not mentioned in the discussion of strength anisotropy. The segregation in the flowing concrete occurs due to asynchronous movements of the cement paste and the aggregate particles during acceleration/deceleration of the concrete flow due to a density difference. The concrete flow, and thereby the relative movements of the aggregate particles, are strongly influenced by the geometry of the formwork. This is illustrated in Figure 5.11 and 5.12, which schematically show the concrete flow for slabs and beams during casting and vibration, respectively. Figure 5.11 shows the concrete flow trajectories and the formation of the weak interfaces during casting of a slab (left) and a beam (right). For the slab, the concrete flow trajectories will follow a radial/horizontal path when moving away from the pouring point. Due to a higher density (higher level of kinetic energy), the aggregate particles move horizontally through the cement paste during deceleration of the concrete flow. Therefore, weak interfaces will form vertically. For the beam, on the other hand, the concrete flow is confined by the boundaries of the narrow formwork. The flow therefore becomes more turbulent than in the case of the slab. Consequently, the relative movements of the aggregates, and thereby the weak interfaces, are randomly orientated.
Figure 5.12 illustrates the movements of the aggregate particles during internal vibration (needle vibrator) for a slab (left) and a beam (right), respectively. In the slab formwork, vibration induces mainly horizontal accelerations in the concrete. Due to the different densities, the aggregate particles will move horizontally through the cement paste and the weak interfaces will form vertically. In a beam formwork, vibration will also mainly induce horizontal accelerations. However, the form sides will confine the concrete flow and induces turbulence locally near these boundaries. Consequently, the weak interfaces are randomly orientated in the zone with turbulent concrete flow.

Figure 5.11: Concrete flow and weak interfaces during casting of slabs (left) and beams (right).

Figure 5.12: Movement of aggregate particles relative to the cement paste and weak interfaces during vibration of slabs (left) and beams (right).

The segregation is a two-phasic phenomenon: first the dynamic segregation during casting and vibration and second the static segregation in the stagnant fresh concrete. In the following, dynamic segregation is considered to have the dominating effect on formation of the weak interfaces (and thereby on the anisotropy) and only in cases where
dynamic segregation leads to randomly orientated weak interfaces (i.e. no anisotropy), the subsequence static segregation may have a governing effect. It follows from this assumption and from the discussions above that in narrow beams, static segregation will be governing and the horizontal weak interfaces will dominate. In slabs and wide beams, on the other hand, the dynamic segregation leads to vertical weak interfaces, which will dominate.

The existence and orientation of the weak interfaces are not directly measurable (Neville, 2011) but are crucial for the compressive strength of drilled cores.

5.2.7.2 The influence of weak interfaces

There exist some explanations on how the weak interfaces induce a strength anisotropy, see e.g. (Van Mier, 1996). The explanation in this thesis is based on Vile’s explanation of stress-carrying mechanisms around a single aggregate particle embedded in cement paste (continuous matrix), see Figure 5.13a (Vile, 1968). In Vile’s explanation, initial micro-cracks are formed between the aggregate particle and the cement paste; parallel to the loading direction. These micro-cracks are stable until load-induced cracks are formed. It is assumed that the aggregate particle has a higher stiffness (or lower Poisson’s ratio) than the cement paste. Therefore, shear stresses between the aggregate particle and the cement paste will be induced during loading. These shear stresses will confine a cement cone above and below the aggregate particle. The confinement increases the strength of these cement cones. First when shear (‘en-echelon’) cracks form between the confined cones and the surrounding cement paste, the development of the initial micro-cracks, and thereby the failure, can proceed. The favourable confinement depends naturally on the interface (bond) between the aggregate particle and the cement paste.

Weak interfaces can easily be included qualitatively in Vile’s explanation. Figure 5.13b shows an embedded aggregate particle where the interface below the particle is weakened. The weak interface has a reduced ability to transfer confining shear stresses between the aggregate particle and the cement cone. Consequently, the compressive strength of the cement cone may be reduced to a degree where a compressive failure of cement cone occurs rather than a shear failure between the cement cone and the unconfined cement. In this case, the weak interface will reduce the compressive strength of the concrete. Figure 5.13c shows an embedded aggregate particle where the interface on the left side of the particle is weakened. Vile’s explanation assumes that initial micro-cracks parallel to the loading direction are stable and the stresses in the aggregate particle are transferred by the confined cones until load-induced cracks form. Therefore, the weak interface does not affect the stress-carrying mechanisms and the compressive strength is thereby unaffected. Overall, cores with weak interfaces perpendicular to the loading direction have a lower compressive strength than cores with weak interfaces parallel to the loading direction.
5.2.7.3 Expected anisotropy

According to the proposed explanation, the geometry of the structural member has a significant influence on the strength anisotropy. In slabs and wide beams, it is expected that $f_{c,0}$ is larger than $f_{c,90}$ (positive anisotropy) due to the vertical weak interfaces from the dynamic segregation. In beams with a small width, it is expected that the horizontal weak interfaces due to static segregation will be dominating. Consequently, $f_{c,90}$ is expected to be larger than $f_{c,0}$ (negative anisotropy). It was assumed that the static segregation does only have a governing effect if the dynamic segregation does not lead to anisotropy. Thus, the anisotropy might be smaller when static segregation is governing (narrow beams) than when dynamic segregation is governing (slabs and wide beams).

Since the static segregation is assumed to have less influence on the anisotropy, it is expected that the strength difference is smaller than for slabs.

Since segregation is based on the movement of the aggregate particles relative to the cement paste in the fresh concrete, many factors may influence the orientation of the weak interfaces, and thereby the anisotropy. For example the density, size and shape of the aggregate particles, viscosity of the cement paste and casting and vibration procedures. Consequently, a large experimental scatter is therefore expected.
5.2.7.4 Relation between experimental tendencies and proposed explanation of anisotropy

The tendency of the test results from Test series 1 and 2 agrees with the proposed explanation; i.e. the anisotropy for the slabs was positive (4.5 MPa) whilst the anisotropy for the beams was negative (-2.5 MPa).

Den Uijl and Yang (2009) found a positive anisotropy (10.3 MPa) for beams with a width of 600 mm. Due to the large width of the section, it is expected that vertically orientated weak interfaces are formed due to dynamic segregation. Therefore, the test results are in line with the proposed explanation.

Van Mier (1984) found no anisotropy in prisms with dimensions 700 x 135 x 135 mm. Due to the small width of the cross section, turbulent concrete flows are expected during casting and vibration. This is in good agreement with the proposed explanation where a negative and small anisotropy is expected.

Ergun and Kurklu (2012) found a slightly positive anisotropy (0.6 MPa) for beams with a cross section of 200 x 200 mm. The fact that the anisotropy was reported to be small (i.e. neglectable) is in good agreement with the explanation due to the turbulent concrete flows during casting and vibration. The small positive anisotropy might be due to scatter or spatial variation, since core_90 and core_90 were drilled from different beams.

Johnston (1973) found overall a positive anisotropy (-2 to 5 MPa) for cubes cut from rectangular specimens with dimensions of 750 x 150 x 150 and 750 x 100 x 100 mm. This is not in good agreement with the proposed explanation. However, Van Mier found no anisotropy for similar prisms cut from rectangular specimens. It should here be noted that Johnston showed a failure surface of a vertical prism after testing in uniaxial tension, where horizontal weak interfaces are easily seen. The fact that the orientation of the weak interfaces is horizontal is in good agreement with the proposed explanation, where static segregation is expected to be governing.

5.3 Comparison of anisotropy

The tested compressive strengths of the ASR-damaged bridges Gammelrand and Lindenborg show a significant anisotropy, see Section 5.1. Significant anisotropy is also seen for sound concrete, see Section 5.2. To investigate to what extent the anisotropy for the ASR-damaged slabs (Gammelrand and Lindenborg) is due to ASR, it is investigated whether it can be explained by the anisotropy in sound concrete.

In Section 5.2.7, it is concluded that static and dynamic segregation are the reason for anisotropy in sound concrete. It is argued that due to dynamic segregation, induced by concrete casting flow and vibration, the vertical interfaces between the aggregates and the cement paste are weakened for slabs. Consequently, the horizontal compressive strength will be lower than the vertical. This finding is confirmed experimentally by Test series 1, see Section 5.2.4. The investigation of the anisotropy of compressive strength of the ASR-damaged slabs shows that the vertical compressive strength is significantly lower than the horizontal compressive strength, i.e. opposite as predicted for sound
concrete. It can therefore be concluded that the anisotropy in ASR-damaged slabs is induced by ASR.

5.3.1 Answer to RQ3

It can be concluded that the anisotropy in ASR-damaged slabs is induced by ASR.

5.4 Analysis of the compressive strength in ASR-damaged slabs

This section analyses the compressive strength in ASR-damaged slabs. The analysis will consist of a comparison of a number of characteristics that describe the behaviour of the concrete during compressive loading. To conduct such a comparison, an extra test series has been carried where a Digital Image Correlation (DIC) system was employed such that strain and deformation fields can be generated. The test series includes compressive tests of prisms cut from the beams from Gammelrand. First, a conceptual description of the aforementioned characteristics for sound concrete is presented as a basis for the analysis, followed by a short introduction of the test series and DIC. Hereafter, the actual analyses of \( f_{c\perp} \) and \( f_{c\parallel} \) are presented.

5.4.1 Characteristics of the compressive behaviour of sound concrete

This section is based on decades of research on the mechanical properties of concrete presented in (Neville, 2011) and (Van Mier, 1996).

It is broadly acknowledged that the compressive behaviour of concrete is governed by crack progression. Since DIC is a novel method within concrete research and the crack progression often occurs inside the concrete (not on the surface where DIC can measure), the crack progression is often quantified by means of axial and lateral deformations presented by stress-strain (\( \sigma-\varepsilon \)) curves. This section provides first a qualitative explanation of the mechanisms behind crack progression and subsequently the characteristics of the compressive behaviour are described by means of \( \sigma-\varepsilon \) curves along with a description of the underlying crack progressions.

If concrete is seen as one continuum (macro-level), there are no reasons of crack progression before failure. Therefore, the crack progression mechanisms are explained on a meso-level (Wittmann, 1983). The essential feature of this level is the mechanical properties of the aggregates and the cement paste and their interfaces. The qualitative explanation of crack progression mechanisms used in this thesis is provided by Vile (1968); the same explanation that was used to explain anisotropy in sound concrete, see Section 5.2.7.2. To ease the reading, the explanation is reintroduced with focus on the crack progression. The explanation simplifies the mechanisms to a 2D process around a single aggregate particle embedded in cement paste, see Figure 5.14a. Already before
external load is applied, cracks are formed at the interface between the coarse aggregate particles and the cement paste. These cracks origin from e.g. shrinkage, segregation or difference in the thermal properties. However, these interfacial cracks are stable until load-induced cracks in the cement paste are formed. For typical concretes, the aggregate particles have a higher stiffness than the cement paste. Therefore, shear stresses will occur between the particle and cement during loading. These shear stresses will confine a cement cone above and below the particle. The confinement increases the strength of the cement cones. First when the shear (en-echelon) cracks form between the confined cones and the surrounding cement paste, the pre-formed cracks can grow further. Figure 5.14b shows a vertical section of a concrete cylinder with a symbolic number of coarse aggregate particles with associated cracks in the cement paste. When additional load is applied, the cracks are bridging, see Figure 5.14c.

Figure 5.14: Schematic illustration of the qualitative explanation on crack progression.

Figure 5.15 shows conceptual $\sigma$-$\varepsilon$ curves for a cylinder during compressive loading for
normal strength concrete where stresses and strains are positive as compression. The left curve shows the lateral strains, $\varepsilon_{lat}$, (perpendicular to the loading direction) and the right curve shows the axial strains, $\varepsilon_{axial}$, (parallel to the loading direction). The grey curve shows the volumetric strains, $\varepsilon_v$, that is defined as:

$$\varepsilon_v = \varepsilon_{axial} + 2 \cdot \varepsilon_{lat}$$  \hspace{1cm} (5.7)

Figure 5.15: Conceptual curves for axial, lateral and volumetric strains in a concrete cylinder during compressive testing. Adapted from (Neville, 2011).

The first characteristic occurs at a low stress stage ($\sigma < \sim0.3f_c$). Here, both the $\sigma$-$\varepsilon_{axial}$ and $\sigma$-$\varepsilon_{lat}$ curve are $\sim$linear. Thus, Possion’s ratio is constant. It is often in this interval that Young’s modulus is determined. At this stage, the pre-formed cracks are stable. The second characteristic occurs at a medium stress stage ($\sim0.3f_c < \sigma < \sim0.7f_c$). Here, the $\sigma$-$\varepsilon_{axial}$ and $\sigma$-$\varepsilon_{lat}$ curve become non-linear. This has been explained by slow and stable growth of the pre-formed cracks. The third characteristic occurs at a high stress stage ($\sigma > \sim0.7f_c$) and is often referred to as the discontinuity point ($\varepsilon_v$ stops increasing). Here, the $\varepsilon_{lat}$ curve bends further and a large lateral expanding occurs. This behaviour is caused by unstable crack growth and bridging as shown on Figure 5.14b and 5.14c. A constant load above this point will lead to failure over time. The last characteristic is the failure ($\sigma = f_c$). Two failure mechanisms are often seen. 1) A splitting mechanism where vertical cracks open horizontally. 2) A sliding mechanism where the failure occurs in an inclined shear plan. Since the compressive strength may be governed by crack progression in the cement paste, it will depend on the strength hereof. Therefore, $f_c$ depends significantly on the water/cement-ratio.
5.4.2 Test series

5.4.2.1 Prisms from ASR-damaged concrete

In the test series that is conducted by Alberg and Petersen (2014), compression tests are conducted with eight prisms cut from Gammelrand with a dimension of $100 \times 100 \times 200$ mm. As for the cores, the prisms were loaded parallel ($\text{prism}_\parallel$) and perpendicular ($\text{prism}_\perp$) to the ASR-induced cracks. For convenience, the four prisms $\perp$ are named H1, H2, H3 and H4 (H for horizontal cracks during loading) and the four prisms $\parallel$ are named V1, V2, V3 and V4 (V for vertical cracks during loading). The prisms are tested as the cores (see Section 5.1.2).

Since the presented method for converting the compressive strength (see Section 5.1.3) does not apply for cut prisms, the compressive strengths of the prisms are shown raw (unconverted). Therefore, the strength of the prisms cannot be compared with the strength of the cores (shown in section 5.1.5) despite that the prisms and cores originate from the same bridge.

5.4.3 Digital Image Correlation (DIC)

In order to generate strain and deformation fields, where cracks can be detected and deformation measured, DIC is employed. DIC is an optical measuring technique, where the test specimen is filmed during the experiment by two cameras. Based on the grey-scale variations in the pictures from the two cameras, DIC can generate a 3D surface of the filmed surface. By means of tracking techniques, DIC can generate strain and deformation fields for the entire surface without having physical contact. In the tests with the prisms, a 4M stereo system (2 cameras of 4 megapixel) was employed. During the tests, a shadowing steel bar was in the front of the prisms. Therefore, the DIC analysis gives either no or wrong results in this area. Consequently, this area is ignored.

5.4.3.1 Definitions

Based on the DIC measurements, a surface of the prisms is generated for each measurement (set of pictures). The reference surface, generated from the first set of pictures, is placed in a reference coordinate system such that the origin of the coordinate system is placed in the bottom left corner of the surface, see Figure 5.16. Later results will refer to this coordinate system.
By comparing generated surfaces from different set of pictures (and thereby different load stages), strain and deformation fields can be generated. When these fields are plotted, grey is neutral, compression is blue and expansion is red. When the strains or deformation are quantified and plotted, compression is defined as positive. The same holds for stresses and forces.

5.4.3.2 Analysis points and sections

Besides using the DIC measurements to generate strain and deformation fields, they are used for calculating relative displacements and sectional analysis. For this purpose, analysis points and sections are applied on the generated surface. Figure 5.17a shows 5 pairs of analysis points consisting of a point in the top and a point in the bottom of the prism with a distance $H'$. The pairs are named $x_1$ to $x_5$. The relative displacement of each pair is used for measuring the overall vertical deformation. Similarly, Figure 5.17b shows 5 pairs of analysis points consisting of a point in each side of the prism with a horizontal distance $B'$. These pairs are named $y_1$ to $y_5$. The relative displacement of these pairs is used for measuring the overall horizontal deformation. Figure 5.17c shows three vertical analysis sections. The red section is placed in the centre of the prism and the green and black sections are shifted 40 mm to the left and to the right, respectively. These are used for measuring how the vertical displacement varies in the height of the prism (as function of the $y$-coordinate). Figure 5.17d shows a pair of analysis points with a point distance $h'$. This pair is placed between two adjacent ASR-induced macro-cracks, and are used for calculating the displacement between the macro-cracks.
5.4.4 Analysis of ASR-damaged concrete

For sound concrete the characteristics were described by load-deformation curves where axial and lateral strain/deformation were considered, see 5.4.1. The same deformations are investigated for ASR-damaged concrete together with crack behaviour (formation, opening and closure). This section consists of 1) a visual inspection, 2) a quantification of $f_c$ and $E$, 3) an analysis of the axial deformation, 4) an analysis of the lateral deformation, 5) an analysis of the volumetric strains and 6) a plausible explanation of the crack formation mechanisms. All is conducted for prism$_{\perp}$ and prism$_{\parallel}$ individually. Finally, the findings for prism$_{\perp}$ and prism$_{\parallel}$ are compared.

5.4.4.1 Analysis of prims$_{\perp}$

5.4.4.1.1 Visual inspection

Figure 5.18 shows the prisms$_{\perp}$ before loading. The ASR-induced cracks are marked with black lines and the thickness of the lines roughly illustrates the crack widths.

- Prism H1 has one coarse through-going crack near the top and some finer cracks, mostly placed at the right side of the prism.

- Prism H2 has one coarse, through-going crack placed in the middle of the lower half of the prism. Additionally, there are finer cracks in more places in the prism.
• Prism H3 does not have any coarse cracks, but has more finer cracks placed around the prism. Most of the finer cracks are placed in the top half.

• Prism H4 has several cracks, both coarse and fine and some through-going. The cracks are spread around, but the main part is placed in the top half of the prism.

Based on the visual inspection, Prism H4 seems more damaged than the other three prisms. Prism H1 and Prism H2 seem to be similar damaged while Prism H3 seems less damaged with only finer cracks.

![Figure 5.18: Prisms] before loading with marking of ASR-induced cracks. From left: H1, H2, H3 and H4.

5.4.4.1.2 Quantification of the compressive strength and Young’s modulus

Table 5.3 shows the measured compressive strength, $f_{c\perp}$, and Young’s modulus, $E_{\perp}$, for the four prisms. The employed strains for calculation of $E_{\perp}$ are found as the relative vertical displacement ($\Delta H'/H'$) derived from analysis points $x_3$, see Figure 5.17. It can be seen that Prism H1, H2 and H3 have similar $f_{c\perp}$ whilst $f_{c\perp}$ for Prism H4 is approximately 20% lower. Besides being the weakest, Prism H4 had the most ASR-induced cracks, see Figure 5.18. Equivalently, Young’s modulus ($E_{\perp}$) is lowest for Prism H4.
<table>
<thead>
<tr>
<th>Prism</th>
<th>Compressive strength $f_{c\perp}$ [MPa]</th>
<th>Young’s modulus $E_{\perp}$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>13.7</td>
<td>2.7</td>
</tr>
<tr>
<td>H2</td>
<td>13.8</td>
<td>3.6</td>
</tr>
<tr>
<td>H3</td>
<td>13.9</td>
<td>3.0</td>
</tr>
<tr>
<td>H4</td>
<td>11.1</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 5.3: Compressive strength and Young’s modulus of prisms $\perp$

5.4.4.1.3 Axial deformation

Figure 5.19 shows the stress-strain curves for the four prisms derived from analysis points $x_3$. It is seen that the fundamental shape of the four curves are identical. However, the curve for Prism H4 is flatter and the peak (the strength) is lower.

![Figure 5.19: Stress-strain ($\sigma$-$\epsilon_{axial}$) curves for the compressive test of prisms $\perp$](image)

Figure 5.20 shows an example of the strain progression during the compressive loading of Prism H1. The figure consists of five subfigures, (A) to (E). Subfigure (A) shows the prism before loading with marked ASR-induced cracks. Subfigures (B) to (E) show a minor strain (compression) plot (left) and a major strain (tension) plot (right) derived from the DIC measurements for four load stages. The four load stages are shown as red dots on the stress-strain curve, see Figure 5.19. Subfigure B) shows the strains when a load of 4.0 MPa is applied. The minor strain plot shows closure of the ASR-induced cracks while the major strain plot only shows noise. Subfigure C) shows the strains when a load of 7.0 MPa is applied. The minor strain plot shows further closure of the ASR-induced cracks, while the major stain plot shows that vertical cracks are formed in the prism (marked with black arrows). In the top of the prism, the vertical cracks are formed near the coarse ASR-induced crack that has been closed due to the loading.
In the bottom, a vertical crack is formed near the tip of another closing ASR-induced crack. Subfigure (D) shows the strains when a load of 11.0 MPa is applied. The vertical cracks shown in Subfigure (C) have grown. Subfigure (E) shows the strains at failure (13.7 MPa). The major strain plot shows that several coarse vertical cracks (splitting cracks) are formed. The minor strain plot shows that beside closure of the ASR-induced cracks, there are also compressive strains in an inclined (almost vertical) load-induced crack.

These four stages occur for all four prims. However, the stages are not as distinct for Prism H3 that had no coarse ASR-induced cracks.

To investigate the closure of ASR-induced cracks and the axial deformations further, the axial deformation, \( \Delta y \), is computed in the three analysis sections shown in Figure 5.17 (C). Figure 5.21 shows how \( \Delta y \) varies along the height of the prism (represented by the \( y \)-coordinate in the reference coordinate system, see Figure 5.16) in the three analysis sections for three load stages. \( y = 0 \) mm is the bottom and \( y = 200 \) mm is the top of the prism and the curve colour matches the colour of the associated analysis section. In the corner of each subfigure, the associated minor strain plot is shown. Figure 5.21a shows \( \Delta y \) when a load of 4.0 MPa is applied. The curves look linear with some discrete jumps. These jumps are closure of ASR-induced cracks. It is seen that the deformations in the left (green) and the centre (red) section are very similar; the majority of the deformation occurs as closure of an ASR-induced crack in the top of the prism (\( y \approx 165 \) mm). The deformation in the right section (black) differs from the deformations in the two other sections. Firstly, the total deformation (the difference between \( \Delta y \) in the top and the bottom) is smaller (approx. 0.2 mm vs. 0.3 mm). Secondly, the deformation occurs as closure of two ASR-induced cracks (\( y \approx 60 \) mm and \( y \approx 150 \) mm). This difference means that an ASR-induced skewness occurs. Figure 5.21b shows that the same behaviour continuous when a load of 7.0 MPa is applied (the load stage where the first vertical load-induced cracks occurs, see Figure 5.20). However, crack closure of more ASR-induced cracks becomes visible. Figure 5.21c shows that the deformation curves have more jumps when a load of 11.0 MPa is applied. This is partly due to closure of ASR-induced cracks and partly due to movements in load-induced \( \sim \)vertical cracks, see load stage (D) in Figure 5.20. All in all, the axial deformations for prisms\( \perp \) are characterised by:

- Closure of ASR-induced cracks
- ASR-induced skewness
- Linear deformation between the ASR-induced cracks

The closure of ASR-induced cracks and the ASR-induced skewness are studied further by comparing stress-strain curves for the five pairs of analysis points (\( x_1 \) to \( x_5 \), see Figure 5.17). The curves are shown in Figure 5.22. The curves have two phases: before and after formation of vertical cracks. The load stage (7.0 MPa) where the first load-induced vertical cracks are formed is shown as a grey horizontal line. When the closure of ASR-induced cracks and ASR-induced skewness are investigated, the first phase, without
influence of load-induced cracks is of interest. The figure shows that the right side of the prism ($x_4$ and $x_5$) is stiffer than the centre ($x_3$) and the left side ($x_1$ and $x_2$). According to Figure 5.21, this is due to closure of ASR-induced cracks; the sum of the crack closure is simply smaller in the right side. From Figure 5.22 it seen that the difference in ‘overall’ strains ($\Delta H'/H'$) between the two sides of the prism, and thereby the crack closure, is approximately proportional to the applied load. This can also be seen on Figure 5.21,
Figure 5.21: Axial deformation ($\Delta y$) of Prism H1 as function of the height of the prism, $y$-coordinate, in three analysis sections at three load stages. The corresponding minor strain plots are shown in the corner.

where the closure of the ASR-induced cracks (the jump in the curves) is increasing for increasing load. Thus, the ASR-induced cracks must have a stiffness and is not just closed fully instantaneously when a small is applied.

Based on the visual inspection, see Figure 5.18, the crack width of the upper ASR-induced crack was larger than the crack width of the rest. The upper crack did also have the largest closure during loading. Thus, it seems that the crack stiffness depends on the crack width.

Figure 5.22: Stress-strain ($\sigma$-$\varepsilon_{\text{axial}}$) curves based on analysis point $x_1$ to $x_5$ for Prism H1.
As proposed by Hiroi et al. (2016), the low Young’s modulus perpendicular to the ASR-induced cracks \( E_\perp \) is due to crack closure of the macro-cracks. To investigate how much this crack closure means for the Young’s modulus, the overall stiffness of the prism is compared with the stiffness between the macro-cracks. This is done by comparing two stress-strain curves. The first employs the ‘overall’ strains \( (\Delta H'/H') \) and the second employs the strain between the macro-cracks \( (\Delta h'/h') \). Since the last curve represents the stiffness of the concrete with micro-cracks it is named ‘micro-cracks’. Figure 5.17 shows the positions of the analysis points in the DIC analysis and Figure 5.23 shows the stress-strain curves. It is seen from the stress-strain curves that the stiffness of the micro-cracked concrete is significantly higher than the overall stiffness for low loads. The first load-induced cracks are seen at \( \sim 7 \) MPa; hereafter, the tangential slope of the curves seems equal.

\[ \begin{align*}
\sigma \text{ [MPa]} & \quad \frac{\Delta h'}{h'}; \frac{\Delta H'}{H'} \text{ [%]} \\
\text{H2 - 'micro-cracks'} & \quad \text{H2 - 'overall'}
\end{align*} \]

*Figure 5.23: Stress-strain curves for Prism H2 measured over the full prism height \( (H') \) and between macro-cracks \( (h') \).*

### 5.4.4.1.4 Lateral deformation

Figure 5.24 shows the curves for the lateral strains \( (\sigma-\varepsilon_{\text{lat}}) \) during compressive loading. The slope of the initial part of the curves is more or less constant, and there are almost no lateral strains. At a point, the curves break and the slope decreases. For Prism H1 and H2, the curves break around 7 MPa while the curves for Prism H3 and H4 break significantly before; namely around 4 and 2 MPa, respectively. The nature of the breaks of the curves does also differ; for Prism H1 and H2, the break appears sudden while it appears more gradual for Prism H3 and H4.
Figure 5.24: ($\sigma$-$\varepsilon_{\text{lat}}$) curves for the compressive test of prisms $\perp$.

Figure 5.25 shows the major strain plots for the four prisms at two load stages: the load stage where the $\sigma$-$\varepsilon_{\text{lat}}$ curves break and the load stage where several of vertical cracks are formed ($\sigma = 0.72f_c$). The upper row of plots shows the major strains for the first load stage (marked with red dots in Figure 5.24). It is seen that vertical cracks are forming. Thus, it is like for sound concrete, the first change of slope of the $\sigma$-$\varepsilon_{\text{lat}}$ curve is caused by crack growth. The vertical cracks are formed around the horizontal ASR-induced cracks, see an example for Prism H1 on Figure 5.24(C). The lower row of plots shows the major strains for the second load stage ($\sigma = 0.72f_c$, marked with black dots in Figure 5.24). It is seen that Prism H1 and H2 have one dominating coarse vertical crack, while Prism H3 has more, but finer and evenly distributed cracks. Prism H4 has several of coarse cracks. Consequently, it seems like there is a correlation between the size of the horizontal ASR-induced cracks and size of the vertical load-induced cracks; the coarser the ASR-induced cracks are, the coarser the load-induced cracks are. Furthermore, it seems like the number of ASR-induced cracks is correlated to the number of load-induced cracks as well.
Figure 5.25: Major strain plots for the four prisms. The upper row corresponds to the load stage where the $\sigma$-$\varepsilon$lat curves break. The lower row corresponds to $\sigma = 0.72f_c$.

5.4.4.1.5 Volumetric strains

Figure 5.26 shows the volumetric strain ($\sigma$-$\varepsilon_v$) curve for the four prisms$_\perp$. It is seen
that the "discontinuity point" occurs when $\sigma \approx 0.35 - 0.55f_c$. For sound concrete it occurs when $\sigma \approx 0.7f_c$. Thus, the compressive behaviour of prism$_\perp$ is not a down-scale of the compressive behaviour for sound concrete where the ratio between the load for the discontinuity point and the failure load is the same. This may be due to a different mechanism for formation of vertical cracks.

![Figure 5.26: ($\sigma$-$\varepsilon_v$) curves for compressive loading of prisms$_\perp$.](image)

5.4.4.1.6 Explanation of crack formation mechanisms

In the DIC analysis two types of vertical load-induced cracks are observed; cracks that originate from an ASR-induced crack and cracks that originate from the area near the tip of an ASR-induced crack, see Figure 5.20 (C). This section gives a plausible mechanical explanation of the formation of these two types of cracks.

Figure 5.27 shows a schematization of the mechanism behind the cracks that originate from an ASR-induced crack. The shown prism has two horizontal, ASR-induced cracks. These cracks are not perfectly horizontal and straight but are curved with local crack tops and the crack width of the upper crack varies. The figure shows the crack formation mechanism in four load stages:

(A) Before loading, there are no other cracks than the two ASR-induced cracks.

(B) When a small load is applied, the majority of the vertical deformation occurs as closure of the ASR-induced cracks. The ASR-induced cracks may contain ASR gel (see e.g. Barbosa et al. (2018a)), which smoothen the crack sides. Consequently, the crack is only able to transfer stresses perpendicular to the crack ($\sigma_{\text{comp}}$). The crack closure will therefore induce horizontal tensile stresses ($\sigma_{\text{split}}$) in the concrete around the local crack tops of the closing ASR-induced crack due to splitting.
These tensile stresses cause the vertical cracks originating from the ASR-induced crack.

(C) As more load is applied, the formed vertical cracks grow and new cracks form.

(D) The vertical cracks grow further and merge into larger cracks (bridging).

\[ \text{ASR-induced crack} \]
\[ \text{Crack top} \]
\[ \text{Load-induced crack} \]

\( \Delta \text{comp} \)
\( \Delta \text{split} \)

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Figure 5.28: Schematization of the FEM model of a prism with an ASR-induced crack.

Figure 5.29 (left) shows the horizontal normal stresses ($\sigma_x$) derived from the FEM analysis. The compressive stresses are shown positive. In the area around the crack tip, there is a concentration of compression stresses. However, the highest compressive stresses, up to 450 MPa, are due to corner singularities and are uninteresting in this investigation. Tensile stresses appear in the areas above and below the crack tip. These stresses induce the vertical cracks that were observed during the test with the prisms $\perp$. These areas with tensile stresses can also be explained by the trajectories of the compressive stresses, see Figure 5.29 (right). If there were no ASR-induced cracks in the prism, the stress trajectories would be straight. When an ASR-induced crack is present, the trajectories have to go around the crack. When the left trajectories bend to the right, the compression force acts with an eccentricity in a horizontal section in the level of the crack. To sustain equilibrium, tension (or reduced compression) will occur in right side of the prism in the $y$-direction. Consequently, the right stress trajectories bend towards to centre as illustrated on Figure 5.29 (right). The fact that the trajectories of the compressive stresses are no longer perfectly vertical means that horizontal stresses are induced. Especially, large compressive stresses are induced in the zone near the crack tip. This is shown on Section A in Figure 5.29 (left) that shows the horizontal normal stresses ($\sigma_x$) in a vertical section to the right of the crack tip. The section also shows the tensile stresses above and below the crack tip. The tensile stresses insure horizontal equilibrium in the vertical section. The cracks from this mechanism may also be included in the bridging with other vertical cracks, see Figure 5.27. All in all, it can be concluded that the formation of the vertical cracks in prisms $\perp$ is caused by the ASR-induced cracks and not due to shear in the cement paste as for sound concrete.
5.4.4.2 Analysis of prims

5.4.4.2.1 Visual inspection

Figure 5.30 shows the prisms before loading. The ASR-induced cracks are marked with black lines and the thickness of the lines roughly illustrates the crack widths.

- Prism V1 has two coarse through-going cracks and some finer cracks that are primarily placed in the top half of the prism.
- Prism V2 has two coarse cracks; one the in the centre and one in the right side of the prism. Additionally, there are some finer cracks, mostly in the right side.
- Prism V3 has several coarse cracks. Generally, the cracks are coarser in the top half of the prism.
- Prism V4 has one through-going crack and some finer cracks around the prism, but mostly in the top half.

Based on the visual inspection, Prism V4 seems the least damaged followed by Prism V2 while Prism V1 and V3 seem more damaged.
5.4.4.2.2 Quantification of the compressive strength and Young’s modulus

Table 5.4 shows the measured compressive strength, $f_{c\parallel}$, and Young’s modulus, $E_{\parallel}$ for the four prisms $\parallel$. As for the prism $\perp$, the employed strains are found as the relative vertical displacement ($\Delta H'/H'$) derived from analysis points $x_3$, see Figure 5.17. It can be seen that $f_{c\parallel}$ varies from 18.9 to 23.3 MPa. It seems that there is a correlation between the visual determined damages and $f_{c\parallel}$; Prism V4, that is least damaged, has the highest compressive strength. Whereas, $E_{\parallel}$ seems to be less correlated to the damages; Prism V3 has a highest $E_{\parallel}$ despite that Prism V4 is less damaged. However, the most damaged prisms (Prism V1 and V3) have the lowest $E_{\parallel}$.
<table>
<thead>
<tr>
<th>Prism</th>
<th>Compressive strength $f_{\text{c} \parallel}$ [MPa]</th>
<th>Young’s modulus $E_{\text{\parallel}}$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>18.9</td>
<td>16.0</td>
</tr>
<tr>
<td>V2</td>
<td>21.1</td>
<td>23.5</td>
</tr>
<tr>
<td>V3</td>
<td>19.6</td>
<td>16.4</td>
</tr>
<tr>
<td>V4</td>
<td>23.3</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Table 5.4: Compressive strength and Young’s modulus of prisms $f_{\text{c} \parallel}$.

5.4.4.2.3 Axial deformation

Figure 5.31 shows the stress-strain curves for the four prisms derived from analysis points $x_3$. It is seen that the fundamental shape of the four curves is similar where the main difference is the peak value, $f_{c \parallel}$.

![Stress-strain curves](image)

Figure 5.31: Stress-strain ($\sigma$-\varepsilon_{axial}) curves for the compressive test of prisms $f_{c \parallel}$.

Figure 5.32 shows an example of the strain progression during the compressive test of Prism V1. The figure consists of five subfigures, (A) to (E). Subfigure (A) shows the prism before loading with marked ASR-induced cracks. Subfigures (B) to (E) show major strain (tension) plot derived from the DIC analysis for four load stages. Since the minor strain (compression) plots only show noise they are left out. The four load stages are shown as red dots on the stress-strain curve, see Figure 5.31. Subfigure (B) shows the major strains (tensile) when a load of 10.0 MPa is applied. The plot shows opening of vertical cracks (marked with black arrows). By comparing (A) and (B), it is seen that these crack openings occur in the ASR-induced cracks (i.e. it is not new-formed cracks). Subfigure (C) and (D) show the major strains when a load of 12.5 and 15.0 MPa are applied, respectively. The plots show that the cracks that opened in plot (B) are growing.
with increasing load. Subfigure (E) shows the major strains at failure (18.0 MPa). The
plot shows that the aforementioned cracks are grown further, both in width and length.
The failure occurs due to splitting. These four stages occur for all four prisms. However,
the number of struts correlates with the number of ASR-induced cracks; Prism V1 and
V3 were split into more struts than Prism V2 and V4. This correlates with $f_{c\parallel}$ as well
(i.e. more struts lead to a lower $f_{c\parallel}$).

![Figure 5.32: Strain progression during compressive test of Prism V1. A) The prism
before loading with marked ASR-induced cracks. B to E) Major strain plots for four load
stages.](image)

The homogeneity of the axial deformation, $\Delta y$, is investigated by means of the DIC
analysis. $\Delta y$ is computed in the three analysis sections shown in Figure 5.17 C). Figure
5.33 shows how $\Delta y$ varies in the height of the prism (represented by the $y$-coordinate
in the reference coordinate system, see Figure 5.16) in the three analysis sections for three
load stages. $y = 0$ mm is the bottom and $y = 200$ mm is the top of the prism and
the curve colour matches the colour of the associated analysis section. In the corner of
each subfigure, the associated minor strain (compression) plot is shown. Figure 5.33a
shows $\Delta y$ when a load of 10 MPa is applied. The curves look linear. The jump around
$y = 80$ mm is due to errors induced by the shadowing steel bar, and is not due to
actual compressive behaviour of the concrete. It is seen that the deformation in the
three sections are very similar and are approximately linear. Figure 5.33b shows that
the curves are still approximately linear when a load of 15 MPa is applied. Figure 5.33c
shows that the shape of the deformation curves changes significantly when a load of 18
MPa is applied. This is due to large displacements in the ASR-induced cracks; the failure
process is started. All in all, the axial deformation for prisms $\parallel$ is characterised by:

- Linear behaviour
5.4.4.2.4 Lateral deformation

Figure 5.34 shows the lateral strains for increasing compressive loading ($\sigma$-$\varepsilon_{\text{lat}}$). The slope of the initial part of the curves is more or less constant, and there are almost no lateral strains. Hereafter, the slope of the curve gradually decreases. This happens at different load stages for the four prisms; $\sim10$ MPa for Prism V1 and V3, $\sim15$ MPa for Prism V2 and $\sim18$ MPa for Prism V4. There is a correlation between the load where the slope starts decreasing and $f_{c\parallel}$; the slope of the curve for Prism V1 and V3 decreases at lowest load and have the lowest $f_{c\parallel}$. 

Figure 5.33: Axial displacement ($\Delta y$) of Prism V1 in a given y-coordinate in three analysis sections at three load stages. The corresponding minor strain plot is shown in the corner.
Figure 5.34: Applied stress, $\sigma$, versus lateral strains, $\varepsilon_{\text{lat}}$, for the compressive test of prisms $\parallel$.

Figure 5.35 shows the major strain plots for two load stages: the load stage where the slope of the $\sigma$-$\varepsilon_{\text{lat}}$-curves start decreasing and the load stage just before failure ($\sigma = 0.95 - 0.98 f_c$). The upper row of plots shows the major strains for the first load stage (marked with red dots in Figure 5.34). It is seen that vertical cracks are forming (marked with black arrows). Thus, it is like for sound concrete, the first change of slope of the $\sigma$-$\varepsilon_{\text{lat}}$ curve is caused by crack growth. The lower row of plots shows the major strains for the second load stage (marked with black dots in Figure 5.34). It is seen that Prism V1 and V3 have several large vertical cracks, while Prism V2 has fewer cracks and Prism V4 has only one dominating crack. For all prisms, the cracks are grown from ASR-induced cracks.
Crack formation

$10 \text{ MPa (0.53} f_c) \%$

$V_1$

$15 \text{ MPa (0.71} f_c)$

$V_2$

$10 \text{ MPa (0.53} f_c)$

$V_3$

$18 \text{ MPa (0.77} f_c)$

$V_4$

$18 \text{ MPa (0.98} f_c)$

$V_3$

$V_4$

$18 \text{ MPa (0.98} f_c)$

$V_2$

$V_1$

$20 \text{ MPa (0.95} f_c) 18 \text{ MPa (0.98} f_c) 22 \text{ MPa (0.95} f_c)$

Figure 5.35: Major strain plots for the four prisms. The upper row is for the load stage where the slope of the $\sigma$-$\varepsilon_{\text{axial}}$-curves start decreasing. The lower row is for $\sigma = 0.95 - 0.98 f_c$.

5.4.4.2.5 Volumetric strain

Figure 5.36 shows the volumetric strain ($\sigma$-$\varepsilon_v$) curve for the four prisms. It is seen that if Prism $V_1$ is left out; the ”discontinuity point” occurs as for sound concrete around $\sigma = 0.7 f_c$. The compressive behaviour for Prism corresponds to a downscale of sound concrete.
5.4.4.2.6 Explanation of crack formation mechanisms

The DIC analysis showed that vertical cracks were formed as opening of the ASR-induced cracks. However, these crack openings were not seen before a significant external load ($\sigma \approx 10$ MPa) was applied, see Figure 5.35. This subsection gives a plausible mechanical explanation of the formation of these crack openings based on the observations done above.

Figure 5.37 shows a schematization of the crack opening mechanism. The shown prism has one through-going and two smaller vertical ASR-induced cracks. The figure shows the crack formation mechanism in three load stages:

(a) Before loading, there are no other cracks than the ASR-induced cracks.

(b) When a significant load is applied, cracks around the coarse aggregate particles form as in sound concrete, see Figure 5.14. This crack formation does not involve the ASR-induced (macro-) cracks. However, the ASR-induced micro-cracks, as mentioned in Chapter 3, reduce the mechanical properties of the cement paste, among others, the tensile strength. The reduced tensile strength of the cement paste means that the load-induced crack around the aggregate particles will form at a lower load stage than for the same concrete without ASR.

(c) As more load is applied, the cracks grow and merge with the ASR-induced macro-cracks (bridging), which will open.

Except for merging with the ASR-induced macro-cracks, this mechanism is very similar to the mechanism for sound concrete. However, the strength is reduced by the ASR-induced micro-cracks.
5.4.4.3 Comparison of prims⊥ and prims∥

It has been shown that there is a significant difference between the compressive strength and the stiffness of prism⊥ and prism∥. The main difference is the influence of the macro-cracks. They play an important role for prism⊥ and a minor role for prism∥. For the stiffness it is possible to measure the influence of the macro- and micro-cracks. It is therefore possible to investigate whether the micro-cracks affect $E_\perp$ and $E_\parallel$ equally, which is expected due the conceptual explanation of ASR-induced cracking in slabs, where the micro-cracks are evenly distributed in the slab and are orientated randomly, see Chapter 3.

Figure 5.38 shows the stress-strain curves for Prism⊥ H2 and Prism∥ V1. For Prism⊥ H2, the strains are both measured between two ASR-induced macro-cracks (named 'micro-crack', red dashed line) and as the average over the entire prism height (named 'overall', red solid line). For Prism∥ V1, the strains are only measured as the average over the
entire prism height (green solid line). It can be seen that ‘micro-crack’ stiffness for 
Prism⊥ H2 is the same as the ‘overall’ stiffness for Prism∥ V1 for stresses lower than 
approximately 7 MPa (marked with a red dot) where the vertical cracks in Prism⊥ H2 
was formed.

It can be concluded that the ‘micro-crack’ stiffness for prism⊥ is the same as the ‘overall’ 
stiffness for prism∥.

5.4.5 Answer to RQ4

$f_{c\perp}$ is highly influenced by closure of macro-cracks that induce vertical cracks, which 
initiate the failure. $f_{c\parallel}$ is mainly influenced by the reduced strength of the cement paste 
due to micro-cracks, and the $\sigma$-ε behaviour in this relation is similar to a down-scale 
response curve for sound concrete.

5.5 Practical application of Young’s modulus

The analysis shows that there is a significant difference between the overall stiffness for 
prisms∥ ($E_{\parallel}$) and prisms⊥ ($E_{\perp}$) for ASR-damaged concrete. The analysis showed further-
more that the low value of $E_{\perp}$ is due to crack closure, while the stiffness between the 
macro-cracks is more or less the same as the overall stiffness for prisms∥. When the ser-
viceability of a bridge slab is assessed, the deformation, accelerations (or eigenfrequency) 
and crack widths are normally of interest. Common for these interests is that they only 
depend on the horizontal stiffness of the concrete, $E_{\parallel}$. Due to the large difference be-
tween $E_{\parallel}$ and $E_{\perp}$ it will be very conservative to base the serviceability calculations on
$E_\perp$. However, it might be troublesome to extract cores$_\parallel$ from a slab since they shall be drilled horizontally. Another method to determine $E_\parallel$ is by employing DIC on tests with vertically drilled cores (cores$_\perp$), such that the stiffness between the ASR-induced cracks can be determined.
Chapter 6

Concrete tensile strength

The literature study in Chapter 2 raised one research question (RQ) regarding the concrete tensile strength:

RQ5 Which test method shall be employed in practice to determine the tensile strength of ASR-damaged concrete for the use in structural calculations?

In this chapter, this question is sought answered. To provide this answer, a closer look at test methods has been taken to examine whether they can be applied for ASR-damaged concrete slabs. Roughly, there exists four well-known test methods for determine tensile strength of sound concrete: *Uniaxial tensile, Brazilian splitting, Flexural testing* and *Wedge splitting*, (Van Mier, 1996; Neville, 2011). Each of these methods and their application for ASR-damaged concrete are studied in the following 4 sections. The chapter closes with a comparison of the test methods and answering the research question.

![Test methods for measuring the tensile strength.](image)

*Figure 6.1: Test methods for measuring the tensile strength.*
6.1 Uniaxial tensile test

In a uniaxial tensile test, also known as a direct tension test, a concrete cylinder is subjected to a uniaxial tension force, see Figure 6.1a. There exists many different methods to apply this tension force and the method has a significant influence on the test results (Van Mier, 1996; Neville, 2011). Common for the methods is, that it is very difficult to avoid eccentricity in the concrete specimen. Sound concrete is a heterogeneous material with a (quasi-) brittle behaviour in tension. Consequently, the entire cross section will not fail simultaneously in uniaxial tensile tests. When the weakest zone in the cross section fails, the stresses in the cross section may be redistributed eccentric which will result in larger stresses in a part of the cross section. Consequently, the test results are often lower than for other test methods and the tensile strength of sound concrete is therefore tested by indirect test methods such as Brazilian split tests instead. ASR-damaged concrete is more heterogeneous than sound concrete - especially when the tension force is applied perpendicular to the ASR-induced cracks. This has been confirmed by Barbosa (2017). By means of DIC, he found that ASR-induced cracks induce significant eccentricities, especially when the force is applied perpendicular to the ASR-induced cracks. Due to the test method’s sensitivity to heterogeneity, the measured tensile strength will only be representative for load scenarios with uniaxial tensile. This load case is often not relevant for slabs.

6.2 Brazilian split test

Brazilian split (BS) test is an indirect method to test the tensile strength of concrete. A concrete cylinder is subjected to two diametrically opposed lateral compressive forces, see Figure 6.1b. The diametrically loading induces stresses normal to the plane of loading ($\sigma_x$ if loaded in the $y$-direction); right under/over the loading, large compressive stresses occur while there are tensile stresses in the rest of the vertical plane. If the load is applied evenly over the height of the cylinder and over an infinitesimal width, the maximum tensile stresses in the center of the cylinder may be found as shown in Equation 6.1, according to the elasticity theory.

$$\sigma_x = \frac{2F}{\pi h_{\text{cylin}} d_{\text{cylin}}}$$  \hspace{1cm} (6.1)

where $h_{\text{cylin}}$ is the height of the cylinder, $d_{\text{cylin}}$ is the cross sectional diameter and $F$ is the applied load.

This calculation is based on a number of assumptions:

1. the material is homogeneous,
2. the material obeys Hooke’s Law up to the point of collapse (elastic and brittle),
3. the material is isotropic,
4. the failure occurs due to tension (horizontal tensile stresses exceeds the tensile strength).

Since, the load is applied over a finite width (larger than zero) and that sound concrete is not perfectly brittle, the stresses are corrected by an empirical factor ($\lambda$) between 0.6 and 0.9 (Olesen et al., 2006). Thereby, the tensile strength may be found as in Equation 6.2.

$$f_t = \lambda \frac{2F_u}{\pi h_{cylin} d_{cylin}}$$  \hspace{1cm} (6.2)

where $F_u$ is the ultimate applied load.

However, researchers have found that the assumption that the failure occur due to tension (assumption 4) may be violated. They found that shear prisms were formed under the loading and the failure was due to a combination of shear and tensile, see e.g. (Darvell, 1990). This has been addressed by Chen and Drucker (1969) who derived a solution based the upper-bound theorem of plasticity theory (rigid plastic material behaviour obeying the modified Mohr-Coulomb failure criteria). Figure 6.2 shows the considered failure mechanism for the derived solution. The failure mechanism consists of two shear prisms that move with the associated load (the upper prism moves down and the lower prism moves up) and two rigid bodies, separated by a vertical crack, that move away from each other, horizontally. Due to sliding between the prisms and the rigid bodies, the measured load depends on the compressive strength of the concrete as well.

![Figure 6.2: Failure mechanism for the upper-bound solution by Chen and Drucker (1969).](image)

Chen and Drucker (1969) replaced the assumption about elasticity and brittleness (assumption 2) with an assumption about rigid plastic behaviour for tensile as well as compression.
There exists more sophisticated models that include the softening branch of the stress-strain curve for concrete in tension, see e.g. (Olesen et al., 2006) and (Hoang et al., 2014). However, they still rely on assumptions about homogeneity and isotropy. For ASR-damaged concrete in 2D-restrained slabs these assumptions are violated. Anisotropy and non-linear compressive behaviour of the concrete may be incorporated in the models by Olesen et al. (2006); Hoang et al. (2014). However, the models cannot handle the discrete distributed ASR-induced macro-cracks.

To be able to use Brazilian split tests to determine the tensile strength of ASR-damaged concrete a new model is needed. In order to develop such model, understanding of the failure mechanism is needed. To achieve this, a few new experiments have been conducted where DIC was employed such that the cracks could be analysed, see a description of DIC in Section 5.4.3. The experiments were conducted with cores drilled from a slab part from Lindenhorg. The slab part had been stored unprotected outside for approximately one year where the ASR may have developed further and the reinforcement may be more corroded. Consequently, the measured strength of the cores may not be representative for the actual bridge. However, it is representative for a severely ASR-damaged slab. Figure 6.3 shows a photo of the slab part from Lindenhorg. It is seen that it has many and course cracks. The cores were drilled with a water-cooled diamond drill and stored inside until testing. During the tests, the load was applied deformation-controlled with a constant rate of 1.0 mm/min and 3 x 10 mm loading strips of masonite were employed. In the tests, the cores were oriented such that the cracks are either vertical or horizontal in the cross section, corresponding to orientation I and II in the experiments by Barbosa (2017), see Figure 2.6.

Figure 6.3: The slab part from Lindenhorg used for the Brazilian split tests

Figure 6.4 shows the cores before loading where the cracks are marked with black lines. Figure 6.5 shows the load-displacement curve for the test in orientation I where the load is applied parallel to the cracks. It is remarked that since the deformation is measured in the compression machine, it includes the deformation of the loading strips. The two red dots indicates the load stages where the strains are investigated.
The vertical ($\varepsilon_y$) and horizontal strains ($\varepsilon_x$) for the two load stages are shown in Figure 6.6. In the strain plots, green is neutral, blue is compression and red is expansion. It is seen for the first load stage (approximately 50% of the failure load) that there are no significant vertical strains (Figure 6.6a) whilst the horizontal strains (Figure 6.6b) show that vertical cracks are already formed; originating from the loading, not from the center of the core. The second load stage is when the failure occur. From the $\varepsilon_x$-plot it is seen...
that a course vertical crack is formed left to the loading. There are no signs of tensile failure in the center of the core; the applied compression force seems to be carried by a vertical strut in pure compression. It is noted that shear prisms are not formed as for sound concrete.

Figure 6.6: Strains in an ASR-damaged cylinder in a Brazilian split test, orientation I, in two load stages, see Figure 6.4a. In the strain plots, green is neutral, blue is compression and red is expansion.

Figure 6.7 shows the load-displacement curve for the test in orientation II where the load is applied perpendicular to the cracks. The three red dots indicate the load stages where the strains are investigated.

The vertical ($\varepsilon_y$) and horizontal strains ($\varepsilon_x$) for the three load stages are shown in Figure 6.8. It is seen for the first load stage (approximately 50% of the failure load), that significant compression occurs in the ASR-induced cracks (Figure 6.8a) whilst the horizontal strains (Figure 6.8b) show that a small vertical crack is formed in the top of the core. In the second load stage (approximately 80% of the failure load), the compression
in the ASR-induced cracks is increased and a vertical crack is formed in the bottom of the core. It is noted that the vertical cracks originate from the ASR-induced horizontal cracks, as it was seen for \textit{prism}_\perp during compressive testing, see Section 5.4.4.1. The third load stage is just before failure. It can be seen that the vertical cracks have grown further and a new is formed in the center of the core. Despite the fact that the vertical crack is almost continuous from bottom to the top of the core, the core is not failing yet. Assuming that the vertical center-section of the cores is fully cracked and has no tensile strength, each of the two core halves must be in load equilibrium. If the tensile strength of the horizontal ASR-induced cracks are low ($\rightarrow 0$), the applied compression force is carried by a vertical strut in pure compression, see Figure 6.9. It is remarked that like for orientation I, shear prisms are not formed.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure6.7.png}
\caption{Load-displacement diagram for an ASR-damaged cylinder in a Brazilian split test, orientation II.}
\end{figure}
Figure 6.8: Strains in an ASR-damaged cylinder in a Brazilian split test, orientation II, at three different load stages, see Figure 6.7. In the strain plots, green is neutral, blue is compression and red is expansion.
All in all, it is hypothesised that the applied load for both orientations is carried by a vertical compression strut. This hypothesis can be tested by the results published by Barbosa (2017) where Brazilian split tests are conducted on cores drilled from Gammeleirand while the associated compressive strengths are presented in Section 5.1.5.1 in this thesis. If the hypothesis is correct, the failure load in the Brazilian split tests will be equal to the compressive strength of the vertical strut, i.e. the failure load in the Brazilian split tests can be found as:

\[
P_{BS,I} = 2a_{strip}h_{core}f_{c||}
\]  \hspace{1cm} (6.3)

\[
P_{BS,II} = 2a_{strip}h_{core}f_{c\perp}
\]  \hspace{1cm} (6.4)

where \(P_{BS,I}\) and \(P_{BS,II}\) are the calculated failure load for the Brazilian split tests in the two orientations, \(2a_{strip}\) is the width of the loading strips, \(h_{core}\) is the height of the cores and \(f_{c||}\) and \(f_{c\perp}\) are the compressive strength.

Barbosa (2017) used 10 mm loading strips and \(f_{c||}\) and \(f_{c\perp}\) are found to be 21.0 MPa and 16.7 MPa, respectively. Figure 6.10 shows a comparison between the experimental failure loads and the calculated failure loads. It can be seen that there is rather good agreement between the experimental and calculated failure loads. What is more worth noticing is that the proposed hypothesis explains why \(P_{BS,I}\) is larger than \(P_{BS,II}\). Evaluating the
experimental results by means of the classic elastic solution (Equation 6.1), the tensile strength perpendicular to the cracks \( f_{t\perp} \) is larger than \( f_{t\parallel} \), which is counter-intuitive. Since the failure load of the Brazilian split tests can be explained by a pure compression strut, Brazilian split tests cannot be employed for testing the tensile strength of concrete from ASR-damaged concrete slabs. This conclusion is in accordance with the recommendations of Clayton et al. (1990) and Siemes et al. (2002).

<table>
<thead>
<tr>
<th>Tests by Barbosa</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{BST,I} )</td>
<td>38.0 kN</td>
</tr>
<tr>
<td>( P_{BST,II} )</td>
<td>27.6 kN</td>
</tr>
</tbody>
</table>

Figure 6.10: Comparison between experimental and calculated results.

### 6.3 Flexural test

Flexural test is another indirect method to test the tensile strength of concrete. Here, an unreinforced concrete beam is subjected to bending induced by two point loads, see Figure 6.1c. The tensile strength, also known as the modulus of rupture, is found as the theoretical maximum tensile stress in the bottom of the beam. This stress is determined on the basis of elastic theory. Since this test requires a rather large test specimen, this test method is not applicable to determining the tensile strength of existing structures (Neville, 2011). Consequently, no further investigation in this method has been carried out.

### 6.4 Wedge splitting test

Wedge splitting (WS) test is a third indirect method to test the tensile strength of concrete. Here is a cube modified such that it has two wedges and a notch. The load is applied as a pair of splitting forces on the wedges, see Figure 6.1d. By means of crack mouth opening displacement (CMOD) and a so-called inverse analysis (see e.g. (Østergaard et al., 2000)) the tensile behaviour, including the tensile strength, can determined. If the method is used for existing structures, it can be performed on modified drilled cores (Van Mier, 1996).

The cubes can be orientated in three directions regarding the ASR-induced cracks, orientation I, II and III, see Figure 6.11. However, unpublished results from Barbosa (2017) have shown that the failure for orientation I occurs in a wedge rather than in a vertical crack originating from the notch. Barbosa (2017) tested orientation II and III for concrete from Lindenborg (ASR-damaged concrete slab). He found that: ”[...] it is still surprising how good the crack hinge model fits the measurements”. Based on his finding, it may be concluded that WS can be used to determine the tensile strength for orientation II and III. Orientation II corresponds to
tension perpendicular to the ASR-induced cracks, \( f_{t\perp} \), and orientation III corresponds to tension parallel to the ASR-induced cracks, \( f_{t\parallel} \).

**Figure 6.11: Crack orientation during the wedge split (WS)**

### 6.5 Comparison

It was found that uniaxial tensile test and Brazilian split test are not appropriate for testing the tensile strength of ASR-damaged concrete slabs while the size of the test specimens for the flexural test makes the method inappropriate for existing structures. In contrast, it was found that wedge splitting test can be used for testing the tensile strength of ASR-damaged concrete; both parallel and perpendicular to the cracks. Despite that wedge splitting test is more complex than uniaxial and Brazilian split test it is concluded to be the best method for ASR-damaged concrete. The tensile strength in Chapter 7 is therefore based wedge splitting tests.

### 6.6 Answer to RQ5

Wedge splitting test is the best way to determine the tensile strength for the use on structural calculations.
Chapter 7

Shear experiments

This chapter provides an experimental investigation on how ASR affects the shear capacity of 2D-restrained slabs. The aim of the investigation is: (i) to give answers to the research questions raised in the literature study in Section 2 and; (ii) to provide experimental evidence that can be used in the development of a mechanical model for ASR-damaged 2D-restrained slabs without shear reinforcement. The raised research questions (RQ) are:

RQ6 To what extent and how does ASR affect the shape of the critical shear crack for 2D-restrained slabs without shear reinforcement?

RQ7 Why does ASR affect the shape of the critical crack for 2D-restrained slabs without shear reinforcement?

To answer RQ6 and RQ7, three experimental programmes are carried out. In the first programme, shear tests are conducted with beams cut from an ASR-damaged 2D-restrained slab bridge - Lindenborg (see Section 7.1.1). In the second programme, reference tests are conducted with new-cast beams (see Section 7.1.2). These beams are identical with the beams from Lindenborg, yet, without ASR-damages. In the third programme, shear tests are conducted with beams cut from another ASR-damaged 2D-restrained slab bridge - Gammelrand (see Section 7.1.3).

7.1 Experimental programmes

This section provides for each experimental programme: (i) a description of the material properties of the concrete, the reinforcement and the prestressing; (ii) a description of how the shear experiments are conducted and; (iii) the test results.

7.1.1 Lindenborg Bridge

Lindenborg is a larger slab bridge located in Denmark that suffers from severe ASR-damages. To test the shear capacity of Lindenborg under controlled conditions, six slab
segments were cut from the bridge slab and brought to laboratory facilities for testing. Each slab segment was cut into three beams and four strips. The beams were used to test the shear capacity, and the strips were used for testing of the mechanical properties of the concrete. The reinforcement bars in the strips were used for testing of the mechanical properties of the steel and the level of prestressing. A more detailed description of the bridge, its condition and how the beams were cut is provided in Section 4.2.

7.1.1.1 Materials

18 beams were cut such that they had two reinforcement bars in the top and three reinforcement bars in the bottom of the cross section, see Figure 7.1. The width of the cross sections varied slightly. The widths and the covers are given in Table 7.2 and 7.3. To determine the exact configuration (size, numbers, type and position) and condition of the reinforcement, the bars were exposed by removing the concrete cover with a hammer after the shear tests. The full outcome of this investigation is provided in C.

![Figure 7.1: Cross section of the beams cut from Lindenborg bridge. All dimensions are in mm.](image)

Concrete

Since the compressive strength of the concrete is presented in Section 5.1.5.2, only the tensile strength of the concrete is presented in this section. According to the conclusion in Chapter 6, the tensile strength is determined by wedge splitting tests. The tests were conducted by Barbosa (2017). Figure 7.2 shows the tensile strength parallel ($f_{t\parallel}$) and perpendicular ($f_{t\perp}$) to the ASR-induced cracks. The tensile strength is only measured on cubes from Slab segment 5.
Reinforcement

Reinforcement from the strips were used to test the mechanical properties. The mean value of the measured yield stress \( f_{ym} \), Young’s modulus \( E_m \) and the ultimate strength \( f_{um} \) are shown in Table 7.1.

<table>
<thead>
<tr>
<th>( f_{ym} ) [MPa]</th>
<th>( E_m ) [GPa]</th>
<th>( f_{um} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>598.6</td>
<td>220.8</td>
<td>685.0</td>
</tr>
</tbody>
</table>

Table 7.1: Mean value of measured yield stress \( f_{ym} \), Young’s modulus \( E_m \) and ultimate strength \( f_{um} \) for the reinforcement steel.

As recommended in the literature (see Section 2.5), the ASR-induced prestressing in the reinforcement was measured directly on the reinforcement bars. The measurements were conducted by mounting strain gauges on the reinforcement bars and cutting the bars free from the concrete, see Figure 7.3. Before mounting the strain gauge, the concrete was locally removed around the rebar and the exposed part of the rebar was grinded and cleaned. The rebar was cut about 10 cm from the strain gauge. The prestressing is found as the measured contractive strain \( \varepsilon_{s,ASR} \).
Figure 7.3: Left photo shows a strain gauge mounted on a reinforcement bar. Right photo shows a cut reinforcement bar.

Figure 7.4 shows the measured prestressing ($\varepsilon_{s,\text{ASR}}$) from the six slab segments. It is seen that the mean prestressing (red dots) varies between $0.90\%_o$ and $1.24\%_o$ corresponding to tensile stresses of 198.7 and 273.8 MPa.

![Graph showing measured ASR-induced prestressing in slab segments from Lindenborg bridge.](image)

**Figure 7.4:** Measured ASR-induced prestressing in the slab segments from Lindenborg bridge.

### 7.1.1.2 Shear experiments

The shear capacity of the beams was tested in two test set-ups: (i) A simply supported beam set-up and; (ii) A continuous beam set-up.

The test set-up for the simply supported beams is shown in Figure 7.5. Details of the supports and the loading are shown in Figure 7.6. The left support ($R_1$) allowed free rotation and the right support ($R_2$) allowed free rotation and horizontal displacements. The supports and the loading were equipped with fibre boards to ensure full contact with the concrete. The load was applied deformation-controlled with a constant rate of
0.5 mm/min. However, the beams from Slab segment 2 (Beam 2.1, Beam 2.2 and 2.3) were loaded differently. They were first loaded until 120 kN with a rate of 6 kN/min (load-controlled). Hereafter they were unloaded until 5 kN with a rate of 12 kN/min and finally loaded with a rate of 0.5 mm/min (deformation-controlled) until failure. The dimensions of each of the nine beams in this test set-up are shown in Table 7.2.

To increase the probability of shear failures, nine beams were tested in a continuous beam set-up, see Figure 7.7. The dimensions of each test set-up are shown in Table 7.3. This set-up had two supports and the load was applied as two point loads, $P_1$ and $P_2$. $P_1$ was applied between the supports and $P_2$ was applied on the left cantilevered part of the beam. The load was applied via a steel spreader beam which ensured the $P_1/P_2$-ratio was constant. Details of the supports and the loadings are shown in Figure 7.8. The principle for the supports is the same as for the simply supported beams. To ensure that the spreader beam only applied a vertical load, the contact point (i.e. load $P_1$) was allowed to rotate as well as move horizontally. Also for this set-up, the supports and loadings were equipped with fibre boards. The load ($P$) was applied as deformation-controlled with a constant rate of 0.5 mm/min until 5 mm deformation whereafter the rate was reduced to 0.35 mm/min. The majority of the first 5 mm deformation occurred as compaction of the fibre wood boards.

Figure 7.9 shows the shear force- and moment diagrams for the continuous beam set-up. The continuous beams have three shear zones, named Left shear Zone, Main shear zone and Right shear zone. The set-up was optimized such that a shear failure should occur in the Main shear zone. In the Main shear zone, the moment changes sign; it is negative in the left end and positive in the right end.

In both test set-ups, DIC measurements were performed (see a short introduction to DIC in Section 5.4.3). The DIC measurement zones for the simply supported beams and the continuous beams are shown in Figure 7.5 and 7.7, respectively. The employed system uses a stereo camera system of 12 MP per camera with a frequency of 0.2 photo/s. In the research in the following sections, only the DIC measurements from some of the beams are used. In Appendix C.3, the DIC measurements for all 18 beams are presented.

![Figure 7.5: Experimental set-up for the simply supported beams, Lindenborg. All dimensions are in mm.](image-url)
Figure 7.6: Supports and loading for the simply supported beam set-up.

Figure 7.7: Experimental set-up for the continuous beams, Lindenborg. All dimensions are in mm.

Figure 7.8: Supports and loadings for the continuous beam set-up.
Figure 7.9: Shear and moment diagram for the continuous beam set-up.

<table>
<thead>
<tr>
<th></th>
<th>Slab segment 2</th>
<th>Slab segment 3</th>
<th>Slab segment 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beam 2.1</td>
<td>Beam 2.2</td>
<td>Beam 2.3</td>
</tr>
<tr>
<td>Width b [mm]</td>
<td>435 410 445</td>
<td>450 450 445</td>
<td>425 440 430</td>
</tr>
<tr>
<td>Left anchorage zone $L_L$ [mm]</td>
<td>999 848 695</td>
<td>1002 850 700</td>
<td>1000 852 700</td>
</tr>
<tr>
<td>Left shear zone $a_L$ [mm]</td>
<td>1005 850 705</td>
<td>998 852 695</td>
<td>1600 854 699</td>
</tr>
<tr>
<td>Right shear zone $a_R$ [mm]</td>
<td>810 720 570</td>
<td>850 703 620</td>
<td>665 600 650</td>
</tr>
<tr>
<td>Right anchorage zone $L_R$ [mm]</td>
<td>3.3 2.8 2.3</td>
<td>3.3 2.8 2.3</td>
<td>3.3 2.8 2.3</td>
</tr>
<tr>
<td>Slenderness $a_L/h$ [-]</td>
<td>60 55 55</td>
<td>60 60 50</td>
<td>55 40 45</td>
</tr>
<tr>
<td>Cover, top $c'$ [mm]</td>
<td>45 50 45</td>
<td>50 45 40</td>
<td>55 50 45</td>
</tr>
<tr>
<td>Cover, bottom c [mm]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2: Dimensions for specimens tested as simply supported beams.
Table 7.3: Dimensions for specimens tested as continuous beams.

### 7.1.1.3 Results

#### Simply supported beams

Table 7.4 shows the test results of the shear tests for the simply supported beams.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Slab segment 1</th>
<th>Slab segment 4</th>
<th>Slab segment 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.1</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>Width</td>
<td>b [mm]</td>
<td>420</td>
<td>435</td>
</tr>
<tr>
<td>Left anchorage zone</td>
<td>L₁ [mm]</td>
<td>595</td>
<td>493</td>
</tr>
<tr>
<td>Left shear zone</td>
<td>L₁ [mm]</td>
<td>415</td>
<td>410</td>
</tr>
<tr>
<td>Main shear zone</td>
<td>x₀ [mm]</td>
<td>1500</td>
<td>1200</td>
</tr>
<tr>
<td>Right shear zone</td>
<td>L₂ [mm]</td>
<td>840</td>
<td>677</td>
</tr>
<tr>
<td>Right anchorage zone</td>
<td>L₉ [mm]</td>
<td>300</td>
<td>320</td>
</tr>
<tr>
<td>Slenderness</td>
<td>x₀/h [-]</td>
<td>5.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Load ratio</td>
<td>P₁/P₂ [-]</td>
<td>0.78</td>
<td>0.53</td>
</tr>
<tr>
<td>Cover, top</td>
<td>c' [mm]</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>Cover, bottom</td>
<td>c [mm]</td>
<td>45</td>
<td>50</td>
</tr>
</tbody>
</table>

Note:
P_max is the maximum applied load, V_max is the maximum applied shear force excl. selfweight, u_max is the deflection when the maximum load is applied.

Table 7.4: Test results for the simply supported beams.

Figure 7.10 shows the associated shear-deflection curves. The depicted deflections are measured at the position of the load and are corrected for the compaction of the wood fibre boards and deformation of the supports. The depicted shear force does not include the self-weight of the beams (less than 5 kN).

The shear-deflection curves show that: (i) the beams from Slab segment 2 and 3 had a higher load-carrying capacity than the beams from Slab segment 6; (ii) the curves show...
a ductile behaviour of the beams with slenderness of $a_L/h = 3.3$ (Figure 7.10c) and; (iii) some beams with a slenderness of $a_L/h = 2.3$ and 2.8 show a more brittle behaviour.

Figure 7.10: Shear-deflection curves for the simply supported beams.

Continuous beams
Table 7.5 shows the test results of the shear tests for the continuous beams. It is noted that another capacity is given in brackets for Beam 4.2. This is the capacity after the first peak. This is discussed later.
<table>
<thead>
<tr>
<th>Beam</th>
<th>$x_0/h$</th>
<th>$P_{\text{max}}$ [kN]</th>
<th>$V_{2,\text{max}}$ [kN]</th>
<th>$u_{2,\text{max}}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>5.0</td>
<td>340.7</td>
<td>95.2</td>
<td>7.6</td>
</tr>
<tr>
<td>1.2</td>
<td>4.0</td>
<td>333.1</td>
<td>103.8</td>
<td>5.0</td>
</tr>
<tr>
<td>1.3</td>
<td>3.0</td>
<td>393.5</td>
<td>148.5</td>
<td>5.2</td>
</tr>
<tr>
<td>4.1</td>
<td>6.0</td>
<td>408.7</td>
<td>87.6</td>
<td>10.8</td>
</tr>
<tr>
<td>4.2</td>
<td>5.0</td>
<td>346.5</td>
<td>97.4 (85.0)</td>
<td>7.1</td>
</tr>
<tr>
<td>4.3</td>
<td>4.0</td>
<td>325.0</td>
<td>101.2</td>
<td>7.2</td>
</tr>
<tr>
<td>5.1</td>
<td>5.0</td>
<td>301.9</td>
<td>84.3</td>
<td>3.8</td>
</tr>
<tr>
<td>5.2</td>
<td>4.0</td>
<td>344.2</td>
<td>107.8</td>
<td>8.4</td>
</tr>
<tr>
<td>5.3</td>
<td>3.0</td>
<td>304.4</td>
<td>115.9</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Note:
$P_{\text{max}}$ is the maximum applied load, $V_{2,\text{max}}$ is the maximum applied shear force in the main shear zone excl. selfweight, $u_{2,\text{max}}$ is the deflection measured at the load $P_2$ when the maximum load is applied.

Table 7.5: Test results for the continuous beams

Figure 7.16 shows the associated shear-deflection curves. The depicted deflections are measured at the position of the load ($P_2$) and corrected for the compaction of the wood fibre boards and deformation of the supports. The depicted shear force does not include the self-weight of the beams (less than 5 kN).

The shear-deflection curves show that, the beams with the same slenderness ($x_0/h$) have approximately the same load-carrying capacity regardless of which slab segment they originate from. The only exception is that Beam 1.3 has a significantly higher load-carrying capacity ($V_{2,\text{max}} = 148.5$ kN) than Beam 5.3 ($V_{2,\text{max}} = 115.9$ kN).
Since several of the beams developed large moment-induced cracks during the tests and the associated shear-deflection curves show a ductile behaviour, the moment capacity may have been compromised, and thereby governing for the failure. Therefore, the moment utilization is studied. The calculation of the moment capacity is based on the same assumptions as in Eurocode 2 (CEN, 2004) where the ASR-induced prestressing is included:

- The ASR-induced prestressing ($\varepsilon_{\text{s,ASR}}$) occurs equally in all rebars, and induces linear equilibrating compressive stresses and strains ($\varepsilon_{c,\text{ASR}}$ in the top and $\varepsilon'_{c,\text{ASR}}$ in the bottom) in the concrete cross section, see Figure 7.12(b).
- The moment induces linear strain distribution to the cross section, see Figure
7.12(c).

- The strains at the extreme fibre of the concrete in compression is equal to the ultimate compressive strains for sound concrete \(\varepsilon_{c,tot} = \varepsilon_{cu} = 0.35\%\), see Figure 7.12(d).

- The compressive stresses in the concrete are rectangular distributed over 0.8x, see Figure 7.12(e).

- According to the conclusions from the study of the concrete compressive strength (see Chapter 5), \(f_{c\|}\) should be used as the concrete compressive strength.

The theoretical moment capacity \(M_{R,\text{mean}}\) is based on the mean values of the concrete compressive strength \(f_{c\|}\), the yield stress and Young’s modulus of the reinforcement \((f_y\text{ and } E_s)\) and the ASR-induced prestressing \(\varepsilon_{s,\text{ASR}}\).

Since the Young’s modulus for the concrete is not measured, the ASR-induced strains in the concrete are neglected \(\varepsilon_{c,\text{ASR}} = \varepsilon_{c,\text{ASR}}' = 0\). This neglect is of no practical importance. Even if Young’s modulus is as low as 1.0 GPa (for Gammelrand \(E_\parallel\) was found between 16.0 and 23.5 GPa, see Table 5.4), the moment capacity was affected with less than 0.1 kNm.

![Figure 7.12: Moment model.](image)

Figure 7.13 shows the moment utilization that is defined as the ratio between the ultimate applied moment \(M_u\) and the theoretical moment capacity \(M_{R,\text{mean}}\). It appears that the moment utilization for the majority of the simply supported beams (left half) exceeds 1.0. Only the utilization for Beam 6.1 is lower than 1.0 (0.79). However, the shear-deflection curve for Beam 6.1 (Figure 7.10c) shows a very ductile behaviour that indicates bending failure. The explanation may be found in the DIC analysis. Figure 7.14 shows a major strain plot for two load stages: an early load stage \((V = 58.7\text{ kN})\) and the failure load stage \((V = 75.0\text{ kN})\). At the plot for the early load stage, it can be seen that the beam had a coarse ASR-induced crack in the top of the beam in the left shear span (marked with a black arrow). However, it can be seen that the applied load has
not caused movements in the crack, as there are no strains. At the plot for the failure load stage, it can be seen that there have been movements in the crack. This crack movement may have led to a lower effective height of the beam, and thereby a lower moment capacity. It is therefore likely that the failure of Beam 6.1 was governed by the moment capacity rather than the shear capacity. Due to the high moment utilization for the simply supported beams, it is concluded that the failures are governed by their moment capacity. Finally, it can be seen that the moment utilization for the continuous beams (right half) in general are lower than 1.0.

![Figure 7.13: Moment utilization ($M_u/M_{R,\text{mean}}$) for the beams from Lindenborg.](image-url)
7.1.2 Reference tests

The reference programme is identical to the Lindenborg programme, yet, without ASR-damages.

7.1.2.1 Materials

Figure 7.15 shows a conceptual drawing of the reference beams. The reference beams contained the same longitudinal reinforcement as the Lindenborg beams (2Y16 in the top and 3Y16 in the bottom) - Figure 7.16a shows the cross section of the reference beam. To avoid anchorage failure, the reference beams were provided with stirrups (ø8 mm) in the anchorage zones. Details on the anchorage zones are shown in Figure 7.16b and 7.16c. Furthermore, additional stirrups were used to prevent the longitudinal reinforcement bars from sagging during casting. These additional stirrups are shown in Figure 7.17 and 7.18. The longitudinal reinforcement was placed with a cover of 45 mm.
Concrete
The concrete mix was determined on the basis of a microscopic examination of the concrete from Lindenborg. The employed concrete had a water/cement-ratio of 0.43, a maximum aggregate size of 16 mm and a desired mean compressive strength of 42.5 MPa. To determine the actual compressive strength, 48 cylinders were cast. All beams and cylinders were produced from the same batch of concrete. Since the beams were tested over 12 days where the concrete strength may increase due to hydration, the compressive strength was determined for each day of testing. The tested cylinders had a diameter of 100 mm and a height of 200 mm, and the measured strengths are corrected by a factor of 0.97 to convert them to a standard cylinder compressive strength, a cylinder with a diameter of 150 mm and height of 300 mm (Nielsen, 2005).

The mean converted compressive strengths are shown in Table 7.6. The table shows furthermore the name of the beams that belong to a given mean strength.

<table>
<thead>
<tr>
<th>Number of cylinders</th>
<th>Days of curing</th>
<th>Mean compressive strength $f_c$ [MPa]</th>
<th>Beam name</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>39</td>
<td>44.9</td>
<td>C3; C4</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>45.2</td>
<td>C2</td>
</tr>
<tr>
<td>12</td>
<td>41</td>
<td>43.6</td>
<td>C1</td>
</tr>
<tr>
<td>12</td>
<td>43</td>
<td>45.1</td>
<td>S1; S2</td>
</tr>
<tr>
<td>8</td>
<td>46</td>
<td>47.0</td>
<td>S3</td>
</tr>
</tbody>
</table>

*Table 7.6: Mean converted compressive strengths.*

Reinforcement
The mean value of the measured yield stress ($f_{ym}$), Young’s modulus ($E_m$) and the ultimate strength ($f_{um}$) are shown in Table 7.7.
<table>
<thead>
<tr>
<th>ø8 (stirrups)</th>
<th>( f_{ym} ) [MPa]</th>
<th>( E_m ) [GPa]</th>
<th>( f_{um} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ø8</td>
<td>611</td>
<td>210</td>
<td>715</td>
</tr>
<tr>
<td>ø16</td>
<td>570</td>
<td>205</td>
<td>669.0</td>
</tr>
</tbody>
</table>

Table 7.7: Mean value of yield stress \( (f_{ym}) \), Young’s modulus \( (E_m) \) and ultimate strength \( (f_{um}) \) for the reinforcement steel.

7.1.2.2 Shear experiments

To test the shear capacity, the same test set-ups as in the Lindenborg programme were employed: (i) A simply supported single span beam set-up and; (ii) A continuous beam set-up. The test set-up for the simply supported beams is shown in Figure 7.17. The figure shows furthermore that the additional stirrup was placed directly under the loading, to ensure that it did affect the shear failure; neither in the left or right shear span. The dimensions of the test set-up are shown in Table 7.8. Details of the supports and the loading are shown in Figure 7.19a and 7.19b, respectively. The supports consisted of: (i) A ø40 mm steel bar that ensured free rotation; (ii) A 10 × 60 mm steel plate that ensured distribution of the reaction force and; (iii) 2 Teflon plates lubricated with copper grease that ensured almost frictionless horizontal movements. The loading consisted of: (i) two 60 mm wide steel plates that ensured that the load was applied evenly distributed over the beam width and; (ii) a 1000 kN load cell with swivel head that ensured precise measurements of the load, and that the load was applied without eccentricities.

The test set-up for the continuous beams is shown in Figure 7.18. The figure shows furthermore that two additional stirrups were provided; one to the left of the reaction, \( R_1 \), (left shear zone) and one to the right of the load, \( P_2 \), (right shear zone). The dimensions of the test set-up are shown in Table 7.9. The continuous beams were supported as the simply supported beams (see Figure 7.19a), whilst the loading was different (see Figure 7.19c). The loadings consisted of: (i) A ø40 mm steel bar that ensures free rotation; (ii) A 10 × 50 mm steel plate that ensured distribution of the reaction force and; (iii) 2 Teflon plates lubricated with copper grease that ensured almost frictionless horizontal movements.

The load was applied deformation-controlled with a constant rate of 0.5 mm/min and DIC measurements were performed in both set-ups. The DIC measurements were performed with a stereo camera set-up with 12 MP per camera with a frequency of 1 photo/s on a 4 m wide zone.
Figure 7.17: Experimental set-up for the simply supported beams. All dimensions are in mm.

Figure 7.18: Experimental set-up for the continuous beams. All dimensions are in mm.

Figure 7.19: Supports and loading for the reference programme.
### Table 7.8: Details for dimensions and the experimental set-up for the simply supported beams.

<table>
<thead>
<tr>
<th>Beam name</th>
<th>( L_1 ) [mm]</th>
<th>( a_L ) [mm]</th>
<th>( a_R ) [mm]</th>
<th>( L_R ) [mm]</th>
<th>( a_L/h ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left anchorage zone</td>
<td>500</td>
<td>700</td>
<td>700</td>
<td>500</td>
<td>2.3</td>
</tr>
<tr>
<td>Left shear zone</td>
<td>500</td>
<td>850</td>
<td>850</td>
<td>500</td>
<td>2.8</td>
</tr>
<tr>
<td>Right shear zone</td>
<td>500</td>
<td>1000</td>
<td>1000</td>
<td>500</td>
<td>3.3</td>
</tr>
<tr>
<td>Right anchorage zone</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

### Table 7.9: Details on dimensions and experimental set-up for the continuous beams.

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left anchorage zone</td>
<td>( L_1 ) [mm]</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Left shear zone</td>
<td>( L_1 ) [mm]</td>
<td>495</td>
<td>420</td>
<td>415</td>
</tr>
<tr>
<td>Main shear zone</td>
<td>( x_0 ) [mm]</td>
<td>900</td>
<td>1200</td>
<td>1500</td>
</tr>
<tr>
<td>Right shear zone</td>
<td>( L_2 ) [mm]</td>
<td>600</td>
<td>675</td>
<td>845</td>
</tr>
<tr>
<td>Right anchorage zone</td>
<td>( L_R ) [mm]</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Slenderness</td>
<td>( x/h ) [-]</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Load ratio</td>
<td>( P_1/P_2 ) [-]</td>
<td>0.38</td>
<td>0.56</td>
<td>0.78</td>
</tr>
</tbody>
</table>

### 7.1.2.3 Results

#### Simply supported beams

Table 7.10 shows the main results of the shear tests for the simply supported beams.

<table>
<thead>
<tr>
<th>( a_L/h )</th>
<th>( P_{max} ) [kN]</th>
<th>( V_{max} ) [kN]</th>
<th>( u_{max} ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam S1</td>
<td>2.3</td>
<td>240.6</td>
<td>120.3</td>
</tr>
<tr>
<td>Beam S2</td>
<td>2.8</td>
<td>173.7</td>
<td>86.9</td>
</tr>
<tr>
<td>Beam S3</td>
<td>3.3</td>
<td>179.0</td>
<td>89.5</td>
</tr>
</tbody>
</table>

Note: 
- \( P_{max} \) is the maximum applied load, \( V_{max} \) is the maximum applied shear force excl. selfweight, \( u_{max} \) is the deflection when the maximum load is applied.

### Table 7.10: Test results for the simply supported beams.

Figure 7.20 shows the associated shear-deflection curves. The depicted deflections are measured at the position of the load, and are corrected for the compaction of the supports. The depicted shear force does not include the self-weight of the beams (less than 5 kN).
The load-deflection curves for the beams with a slenderness \( (a_L/h) \) of 2.3 and 2.8 are alike whilst the beam with a slenderness of \( a_L/h = 3.3 \) is significantly more ductile.

![Load-deflection curves for the simply supported beams.](image)

**Figure 7.20:** Load-deflection curves for the simply supported beams.

**Continuous beams**

Table 7.11 shows the main results of the shear tests for the continuous beams.
<table>
<thead>
<tr>
<th>$x_0/h$</th>
<th>$P_{\text{max}}$ [kN]</th>
<th>Failure zone</th>
<th>$V_{\text{fail, max}}$ [kN]</th>
<th>$u_{2,\text{max}}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam C1</td>
<td>3.0</td>
<td>433.3</td>
<td>Main</td>
<td>164.9</td>
</tr>
<tr>
<td>Beam C2</td>
<td>4.0</td>
<td>351.7</td>
<td>Main</td>
<td>109.5</td>
</tr>
<tr>
<td>Beam C3</td>
<td>5.0</td>
<td>396.0</td>
<td>Main</td>
<td>110.8</td>
</tr>
<tr>
<td>Beam C4</td>
<td>6.0</td>
<td>423.5</td>
<td>Main</td>
<td>90.7</td>
</tr>
</tbody>
</table>

Note:
$P_{\text{max}}$ is the maximum applied load, $V_{\text{fail, max}}$ is the maximum applied shear force in the shear zone where the fail occur excl. selfweight, $u_{2,\text{max}}$ is the deflection measured at the loading $P_2$ when the maximum load was applied.

Table 7.11: Test results for the continuous beams.

Figure 7.21 shows the associated shear-deflection curves. The depicted deflections are measured at the position of the load ($P_2$) and corrected for deformation of the supports. The depicted shear force does not include the self-weight of the beams (less than 5 kN). The load-deflection curves for the beams with a slenderness ($x_0/h$) of 3.0, 4.0 and 5.0 are alike, while the curve for the beam with a slenderness of $x_0/h = 6.0$ is significantly more ductile.
7.1.3 Gammelrand bridge

Gammelrand was a three-span concrete bridge (located in Denmark) that suffered from severe ASR-damages. Due to uncertainties regarding the load-carrying capacity, the bridge was replaced by a new bridge. In connection with the demolition, four beams were cut from the bridge deck (a slab without shear reinforcement). The beams were used for testing of the shear capacities. A more detailed description of the bridge, its condition and how the beams were cut is provided in Section 4.1. The experiments are planned and conducted by Jensen and Rask (2014).

7.1.3.1 Material

In order to test the shear capacity, the four beams from Gammelrand bridge, were brought to laboratory facilities. To ensure shear failures (i.e. avoid bending failure), the cross sections were changed by cutting. Figure 7.22 shows the cross sections of the beams after the cutting. Table 7.12 shows the amount of reinforcement and the size of
the concrete cover for each of the four beams. To determine the exact configuration (size, numbers, type and position) and the condition of the reinforcement, the reinforcement bars were exposed by removing the concrete with a hammer after the shear tests. The full outcome of this determination is provided in Appendix B.

![Beam 1 Beam 2 Beam 3 Beam 4](image)

*Figure 7.22: Cross sections for the four beams. All dimensions are in mm.*

<table>
<thead>
<tr>
<th>Beam</th>
<th>Reinforcement</th>
<th>Cover [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top</td>
<td>Bottom</td>
</tr>
<tr>
<td>Beam 1</td>
<td>1ø25</td>
<td>3ø25</td>
</tr>
<tr>
<td>Beam 2</td>
<td>1ø20 + 1ø25</td>
<td>3ø25</td>
</tr>
<tr>
<td>Beam 3</td>
<td>1ø20</td>
<td>3ø25</td>
</tr>
<tr>
<td>Beam 4</td>
<td>2ø20</td>
<td>3ø25</td>
</tr>
</tbody>
</table>

*Table 7.12: Reinforcement and cover. All dimensions are in mm.*

**Concrete**

Since the compressive strength of the concrete has been presented in Section 5.1.5.1, only the tensile strength of the concrete is presented in this section. According to the conclusion in Chapter 6, the tensile strength is determined by wedge splitting tests. The tests were conducted by Barbosa (2017). Figure 7.23 shows the tensile strength parallel ($f_{\parallel}$) and perpendicular ($f_{\perp}$) to the ASR-induced cracks.
Reinforcement

The mechanical properties and the prestressing were measured on undamaged reinforcement bars after the shear experiments were conducted. Table 7.13 shows the mean value of the measured yield stress ($f_{ym}$), Young’s modulus ($E_m$) and the ultimate strength ($f_{um}$) for both the $\varnothing20$ and the $\varnothing25$ mm reinforcement bars.

<table>
<thead>
<tr>
<th></th>
<th>$f_{ym}$ [MPa]</th>
<th>$E_m$ [GPa]</th>
<th>$f_{um}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varnothing20$</td>
<td>586.9</td>
<td>184.2</td>
<td>742.9</td>
</tr>
<tr>
<td>$\varnothing25$</td>
<td>657.3</td>
<td>191.4</td>
<td>809.4</td>
</tr>
</tbody>
</table>

*Table 7.13: Mean value of measured yield stress ($f_{ym}$), Young’s modulus ($E_m$) and ultimate strength ($f_{um}$) for $\varnothing20$ and $\varnothing25$ reinforcement bars.*

The prestressing in the reinforcement was measured directly on the reinforcement bars as it was done for Lindenborg. The employed test method is described in Section 7.1.1. Figure 7.24 shows the measured prestressing strain ($\varepsilon_{s,ASR}$). Each measurement is shown as a black cross and the minimum and maximum value are written with black numbers. The mean values are shown with red dots and red numbers. It is seen that the $\varnothing20$mm rebars have a mean prestressing of 1.24% while the mean prestressing is 0.83% for the $\varnothing25$mm rebars. The prestressings correspond to tensile stresses of 228.4 and 158.9 MPa, respectively.
7.1.3.2 Shear experiments

The shear capacity of the four beams was tested in the same asymmetric three-point-bending experimental set-up, see Figure 7.25. The beam was simply supported and loaded by one point load ($P$), 1868 mm from the left support ($R_1$). Details of the supports and the loading are shown in Figure 7.26. The left support ($R_1$) allowed free rotations and the right support ($R_2$) allowed free rotation and horizontal displacements. The supports and the loading are conducted with wood fibre boards to ensure full contact to the concrete. The load on Beam 1 to 3 was applied by manual controlled oil pressure (not fully load-controlled). The load was applied continuously until approximately 140 kN and subsequently applied stepwise until failure. In each step, the load was increased by 15 kN over 1 minute and kept constant for 2 minutes. Due to technical issues the load-applying actuator was changed during the loading of Beam 3. For Beam 4, the load was applied deformation-controlled with a constant rate of 0.35 mm/min.

In the experiments with Beam 1 and 2, DIC measurements were performed in the zone between the loading and the left support, see Figure 7.25. The employed DIC system uses a stereo camera set-up of 4 MP per camera with a frequency of 0.1 photo/s.
7.1.3.3 Results

Table 7.14 shows the results and the moment utilization of the four shear tests. The first issue that leaps out from the results is that Beam 3 failed in the right shear span despite that the shear force in the left shear span was larger. That the left shear span was stronger than the right can be due to many unknown factors, such as unexpected reinforcement. During a post-examination of the failure of Beam 3, a straight vertical rebar was found. Since it crossed the failure, it is likely that it had an influence on tested shear capacity. This will be kept in mind in the later comparison. The second issue that leaps out is the variation of the shear capacity. Despite that the experimental set-up was identical for Beam 1, 2 and 4 and the beams were cut from the same area of the bridge, the shear capacity varies between 243.9 kN to 305.8 kN. The moment utilizations are all below 1.0. This indicates that the moment capacity has not been governing for the failure.

Figure 7.28 shows the shear-deflection curves from the tests. The deflection is measured at the loading point and for Beam 1, 3 and 4 the deformation is corrected for the compaction of the wood fibre boards and the deformation of the supports. Beam 2 had a horizontal crack in the end before testing. This crack opened further during loading, see Figure 7.27. Since the deformation of the support was measured on the top of the
beam, the opening of the end crack influences the measured deformation. Consequently, the deformation used for Beam 2 is not corrected for the support deformations. The depicted shear force corresponds to the shear span where the failure occurred, i.e. for Beam 1, 2 and 4, the shear force in the left shear span is depicted, whilst the shear force in the right shear span is depicted for Beam 3.

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{max}}$ [kN]</th>
<th>$V_{\text{max}}$ [kN]</th>
<th>$u_{\text{max}}$ [mm]</th>
<th>Failure side</th>
<th>$M_u/M_{R,\text{mean}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1</td>
<td>425.1</td>
<td>305.8 (283.3)</td>
<td>21.2</td>
<td>Left</td>
<td>0.89</td>
</tr>
<tr>
<td>Beam 2</td>
<td>388.1</td>
<td>281.1 (258.6)</td>
<td>22.5$^A$</td>
<td>Left</td>
<td>0.79</td>
</tr>
<tr>
<td>Beam 3</td>
<td>418.1</td>
<td>162.0 (139.5)$^B$</td>
<td>13.8</td>
<td>Right</td>
<td>0.84</td>
</tr>
<tr>
<td>Beam 4</td>
<td>332.2</td>
<td>243.9 (221.4)</td>
<td>13.4</td>
<td>Left</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Note:

$P_{\text{max}}$ is the maximum applied load, $V_{\text{max}}$ is the maximum applied shear force incl. selfweight while the applied shear force excl. selfweight is given in brackets and $u_{\text{max}}$ is the deflection when the maximum load was applied.

A: Not corrected for support compression

B: For the right shear span

**Table 7.14:** Test results for the four shear tests.

**Figure 7.27:** Horizontal crack at the end of Beam 2 after testing.
Figure 7.28: Shear-deflection curves for the four shear tests. For beam 1, 2 and 4, the shear force in the left shear span is depicted, whilst the shear force in the right shear span is depicted for Beam 3.

7.2 Shape of the critical shear crack

To answer to what extent and how ASR affects the shape of the critical crack (RQ6), the critical cracks for the ASR-damaged beams (Lindenborg) and the reference beams are compared, see Section 7.2.1. However, since the failure of the simply supported beams from Lindenborg was governed by the moment capacity, only a comparison of the critical shape is made for the continuous beams. To investigate how ASR affects the shape of the critical crack for simply supported beams that fail in shear, the shape of the critical crack for the beams from Gammelrand is compared with the expected shape of the critical crack for corresponding beams without ASR-damages, see Section 7.2.2. To answer why ASR affects the shape of the critical crack (RQ7), the crack formation has
been investigated. The crack formation for two beams from Lindenborg (one continuous and one simply supported) are compared with the crack formation of the corresponding reference beams, see Section 7.2.4.

### 7.2.1 Comparison of the critical cracks, Lindenborg bridge

The crack patterns are shown by means of major strain plots derived from the DIC measurements. In the major strain plots, a colour scale indicates the level of the tensile strains: From blue, which is neutral, via green and yellow to red, which is the largest tensile strains. The beams are grouped according to their slenderness ($x_0/h$).

Figure 7.29 to 7.31 show the beams with a slenderness of $x_0/h = 3.0$. Figure 7.29 and 7.30 show the crack patterns for Beam 1.3 and 5.3 from Lindenborg (ASR-damaged), whilst Figure 7.31 shows the crack pattern for the reference beam. It appears that the critical crack in the ASR-damaged beams is a straight diagonal crack that runs from $R_1$ to $P_2$ within the zone between the two layers of reinforcement. The diagonal crack is accompanied by horizontal cracks in the compression zones. In contrast, the crack pattern for the reference beam consists of vertical flexural cracks over $R_1$ and under $P_2$ and two flexural-shear (non-vertical flexural cracks) cracks under $P_2$. The left flexural-shear crack (marked with an arrow) is the critical crack (where the failure occurs).

![Figure 7.29: Major strain plot: Crack pattern at failure for Beam 1.3 from Lindenborg ($x_0/h = 3.0$).](image1)

![Figure 7.30: Major strain plot: Crack pattern at failure for Beam 5.3 from Lindenborg ($x_0/h = 3.0$).](image2)
Figure 7.31: Major strain plot: Crack pattern at failure for Beam C1 from the reference programme ($x_0/h = 3.0$).

Figure 7.32 to 7.35 show the crack patterns for the beams with a slenderness of $x_0/h = 4.0$. It appears that the critical crack in Beam 1.2 (Figure 7.32) and 4.3 (Figure 7.33) is similar to the critical crack in the ASR-damaged beams with a slenderness of $x_0/h = 3.0$: the critical crack is a straight diagonal crack that runs between the two layers of reinforcement, and is accompanied by horizontal cracks in the compression zones. The critical crack in Beam 5.2 (Figure 7.34) differs a bit; it consists of two diagonal cracks. Furthermore, the crack patterns are similar to the ones for Beam 1.2 and 4.3. The crack pattern for the reference beam (Figure 7.35) differs significantly from the crack patterns in the ASR-damaged beams. It consists of a number of vertical flexural cracks, where the critical crack is formed in one of them. The critical crack consists of three parts: (i) a horizontal part that follows the lower layer of reinforcement; (ii) a steep part, that follows the flexural crack and; (iii) a flat part in the top of the beam.

Figure 7.32: Major strain plot: Crack pattern at failure for Beam 1.2 from Lindenborg ($x_0/h = 4.0$).
Figure 7.33: Major strain plot: Crack pattern at failure for Beam 4.3 from Lindenborg ($x_0/h = 4.0$).

Figure 7.34: Major strain plot: Crack pattern at failure for Beam 5.2 from Lindenborg ($x_0/h = 4.0$).

Figure 7.35: Major strain plot: Crack pattern at failure for Beam C2 from the reference programme ($x_0/h = 4.0$).

Figure 7.36 to 7.39 show the crack patterns for the beams with a slenderness of $x_0/h = 5.0$. While the crack pattern for 5.1 (Figure 7.38) is similar to the crack pattern observed for the ASR-damaged beams with a lower slenderness, the crack pattern for Beam 1.1 and 4.2 (Figure 7.36 and 7.37) differs. Instead of having a straight diagonal crack, they have an inclined crack that runs to the midspan (marked with an arrow on the figures) and subsequently follow the reinforcement. On Figure 7.37 it appears that the inclined crack is accompanied by horizontal cracks in compression zone near $P_2$ as for the other ASR-damaged beams. It is remarked that a course horizontal crack was formed along the reinforcement in Beam 4.2; both along the upper and lower layer. Also for a
slenderness of $x_0/h = 5.0$, the crack pattern for the reference beam differs significantly from the crack pattern for the ASR-damaged beams. The critical crack here is similar to one developed in the reference beams with a slenderness of $x_0/h = 4.0$ where it partly follows a flexural-shear crack, see Figure 7.39.

**Figure 7.36: Major strain plot: Crack pattern at failure for Beam 1.1 from Lindenborg ($x_0/h = 5.0$).**

**Figure 7.37: Major strain plot: Crack pattern at failure for Beam 4.2 from Lindenborg ($x_0/h = 5.0$).**

**Figure 7.38: Major strain plot: Crack pattern at failure for Beam 5.1 from Lindenborg ($x_0/h = 5.0$).**
Figure 7.39: Major strain plot: Crack pattern at failure for Beam C3 from the reference programme ($x_0/h = 5.0$).

Figure 7.40 and 7.41 show the crack pattern for the beams with a slenderness of $x_0/h = 6.0$. The critical crack in the ASR-damaged beam is similar to the one in Beam 4.2, see Figure 7.37. The inclined crack is, however, shifted towards the midspan such that it does not run to the loading point. As for Beam 4.2, the inclined crack is accompanied by horizontal crack in the compression zone near the loading point $P_2$. The shape of the critical crack in the reference beam is similar to what was observed for the reference beams with a lower slenderness, however, it is formed near $R_1$ rather than near $P_2$, see Figure 7.41.

Figure 7.40: Major strain plot: Crack pattern at failure for Beam 4.1 from Lindenborg ($x_0/h = 6.0$).

Figure 7.41: Major strain plot: Crack pattern at failure for Beam C4 from the reference programme ($x_0/h = 6.0$).

Generally, it can be concluded that the shape of the critical crack in the continuous beams is significantly affected by ASR. Instead of being formed in a flexural/flexural-shear crack, the critical crack is formed as an almost straight diagonal crack between $R_1$ and $P_2$ within the zone between the two layers of reinforcement and accompanied by horizontal cracks in the compression zones.
7.2.2 Shape of the critical crack, Gammelrand bridge

The shape of the critical crack is investigated by means of major strain plots derived from DIC measurements for Beam 1 and 2. Since DIC measurements were not performed for Beam 4, the shape of the critical crack is investigated by means of a photo of the crack pattern after the failure occurred. Since the failure of Beam 3 was influenced by an unexpected vertical reinforcement bar, Beam 3 is not included in this investigation. Figure 7.42 to 7.44 show the failure crack pattern for Beam 1, 2 and 4. It appears that the failure occurs in a composition of more cracks rather than in one critical shear crack. The first part is a crack with a low inclination running from the lower layer of reinforcement to approximately 180 mm under the loading (the beam height is approximately 750 mm), this crack is marked with an ”A” on the figures. Figure 7.42 and 7.43 show that the width of this crack is rather large. The second part of the failure occurs in almost horizontal cracks in the top and the bottom of the beam, these cracks are marked with a ”B” on the figures. The cracks are formed on the onset of the failure. The cracks merge with the large (”A”-) crack around the midspan.

![Figure 7.42: Major strain plot: Crack pattern at failure for Beam 1 from Gammelrand ($a_L/h = 2.5$).](image1)

![Figure 7.43: Major strain plot: Crack pattern at failure for Beam 2 from Gammelrand ($a_L/h = 2.5$).](image2)
Since no reference beams were tested for Gammelrand, it is not possible to show exactly how ASR affects the shape of the critical crack. However, it is expected that the critical crack for a shear failure for a simply supported beam with a slenderness larger than $a_L/h = 2.5$ develops via an existing flexural-shear crack (Nielsen and Hoang, 2010). Consequently, it can be concluded that ASR affects the shape of the critical crack for simply supported beams as well.

### 7.2.3 Answer to RQ6

It can be concluded that ASR affects the shape of the critical crack for simply supported beams as well as for continuous beams. For beams without ASR-damages, the critical crack develops by propagation of an existing flexural-shear crack whilst for beams with ASR-damages, the critical crack develops as an inclined crack that is accompanied with cracks near the loading (top) and the reaction (bottom).

### 7.2.4 Comparison of the crack formation

Figure 7.45 and 7.46 show the crack formation for a continuous beam with a slenderness of $x_0/h = 3.0$ with and without ASR-damages, respectively. The crack formation is shown in four selected load stages. Starting with the beam with ASR-damages, Beam 1.3 from Lindenborg. No cracks were formed before a load corresponding to a shear force of $V_2 = 110.9$ kN was applied. Figure 7.45a shows that small vertical flexural cracks under $P_2$ (marked with arrows) and a diagonal crack between $R_1$ and $P_2$ (marked with an arrow) are formed simultaneously. Since the load was applied deformation-controlled, the load decreased due to the formation of these cracks. When the load was recovered, the diagonal crack propagated in length and width, see Figure 7.45b. When the load was increased to what corresponds to a shear force of $V_2 = 144.6$ kN, the diagonal crack opened further, and new flexural cracks were formed in the midspan between $R_1$ and $P_2$ both in the top and the bottom of the beam, see Figure 7.45c. It is remarked that in the midspan, flexural cracks are formed at both the top and the bottom of the beam in the same section. This cannot be explained by simple beam theory. Figure 7.45d shows the crack pattern when the failure occurred. It appears that the diagonal crack and the
midspan flexural cracks have grown and they are accompanied by horizontal cracks in the compression zones. It is remarked that the flexural cracks under $P_2$ are only present in the lower third of the beam. Figure 7.46 shows the crack formation for an identical beam without ASR-damages. The first flexural crack is formed when a load corresponding to a shear force of $V_2 = 71.8$ kN is applied, see Figure 7.46a. It is noted that the first flexural cracks were formed when $V_2 = 110.9$ kN in the beam with ASR-damages. When the load is increased to what corresponds to $V_2 = 88$ kN, two significant flexural cracks are formed, see Figure 7.46b. Already at this stage, the height of the flexural cracks is almost equal to the beam height. When the load is increased to what corresponds to $V_2 = 110.6$ kN, a flexural-shear crack is formed (marked with an arrow on Figure 7.46c). It is this flexural-shear crack that develops into the critical crack (see Figure 7.46d).
Figure 7.45: Major strain plot: Crack propagation for Beam 1.3 from Lindenborg ($x_0/h = 3.0$).
A similar comparison is conducted for a simply supported beam with a slenderness of $a_l/h = 2.3$. Figure 7.47 and 7.48 show the crack formation for Beam 3.3 from Lindenborg (ASR-damaged) and the reference beam, respectively. Considering the ASR-damaged beam. The first flexural cracks occur when a load corresponding to a shear
The force of $V = 93.6$ kN is applied, see Figure 7.47a. The height of the flexural cracks is approximately 100 mm (the beam height is 300 mm). When the load is increased to what corresponds to $V = 112.9$ kN, inclined cracks are formed, see Figure 7.47b. These cracks seem to run from the lower layer of reinforcement and up to approximately 100 mm under the load plate. As the load is increased to failure, more inclined cracks are formed, and accompanied by horizontal cracks in the compression zone, see Figure 7.47c and 7.47d. Considering the crack formation for the reference beam. The first load stage ($V = 93.6$ kN) corresponds to the load stage where the first flexural cracks were formed in the ASR-damaged beam. It appears that both large flexural and flexural-shear cracks are formed, see Figure 7.48a. These cracks run almost to the top of the beam. When the load is increased, one of these flexural-shear cracks develops into the critical crack, see Figure 7.48b and 7.48c. Finally, it is remarked that despite the fact that the failure of the ASR-damaged beam was governed by the moment capacity, it had a higher load-carrying capacity than the reference beam.
Figure 7.47: Major strain plot: Crack propagation for Beam 3.3 from Lindenborg ($a_L/h = 2.33$).
Regarding crack formation, it appears that there are three main differences between the beams with and without ASR-damages:

- In the ASR-damaged beams, the flexural cracks are formed at a higher load and the height of the cracks are smaller. This may be explained by the presence of the ASR-induced prestressing.

- In the ASR-damaged beams, flat inclined cracks were formed. For the continuous beams, a diagonal crack between the two layers of reinforcement was formed, whilst an inclined crack in the bottom of the beam was formed for the simply supported beams. This may be due to a very low tensile strength perpendicular to the ASR-induced crack, $f_{t\perp}$ (see Section 7.1.1).

Figure 7.48: Major strain plot: Crack propagation for Beam S2 from reference programme ($a_L/h = 2.33$).
- The flexural-shear cracks that were observed in the beams without ASR-damages were not observed in the beams with ASR.

The crack formation has a large impact on the shape of the critical crack. For the reference beams, the critical crack formed through an existing flexural/flexural-shear crack. This happens because, the resistance from aggregate interlocking decreases with increasing crack width (see Section 2.2); the existing flexural-shear crack is simply the weakest path.

Since the flexural cracks in ASR-damaged beams have a smaller height, the length with reduced resistance from aggregate interlocking is shorter. Consequently, other failure paths than through the existing flexural cracks seem to be critical.

For continuous ASR-damaged beams, a course diagonal crack is formed. Due to the large crack width of this crack, the weakest path includes the diagonal crack. To complete the critical crack, cracks are formed in the compression zones during failure.

For simply supported ASR-damaged beams, an inclined crack was formed in the bottom of the beam before the failure occurred. In the beams from Gammelrand, crack originating from the reaction and the loading merged with this inclined crack.

### 7.2.5 Answer to RQ7

The two main reasons why ASR affects the shape of the critical crack are: (i) the flexural cracks are smaller (width and height) and; (ii) flat inclined cracks are formed in the ASR-damaged beams.
Chapter 8

Shear Model

One of the main aims of this thesis is to establish a mechanical model for the shear capacity of ASR-damaged slabs without shear reinforcement. Such a model is established in this chapter. The model will be based on the upper bound theorem of the theory of plasticity.

In the first section, a short introduction to the theory of plasticity for concrete structures is given. In the second and third section, a model for the shear capacity for ASR-damaged simply supported and a model for the continuous beams are established. The fourth section provides a comparison between proposed models and the experimental results obtained from Lindenborg and Gammelrand (see Section 7.1.1 and 7.1.3). In the last section, recommendations on practical applications of the proposed models are provided.

8.1 Plasticity theory for concrete structures

This section provides a brief introduction to the theory for rigid-plastic materials, plasticity theory, and its application to concrete structures. A rigid-plastic material is defined as a material having a stress-strain relationship as shown in Figure 8.1. The material experiences zero strains before the stress reaches a certain threshold, the so-called yield stress, denoted $f_y$. Hereafter, unlimited strains can occur at constant stress, $f_y$, i.e. neither strain hardening nor softening occur.

Figure 8.1: Stress-strain relationship for a rigid-plastic material.
The rigid-plastic material behaviour is a simple idealization of reality and the behaviour of concrete subjected to uniaxial compression differs, in fact, significantly from that of a rigid-plastic material. However, plasticity theory and thereby the assumption of rigid-plastic behaviour, has proven to work well for determination of the load-carrying capacity of reinforced concrete structures when effective strength parameters are employed.

To use plasticity theory for determination of the load-carrying capacity (also known as limit analysis), the following aspects must be considered: (i) the yield condition; (ii) the normality condition and; (iii) the extremum principles. These aspects will be introduced individually in the following sections.

It is remarked that this thesis does not aim at providing a full explanation on the topic of plasticity theory; rather, it aims at providing the essential background knowledge for the understanding of the models that are established within this chapter. For more detailed information on the topic and a historical overview, the reader is referred to established literature (e.g. (Nielsen and Hoang, 2010) or (Chen and Han, 2007)).

8.1.1 Yield condition

A yield condition describes the criterion for yielding of a material. By means of a yield function, the yield condition describes the combinations of stresses that lead to yielding:

\[ f(\sigma_x, \sigma_y, \ldots, \tau_{xx}) = 0 \] (8.1)

where \((\sigma_x, \sigma_y, \ldots, \tau_{xx})\) are the applied stresses. The yield condition is assumed to be a convex surface in the stress space enclosing the point of no stresses. This surface is known as the yield surface. If \(f < 0\), the applied stresses lie within the yield surface and the material is not yielding and is able to carry the applied stresses. If \(f = 0\), the material is yielding. Since \(f > 0\) means that the applied stresses are larger than the yield strength of the material, this stress state cannot occur.

8.1.2 Normality condition

When the yield condition (Equation 8.1) is satisfied, the material will experience plastic strains, which lead to dissipation of energy. For a structure with a certain volume, \(V\), the dissipated energy, \(D\), can be found as:

\[ D = \int_V \sigma_i \cdot \varepsilon_i dV = \int_V W dV \] (8.2)

where \(\sigma_i\) are the normal- or shear stresses and \(\varepsilon_i\) are the associated strain components. According to Von Mises's hypothesis on maximum work, any stress state on the yield surface will result in a strain state that make the work, \(W\), becomes as large as possible, i.e. the material conducts the largest possible resistance to a given deformation (Mises,
Consequently, the strain vector will be a normal to the yield surface. When the yield function is differentiable, the strain vector can be described as:

$$
\varepsilon_i = \lambda \frac{\partial f}{\partial \sigma_i}
$$

(8.3)

where $\lambda$ is an indeterminate non-negative factor. This relation is known as the normality condition or the associated flow rule.

8.1.3 Extremum principles

Drucker et al. (1952) introduced two so-called extremum principles by formulating a number of theorems. The theorems are essential for employing limit analysis in practice. In the following, the upper bound- and lower bound theorem are shortly introduced.

8.1.3.1 The lower bound theorem

The first extremum principle is derived from the lower bound theorem, which states that: "If a safe statically admissible state of stress can be found at each stage of loading, collapse will not occur under the given loading schedule." (Drucker et al., 1952)

In practice, this means that if, for a given load it is possible to find a stress distribution that satisfies the equilibrium- and the static boundary conditions and the stresses are within the yield surface (i.e. $f < 0$, see Equation 8.1), the load will not lead to failure. This stress distribution is denoted a safe and statically admissible stress distribution. Since the load-carrying capacity obtained by this principle is smaller than, or equal to the yield load, the theorem is called the lower bound theorem.

8.1.3.2 The upper bound theorem

The second extremum principle is derived from the upper bound theorem, which states that: "If a kinematically admissible collapse state can be found at any stage of loading, collapse must impend or have taken place previously." (Drucker et al., 1952)

In practice, this means that the load that is required to form the considered kinematically admissible failure mechanism is equal to, or larger than the yield load. Since the load-carrying capacity obtained by this principle is larger than or equal to the yield load, the theorem is called the upper bound theorem.

8.1.4 Plasticity theory for concrete structures

This section provides an introduction to the special conditions that should be applied when the load-carrying capacity of concrete structures is determined by means of plasticity theory. Since upper bound solutions are based on a failure mechanism, it is possible to incorporate the experimental observations (observed failure mechanisms). The models
that are established later in this thesis are therefore based on upper bound solutions. Consequently, this section only introduces the upper bound method.

### 8.1.4.1 Yield condition for Concrete

The first step in the derivation of the load-carrying capacity is to determine the yield condition for concrete. Several yield conditions for different materials exist. For concrete, it is generally accepted that the Modified Mohr-Coulomb Failure Criterion can be adopted (Nielsen and Hoang, 2010). As the name reveals, the criterion is a modification of Coulomb’s well-known frictional criterion (Coulomb, 1776). The modified criterion is shown in Figure 8.2. The figure shows the combinations of shear- and normal stresses (positive as tension) that lead to failure. The inclined lines represent the criterion for sliding failure, and are described by the angle of friction $\varphi$ and the cohesion $c$, which are material parameters. The vertical line is the criterion for separation failure, and is described by the concrete tensile strength $f_t$. To find the critical section and thereby the critical set of stresses $(\sigma, \tau)$ for a given load case, Mohr’s circles are employed. The example shown in Figure 8.2 corresponds to uniaxial compressive loading where the smallest principal stress ($\sigma_3$) corresponds to the compressive strength of concrete ($f_c$).

![Figure 8.2: Modified Mohr-Coulomb failure criterion](image)

The criteria for the sliding and separation failure can be described as:

$$|\tau| = c - \sigma \tan \varphi \quad (8.4)$$

$$\sigma = f_t \quad (8.5)$$

### 8.1.4.2 Upper bound solutions

An upper bound solution is established by considering a kinematically admissible failure mechanism (see Section 8.1.3.2). Since the material behaves rigid-plastic, all deformations occur in the area with yielding. Figure 8.3 shows an example of a shear failure
mechanism for a simply supported beam where the failure occurs as sliding in a diagonal yield line. The relative displacement in the yield line is described by the length of a displacement vector, $|\mathbf{u}|$, and the angle between the yield line and the displacement vector, $\alpha$.

![Figure 8.3: Kinematically admissible failure mechanism for a simply supported concrete beam.](image)

The load, $P_u$, that is required to form this failure mechanism may be determined by means of the work equation, which states that the work conducted by the external loads (the external work, $W_E$) is the same as the work conducted by the internal forces (the internal work, $W_I$):

$$W_E = W_I \tag{8.6}$$

The work, internal as well as external, is determined from the considered failure mechanism. Generally, the external work can be found as:

$$W_E = \sum_{i=1}^{n} P_i \cdot u_i \tag{8.7}$$

where $P_i$ is the external load at point $i$ and $u_i$ is the displacement in the direction of $P_i$. For the example in Figure 8.3, the external work is $W_E = P_u u_v$, where $u_v$ is the vertical component of the displacement vector.

In general, the internal work can be found as:

$$W_I = D = \int_V \sigma_i \cdot \epsilon_i dV \tag{8.8}$$

Since deformations only occur in the yield line, the internal work may be found by integrating over the length of the yield line. In practice, the stresses and strains in the yield line are derived by means of the yield condition and the normality condition. In this
thesis, the formulas for the internal work are provided without any derivation (Nielsen and Hoang, 2010). For plane strain problems, the internal work per unit area of the yield line (also called the dissipation) may be found as follows (Nielsen and Hoang, 2010):

\[ W_i = \frac{1}{2} \nu f_c (l - m \sin(\alpha)) |u|, \quad \varphi \leq \alpha \leq \pi - \varphi \]  

(8.9)

where:

\[ l = 1 - 2 \frac{f_{t,ef}}{\nu f_c} \sin \varphi \]  

(8.10)

\[ m = 1 - 2 \frac{f_{t,ef}}{\nu f_c} \frac{1}{1 - \sin \varphi} \]  

(8.11)

where \( \nu \) is the effectiveness factor which accounts for the fact that concrete does not behave rigid-plastic, \( f_{t,ef} \) is the effective tensile strength of concrete, \( \varphi \) is the angle of friction. The angle of friction is a material parameter, which depends on the compressive strength. However, it is simplified to \( \tan \varphi = 0.75 \) in this thesis. This will be discussed in Section 8.3.1.2. For plane stress problems, the internal work per unit area of the yield line may be found from Equation 8.9, yet without the restrictions for \( \alpha \). In cases where the tensile strength of concrete, \( f_t \), is neglected, the dissipation formula is simplified as:

\[ W_i = \frac{1}{2} \nu f_c (1 - \sin(\alpha)) |u|, \quad \varphi \leq \alpha \leq \pi - \varphi \]  

(8.12)

The dissipation for pure tension (\( \alpha = 90^\circ \)) can be simplified to:

\[ W_{i,\text{tensile}}(\alpha = 90^\circ) = f_{t,ef} |u| \]  

(8.13)

The dissipation for pure compression (\( \alpha = -90^\circ \)) may also be found by the dissipation formula, easiest by considering it as a plane stress problem:

\[ W_{i,\text{compressive}}(\alpha = -90^\circ) = \nu f_c |u| \]  

(8.14)

8.2 Simply supported beams

In this section, a model for the shear capacity of simply supported ASR-damaged beams is established. The model is based on a failure mechanism that is inspired by observations of the shape of the critical crack that were made during the Gammelrand experiments, see Section 7.2.2. Figure 8.4 shows a schematic illustration of the observed cracks. Figure 8.4a shows the crack that is observed first, illustrated as a black line. It is a straight crack with a low inclination. It runs from the lower layer of reinforcement over reaction \( R_1 \) to approximately 180 mm under the load \( P \). Figure 8.4b shows the cracks that were formed during failure. It is two straight cracks, one in the top of the beam, and one in the bottom of the beam, illustrated as red lines. The crack in the top of the beam runs from the load \( P \) to a point where it merges with the first-formed crack (black line). The crack in the bottom of the beam runs from reaction \( R_1 \) to a point where it merges with
the first-formed crack. Figure 8.4c shows the observed failure mechanism. The yield line consists of three parts that are numbered from 1 to 3: (i) part 1 follows the crack in the bottom of the beam near reaction \( R_1 \) that was formed during failure; part 2 follows a part of the first-formed crack and; (iii) part 3 follows the crack in the top of the beam near load \( P \) that was formed during failure. During the experiments, it has been observed that the distance where the yield line follows the first-formed crack (part 2) is rather small. This part is therefore neglected in the failure mechanism that is used for establishing an upper bound solution for the shear capacity. Hence, the considered failure mechanism has a diagonal yield line that runs from reaction \( R_1 \) to load \( P \), see Figure 8.5. Bræstrup (1979) has derived a model for the shear capacity of beams without shear reinforcement based of the same failure mechanism (see also (Nielsen and Hoang, 2010)). Since the model was developed for beams without ASR-damages, the concrete strength was assumed isotropic. Consequently, the model must be adapted for the use for ASR-damaged concrete by considering the anisotropic compressive strength.

In Chapter 5 it is found that:

- \( f_{\perp} \) is highly influenced by closure of the ASR-induced macro-cracks.
- \( f_{\parallel} \) is influenced by the ASR-induced micro-cracks and reduced strength of the cement paste. Furthermore, it behaves like a downscale of sound concrete.

It can therefore be concluded that \( f_{\perp} \) is only representative for load scenarios where compressive stresses are applied perpendicular to the horizontal ASR-induced macro-cracks. In contrast, it can be concluded that \( f_{\parallel} \) represents a general compressive strength of the ASR-damaged concrete for load scenarios where the macro-cracks play a minor role. In the considered failure mechanism, the shear is transferred as sliding in a diagonal yield line, i.e. the sliding occurs independent of the horizontal ASR-induced cracks. Consequently, \( f_{\parallel} \) is adopted in the solution proposed by Bræstrup (1979):

\[
V_R = \begin{cases} 
\frac{1}{2} f_{\parallel} \nu bh \left( \sqrt{1 + \left( \frac{a}{h} \right)^2} - \frac{a}{h} \right), & \Phi \geq \frac{1}{2} \nu \\
\frac{1}{2} f_{\parallel} \nu bh \left( \frac{a}{h} \right)^2 + 4 \frac{\Phi}{\nu} \left( 1 - \frac{\Phi}{\nu} \right) - \frac{a}{h}, & \Phi \leq \frac{1}{2} \nu 
\end{cases}
\] (8.15)

where \( \nu \) is the effectiveness factor, \( b \) and \( h \) are the average width and the height of the beam, respectively, \( a \) is the shear span while \( \Phi \) is the mechanical reinforcement degree, see Equation 8.16.

\[
\Phi = \frac{f_y A_s}{f_{\parallel} bh} \] (8.16)

According to Nielsen and Hoang (2010) the effectiveness factor for beams without shear reinforcement, \( \nu \), can be found as:

\[
\nu = \frac{0.88}{\sqrt{f_c}} \left( 1 + \frac{1}{\sqrt{h}} \right) (1 + 26 \rho) \neq 1.0 \] (8.17)
where $\rho$ is the reinforcement degree ($\rho = A_s/bh$) which includes both the reinforcement in the top and the bottom of the beam, $f_c$ is in MPa and $h$ is in meters. When use in context of Equation 8.15, $f_c$ is replaced by $f_{c||}$.

\[ \rho = \frac{A_s}{bh} \]

(a) First crack: inclined crack in the bottom of the beam.

(b) Cracks formed during failure (red).

(c) Failure mechanism.

Figure 8.4: Schematic illustration of the observed cracks in the simply supported ASR-damaged beams.
8.2.1 Discussion of proposed model

8.2.1.1 Limitations

The proposed model is based on observations from the experiments, namely that a crack with a low inclination in the bottom of the beam (black line in Figure 8.4a) was formed instead of flexural-shear cracks, which are observed for similar beams without ASR-damages. Despite the fact that the failure of the simply supported beams from Lindenborg was governed by the moment capacity, the same crack with low inclination was formed in all beams ($a_l/h = 2.3-3.3$). Since the proposed model does not include a criterion for formation of the crack with low inclination, it is not possible to determine when the failure occurs through this crack, and when the failure occurs through a flexural-shear crack.

In the proposed model, the length of part 2 in the failure mechanism (see Figure 8.4c) is neglected based on observations from the Gammelrand experiments. Since the failure of the beams from Lindenborg were governed by the moment capacity, these experiments cannot confirm or deny the assumed failure mechanism for shear. One could imagine that the length of part 2 will increase for increasing slenderness ($a/h$). In part 2 of the failure mechanism, the yield line follows and existing crack with a considerable crack width. Since it is well known that the aggregate interlock is decreasing for increasing crack width (see Section 2.2), the dissipation for part 2 may be reduced as well. This is for example seen in the Crack Sliding Model where the dissipation is reduced by an additional effectiveness factor of $\nu_s = 0.5$ if the yield line follows an existing flexural-shear crack (Zhang and Nielsen, 1997). This effect is not included in the proposed model.

8.2.1.2 Influencing parameters

From Equation 8.15 it appears that the shear capacity according to the proposed model depends on the beam dimensions ($b$ and $h$), the slenderness ($a/h$), the reinforcement ($f_y$ and $A_s$) and the concrete compressive strength parallel to the ASR-induced cracks ($f_{c\|}$). It is remarked that the prestressing is not included in the proposed model. At first glance, this seems to be in contrast to what was found in the literature where the
prestressing was given as explanation for surprisingly high shear capacity that was found experimentally (not 2D-restrained beams), see Section 2.5.

8.3 Continuous beams

In this section, a model for the shear capacity of continuous ASR-damaged beams is established. The model is based on observations of crack formation and crack shape made during the Lindenborg experiments, see Section 7.2.1 and 7.2.4. A representative example of crack formation and shape of the critical shear crack in continuous ASR-damaged beams is shown in Figure 8.6 (Beam 5.1 from Lindenborg). The first load stage corresponds to a shear force in the main shear zone of $V_2 = 65.6 \text{kN}$, whilst the second load stage is when the failure occurs ($V_2 = 81.7 \text{kN}$). From the crack pattern for the first load stage (Figure 8.6a), it appears that the diagonal crack is formed between the two layers of reinforcement, and it is accompanied by vertical cracks in the midspan (marked with arrows in the figure). From the crack pattern for the failure load stage (Figure 8.6a), it appears that the crack width of the diagonal crack is rather larger and flat cracks are formed in the top of the beam near load $P_2$ and in the bottom of the beam near reaction $R_1$. After the formation of the coarse diagonal crack, the beam finds a new way to carry the applied moment as well as the applied shear force where the diagonal crack does not contribute. Consequently, the failure mechanism is changed, either will the cracking mechanism that causes the formation of the diagonal crack develop further into an actual flexural collapse, or the load can be increased until a shear collapse occurs. The established model will therefore consist of two criteria: (i) a criterion for the formation of the diagonal crack (without collapse) and; (ii) a criterion for collapse when the diagonal crack has been formed. This criterion considers both a shear collapse and a flexural collapse. The criterion for formation of the diagonal crack as well as the criterion for collapse will be established by means of the upper bound method.
8.3.1 Criterion for formation of the diagonal crack

The considered cracking mechanism is illustrated in Figure 8.7. The mechanism consists of a diagonal crack between $R_1$ and $P_2$, vertical cracks in the shear span (Crack I and III) and vertical cracks near $P_2$ (Crack II) and $R_1$ (Crack IIII). These cracks enable a rotational mechanism consisting of 4 rotating rigid bodies called rotation parts. Part numbers are shown in circles in the figure, 1⃝-4⃝. The rotations of the four parts are named $\theta_1$ to $\theta_4$. The adjacent parts are connected via a rotation point in each vertical crack. The rotation points are shown as crosses in the figure and are named after the affiliated crack, i.e. $R_I$ to $R_{III}$. The rotations and displacements are all derived relative to a Cartesian ($x, y$)-coordinate system with origin in $R_{III}$. Therefore, the displacements (horizontally and vertically) of $R_{III}$ are zero. Furthermore, it is assumed that $R_{II}$ is fixed against vertical displacements. In this way, the relative displacements of the loads $P_1$ and $P_2$ and reaction $R_2$ can be described. The displacement vectors for $R_1$ and $R_{III}$ ($\delta_1$ and $\delta_{III}$), $P_1$ ($\delta_{P_1}$), $P_2$ ($\delta_{P_2}$) and $R_2$ ($\delta_{R_2}$) are shown in the upper figure.

It is remarked that the diagonal crack in the proposed mechanism has the same nature as the diagonal crack observed in the experiments; the largest crack width is in the midspan.
To establish an upper bound solution, all displacement vectors and rotations are needed. Therefore, this section starts with a derivation of displacement vectors and rotations. Subsequently, the upper bound solutions are set up and derived.

### 8.3.1.1 Displacement vectors and rotations

Figure 8.7 shows free body diagrams of the four rotation parts. On the diagrams, the positive directions of the displacement vectors and rotations of the rotation parts (θ₁ to θ₄) are shown. Since parts 1 and 4 rotate about the fixed point (R₃), the shown displacement vectors are absolute and global, while the displacement vectors for parts 2 and 3 are shown relative to R₃ and R₁, respectively. The relative displacement vectors are indexed with the name of the rotation point where the displacement occurs, and the name of the rotation point to which the displacement is relative to, e.g. δ₁₂ is the displacement vector of R₁ relative to R₂.
Figure 8.8: Free body diagrams for the four rotation parts.
In the following derivation, the horizontal and vertical projections of the displacement vectors are indexed with "x" and "y", respectively.

The displacement in $R_I$ can be expressed as (see Figure 8.8 A):

$$\delta_{l,y} = \theta_1(x_0 - x_2) \quad (8.18)$$

$$\delta_{l,x} = \theta_1(h - h_{III} - h_I + y_I) \quad (8.19)$$

The vertical displacement in $R_I$ relative to $R_{II}$ (see Figure 8.8 B) can be found as:

$$\delta_{l,II,y} = \theta_3 x_2 \quad (8.20)$$

Since $R_{II}$ is fixed against vertical displacements, the vertical displacement in $R_I$ relative to $R_{II}$ ($\delta_{l,II,y}$) corresponds to the global vertical displacement in $R_I$ ($\delta_{l,y}$). Thereby can the angle of rotation $\theta_3$ can be expressed as:

$$\theta_3 = \delta_{l,y}/x_2 \quad (8.21)$$

The horizontal displacement of $R_I$ relative to $R_{II}$ is (see Figure 8.8 B):

$$\delta_{l,II,x} = \theta_3(h_I - y_I - h_{II}) \quad (8.22)$$

The difference between the horizontal displacement in $R_I$ relative to $R_{III}$ (global displacement) and the horizontal displacement in $R_I$ relative to $R_{II}$ corresponds to the global horizontal displacement of $R_{II}$ (see Figure 8.8 A and B):

$$\delta_{II,x} = \delta_{l,x} - \delta_{l,II,x} \quad (8.23)$$

Substituting Equation 8.19 and 8.22 in Equation 8.23 leads to:

$$\delta_{II,x} = \theta_1(h - h_{III} - h_I + y_I) - \theta_3(h_I - y_I - h_{II}) \quad (8.24)$$

which takes the following form when inserting the right hand side of Equation 8.21:

$$\delta_{II,x} = \theta_1(h - h_{III} - h_I + y_I) - \frac{\delta_{l,y}}{x_2}(h_I - y_I - h_{II}) \quad (8.25)$$

By inserting Equation 8.18, Equation 8.25 becomes:

$$\delta_{II,x} = \theta_1(h - h_{III} - h_I + y_I) - \frac{\theta_1(x_0 - x_2)}{x_2}(h_I - y_I - h_{III}) \quad (8.26)$$

$$\delta_{II,x} = \theta_1 \left( h - h_{III} - h_{II} + \frac{x_0}{x_2}(y_I + h_{II} - h_I) \right) \quad (8.27)$$
The height of crack I can be found based on simple geometry (see Figure 8.8):

\[ h_I = h_{II} + \frac{h - h_{III} - h_{II}}{x_0} x_2 \]  

(8.28)

which when inserting into Equation 8.27 gives:

\[ \delta_{II,x} = \theta_1 \left( h - h_{III} - h_{II} + \frac{x_0}{x_2} (y_I + h_{II} - h_{III} - h_{II}) \right) \]  

(8.29)

which can be simplified to:

\[ \delta_{II,x} = \theta_1 \left( h - h_{III} - h_{II} - h + h_{III} + h_{II} + \frac{x_0}{x_2} (y_I + h_{II} - h_{III}) \right) \]  

(8.30)

and further simplified to:

\[ \delta_{II,x} = \theta_1 \frac{x_0}{x_2} y_I \]  

(8.31)

The angle of rotation \( \theta_4 \) can by described as (see Figure 8.8 C):

\[ \theta_4 = \frac{\delta_{III,y}}{x_1} \]  

(8.32)

The vertical displacement of \( R_{III} \) relative to \( R_{II} \) may be found as (see Figure 8.8 D):

\[ \delta_{III,y} = \theta_2 (x_0 - x_1) \]  

(8.33)

Since \( R_{II} \) is fixed against vertical displacements, the vertical displacement in \( R_{III} \) relative to \( R_{II} (\delta_{III,y}) \) corresponds to the global vertical displacement in \( R_{III} (\delta_{III,y}) \):

\[ \delta_{III,y} = \delta_{III,y} \]  

(8.34)

By inserting Equation 8.33, the vertical displacement in \( R_{III} \) takes the following form:

\[ \delta_{III,y} = \theta_2 (x_0 - x_1) \]  

(8.35)

By inserting the new expression for the vertical displacement in \( R_{III} \) (Equation 8.35) in Equation 8.32, the angle of rotation \( \theta_4 \) can be expressed by \( \theta_2 \):

\[ \theta_4 = \frac{\theta_2 (x_0 - x_1)}{x_1} \]  

(8.36)

The horizontal displacement of \( R_{III} \) may be found as (see Figure 8.8 C):

\[ \delta_{III,x} = \theta_4 (h_{III} - h_{III} - y_{III}) \]  

(8.37)

Similarly, the horizontal displacement of \( R_{III} \) relative to \( R_{II} \) may be found as (see Figure 8.8 D):

\[ \delta_{III,II,x} = \theta_2 (h - h_{II} - h_{III} + y_{III}) \]  

(8.38)
The difference between the horizontal displacement in R_{III} relative to R_{IIII} (global displacement) and the horizontal displacement in R_{III} relative to R_{II} corresponds to the global horizontal displacement of R_{II} (see Figure 8.8 C and D):

\[ \delta_{II,x} = \delta_{III,x} - \delta_{III,x} \]  

(8.39)

By inserting Equation 8.37 and 8.38 the horizontal displacement in R_{II} becomes:

\[ \delta_{II,x} = \theta_2(h - h_{II} - h_{III} + y_{III}) - \theta_4(h_{III} - h_{IIII} - y_{III}) \]  

(8.40)

By substituting \( \theta_4 \) expressed by \( \theta_2 \) (Equation 8.36), the expression for the horizontal displacement in R_{II} can be found as:

\[ \delta_{II,x} = \theta_2(h - h_{II} - h_{III} + y_{III}) - \frac{\theta_2(x_0 - x_1)}{x_1}(h_{III} - h_{IIII} - y_{III}) \]  

(8.41)

which can be simplified to:

\[ \delta_{II,x} = \theta_2 \left( h - h_{II} - h_{III} + x_0 \left( h_{III} + y_{III} - h_{IIII} \right) \right) \]  

(8.42)

The height of Crack III can be found based on simple geometry (see Figure 8.8):

\[ h_{III} = h_{IIII} + \frac{h - h_{IIII} - h_{II}}{x_0} \]  

(8.43)

When inserting this into Equation 8.43, the expression for the horizontal displacement in R_{II} may be simplified:

\[ \delta_{II,x} = \theta_2 \left( h - h_{II} - h_{III} + x_0 \left( h_{III} + y_{III} - h_{IIII} - \frac{h - h_{IIII} - h_{II}}{x_0} \right) \right) \]  

(8.44)

which can be simplified to:

\[ \delta_{II,x} = \theta_2 \left( h - h_{II} - h_{III} - h + h_{III} + h_{II} + \frac{x_0}{x_1} \left( h_{III} + y_{III} - h_{III} \right) \right) \]  

(8.45)

and further simplified to:

\[ \delta_{II,x} = \theta_2 \frac{x_0}{x_1} y_{III} \]  

(8.46)

When inserting Equation 8.31, the angle of rotation \( \theta_2 \) can be expressed by \( \theta_1 \):

\[ \frac{\theta_1 x_0}{x_2} y_{II} = \frac{\theta_0 x_0}{x_1} y_{III} \]  

(8.47)

\[ \theta_2 = \frac{\theta_1 x_1}{x_2} \frac{y_{II}}{y_{III}} \]  

(8.48)

To simplify the expression, \( c_0 \) is introduced:
\[ c_\theta = \frac{x_1 y_1}{x_2 y_{III}} \]  

(8.49)

Thereby, the angle of rotation \( \theta_2 \) can be expressed as:

\[ \theta_2 = \theta_1 c_\theta \]  

(8.50)

The angle of rotation \( \theta_3 \) can be expressed by \( \theta_1 \) by substituting Equation 8.18 into Equation 8.21:

\[ \theta_3 = \theta_1 \frac{x_0 - x_2}{x_2} = \frac{\theta_1 x_0}{x_2} - \theta_1 \]  

(8.51)

The angle of rotation \( \theta_4 \) can be expressed by \( \theta_2 \) by substituting Equation 8.35 into Equation 8.32:

\[ \theta_4 = \theta_2 \frac{x_0 - x_1}{x_1} = \frac{\theta_2 x_0}{x_1} - \theta_2 \]  

(8.52)

By inserting \( \theta_2 \) expressed by \( \theta_1 \) (Equation 8.50), the angle of rotation \( \theta_4 \) can be expressed by \( \theta_1 \) as well:

\[ \theta_4 = \theta_1 \left( \frac{c_\theta x_0}{x_1} - c_\theta \right) = \frac{\theta_1 c_\theta x_0}{x_1} - \theta_1 c_\theta \]  

(8.53)

Thus, all rotations are expressed by \( \theta_1 \). Similarly, the displacement vectors for the loading- and reaction points can be expressed by \( \theta_1 \).

The vertical displacement of load \( P_1 \) (see Figure 8.8 A) can be found as:

\[ \delta_{P1,y} = \theta_1 L_1 \]  

(8.54)

The horizontal displacement of load \( P_2 \), measured in the rotation point, \( R_{II} \) (see Figure 8.7) can be expressed as follows when Equation 8.46, 8.50 and 8.49 are substituted.

\[ \delta_{P2,x} = \delta_{II,x} = \theta_2 \frac{x_0}{x_1} - y_{III} = \theta_1 c_\theta \frac{x_0}{y_{III}} = \theta_1 \frac{x_1}{y_{III}} x_0 = \theta_1 \frac{y_1}{x_2} \]  

(8.55)

The vertical displacement of reaction \( R_2 \) becomes as follows when Equation 8.50 is substituted (see Figure 8.8 D):

\[ \delta_{R2,y} = \theta_2 L_2 = \theta_1 c_\theta L_2 \]  

(8.56)

The displacement vectors in the diagonal crack are derived and expressed by \( \theta_1 \) as well. For convenience in the further derivation, the inclination (relative to the \( x \)-axis) of the diagonal crack is expressed by:

\[ \alpha_{dia} = \arctan \frac{h - h_{III} - y_{III}}{x_0} \]  

(8.57)

The relative displacement vector between parts \( 1 \) and \( 4 \), \( u_{1/4} \) (see Figure 8.14) is easily found since both parts rotate around \( R_{III} \). Since the common rotation point (\( R_{III} \)) is on the diagonal crack, \( u_{1/4} \) will be perpendicular to the diagonal crack. When Equation 8.53 is inserted, \( u_{1/4} \) can be expressed by \( \theta_1 \):
The relative displacement vector between parts 2 and 3, \( u_{2/3} \), can similarly be found as they both rotate around \( R_{II} \), and by employing Equation 8.50 and 8.51 it can be expressed by \( \theta_1 \):

\[
u_{2/3} = (\theta_2 + \theta_3) \frac{x_0 - x}{\cos \alpha_{\text{dia}}} = \theta_1 \left( \frac{c_\theta x_0}{x_1} - c_\theta \right) \frac{x}{\cos \alpha_{\text{dia}}} \quad (8.59)
\]

The relative displacement vector for the remaining part of the diagonal crack depends on \( x_1 \) and \( x_2 \); if \( x_0 - x_2 < x_1 \) parts 3 and 4 are overlapping and if \( x_0 - x_2 > x_1 \) it is parts 1 and 2 that are overlapping, see Figure 8.7. Consequently, the two scenarios are considered individually.

### Scenario 1

In this scenario, \( x_0 - x_2 \) is smaller than \( x_1 \) which means that it is the relative displacement vector between parts 3 and 4, \( u_{3/4} \) that is considered. Figure 8.9 shows the two parts. Part 4 rotates around \( R_{III} \), while part 3 rotates around \( R_{II} \) and at the same time moves \( \delta_{P2} \) to the left. The relative displacement vector is divided into contributions from the two parts, and is expressed by the \( x \)- and \( y \)-component. The horizontal contribution from part 3 appears as follows when the expression for \( \delta_{P2} \) (Equation 8.51) and \( \theta_3 \) (Equation 8.55) are substituted:

\[
u_{3,x} = \delta_{P2} + \theta_3 (x_0 - x) \tan \alpha_{\text{dia}} = \theta_1 \left( \frac{y_1}{x_2} x_0 + \frac{x_0 - x_2}{x_2} (x_0 - x) \tan \alpha_{\text{dia}} \right) \quad (8.60)
\]

When \( \theta_3 \) expressed by \( \theta_1 \) (Equation 8.51) is inserted, the vertical contribution from part 3 takes the following form:

\[
u_{3,y} = \theta_3 (x_0 - x) = \theta_1 \frac{x_0 - x_2}{x_2} (x_0 - x) \quad (8.61)
\]

The horizontal and vertical contribution from part 4 can similarly be expressed by \( \theta_1 \) when Equation 8.53 is employed:

\[
u_{4,x} = \theta_4 x \tan \alpha_{\text{dia}} = \theta_1 \left( \frac{c_\theta x_0}{x_1} - c_\theta \right) x \tan \alpha_{\text{dia}} \quad (8.62)
\]

\[
u_{4,y} = \theta_4 x = \theta_1 \left( \frac{c_\theta x_0}{x_1} - c_\theta \right) x \quad (8.63)
\]

The length of the relative displacement vector between parts 3 and 4, \( u_{3/4} \), may be found as (see Figure 8.10a):

\[
|u_{3/4}| = \sqrt{(u_{3,x} + u_{4,x})^2 + (u_{3,y} + u_{4,y})^2} \quad (8.64)
\]
Whilst the inclination (relative to the x-axis) of $\mathbf{u}_{3,4}$ may be found as (see Figure 8.10a):

$$\alpha_{3/4} = \arctan \left( \frac{u_{3,y} + u_{1,y}}{u_{3,x} + u_{4,x}} \right)$$  \hfill (8.65)

Thereby can the angle between $\mathbf{u}_{3/4}$ and the diagonal crack be found as (see Figure 8.10a):

$$\alpha_{3/4-dia} = \alpha_{3/4} + \alpha_{dia}$$  \hfill (8.66)

**Figure 8.9:** Parts 3 and 4.

**Scenario 2**

In this scenario, $x_0 - x_2$ is larger than $x_1$, which means that it is the displacement vector between parts 1 and 2, $\mathbf{u}_{1/2}$ that is considered.

Figure 8.11 shows the two parts. Part 1 rotates around $R_{III}$ while part 2 rotates around $R_{II}$ and moves $\delta_{P2}$ to the left as well. The components of the relative displacement
vector between parts 1 and 2 are determined individually. The horizontal and vertical contribution from part 1 may be found as:

\[ u_{1,x} = \theta_1 x \tan \alpha_{\text{dia}} \tag{8.67} \]
\[ u_{1,y} = \theta_1 x \]  

The horizontal contribution from part 2 appears as follows when Equation 8.50 and 8.55 are substituted:

\[ u_{2,x} = \delta P_2 + \theta_2 (x_0 - x) \tan \alpha_{\text{dia}} = \theta_1 \left( \frac{y_I}{x_2} x_0 + c_\theta(x_0 - x) \tan \alpha_{\text{dia}} \right) \tag{8.69} \]

The vertical contribution from part 2 can found as shown below when \( \theta_2 \) expression by \( \theta_1 \) (Equation 8.50) is substituted:

\[ u_{2,y} = \theta_2 (x_0 - x) = \theta_1 c_\theta (x_0 - x) \]  

\[ \theta_1 \]  

\[ \text{Figure 8.11: Parts (1) and (2).} \]

The length of the relative displacement vector, \( |\mathbf{u}_{1/2}| \), may be found as (see Figure 8.10b):

\[ |\mathbf{u}_{1/2}| = \sqrt{(u_{1,x} + u_{2,x})^2 + (u_{1,y} + u_{2,y})^2} \tag{8.71} \]

The inclination (relative to the x-axis) of \( \mathbf{u}_{1/2} \) may be found as (see Figure 8.10b):

\[ \alpha_{1/2} = \arctan \left( \frac{u_{1,y} + u_{2,y}}{u_{1,x} + u_{2,x}} \right) \tag{8.72} \]

The angle between \( \mathbf{u}_{1/2} \) and the diagonal crack can be found as (see Figure 8.10b):

\[ \alpha_{1/2,\text{dia}} = \alpha_{1,2} + \alpha_{\text{dia}} \]  

The relative rotation, i.e. the rate of crack opening, in the vertical cracks can be found as shown below, when the relevant rotations are expressed by \( \theta_1 \) (see Figure 8.8):

The relative rotation in Crack I:
\[ \alpha_I = \theta_1 + \theta_3 = \theta_1 + \frac{\theta_1 x_0}{x_2} - \theta_1 = \frac{\theta_1 x_0}{x_2} \]  

(8.74)

The relative rotation in Crack II:

\[ \alpha_{II} = \theta_2 + \theta_3 = \theta_1 c_\theta + \frac{\theta_1 x_0}{x_2} - \theta_1 = \theta_1 \left( c_\theta + \frac{x_0}{x_2} - 1 \right) \]  

(8.75)

The relative rotation in Crack III:

\[ \alpha_{III} = \theta_2 + \theta_4 = \theta_2 + \theta_2 \frac{x_0}{x_1} - \theta_2 = \theta_2 \frac{x_0}{x_1} = \theta_1 \frac{c_\theta x_0}{x_1} \]  

(8.76)

The relative rotation in Crack IIII:

\[ \alpha_{III} = \theta_1 + \theta_4 = \theta_1 + \theta_1 \frac{c_\theta x_0}{x_1} - \theta_1 c_\theta = \theta_1 \left( 1 + \frac{c_\theta x_0}{x_1} - c_\theta \right) \]  

(8.77)

8.3.1.2 Upper bound solution for formation of the diagonal crack

In this section, an upper bound solution for the formation of the diagonal crack is established based on the presented cracking mechanism (see Figure 8.7). The section consists of four parts where: (i) the internal work is determined; (ii) the external work is determined; (iii) a solution is set up by the work equation and; (iv) the solution is optimized.

8.3.1.2.1 Internal work

The internal work for the cracking mechanism has contribution from five cracks: the four vertical cracks (Crack I to IIII) and the diagonal crack. The internal work for each crack is derived in the following. Due to similarities, the internal work for Crack I is determined together with the internal work for crack III, and the internal work for Crack II and IIII is determined together.

Internal work for Crack I and III

Figure 8.12 shows the assumed normal stress distribution in Crack I and III. Due to the direction of the stresses, the effective plastic strength parallel to the ASR-induced cracks is adopted, i.e. \( f_{c||,ef} \) is adopted for the compression zones and \( f_{t||,ef} \) for the tensile zones. Since it is the crack formation that is considered (not the load-carrying capacity), the ASR-induced prestressing, \( \sigma_{ASR} \), is adopted as stress level for the reinforcement instead of the yield stress. It is remarked that a local vertical axis, \( y^* \), is applied for later integration.
The effective concrete compressive strength can be found as:

\[ f_{c,\text{eff}} = \nu_c f_c \]

(8.78)

where \( \nu_c \) is effectiveness factor, which in this context must be taken as the one used for bending problems for normally reinforced beams. \( \nu_c \) may according to Nielsen and Hoang (2010) be taken as:

\[ \nu_c = 0.98 - \frac{f_c}{500}, f_c \text{ in [MPa]} \]

(8.79)

When used for here, \( f_c \) must be replaced by \( f_{c,\text{eff}} \).

The effective concrete tensile can be found as:

\[ f_{t,\text{eff}} = \nu_t f_{t,\text{eff}} s(h_{\text{crack}}) \]

(8.80)

where \( \nu_t \) is effectiveness factor for the tensile strength and \( s(h_{\text{crack}}) \) is the size effect factor (Weibull effect). Zhang and Nielsen (1997) investigated the effectiveness factor for the tensile strength in beams subjected to pure bending. They compared the moment capacity of rectangular unreinforced beams, determined by experiments, with a theoretical moment capacity where the tensile stresses were plastic distributed and the depth of the compression zone was neglected. They found an effectiveness factor of \( \nu_t = 0.6 \). Furthermore, they recommend to employ the following size effect (see also Nielsen and Hoang (2010)):

\[ s(h) = \left( \frac{h_{\text{crack}}}{0.1} \right)^{-0.3} \leq 1.0, h_{\text{crack}} \text{ in meters} \]

(8.81)

where \( h_{\text{crack}} \) is the height of the tension zone, i.e. in their case the beam height. For the vertical crack in the mechanism considered in this section, the height of the tension zone is employed as \( h_{\text{crack}} \), i.e. \( h_{\text{III}} - y_{\text{III}} \) in Figure 8.12.

The ASR-induced prestressing in the reinforcement may be found as:

\[ \sigma_{\text{ASR}} = \varepsilon_{\text{ASR}} E_s \]

(8.82)
where $\varepsilon_{\text{ASR}}$ is the ASR-induced prestressing (strain) and $E_s$ is Young’s modulus of the reinforcement steel.

The internal work for Crack I can then be found as:

$$W_{i,1} = \int_0^{y_1} f_{c||,ef} b |u| dy^* + \int_{y_1}^{h_1} f_{t||,ef} b |u| dy^* + \sigma_{\text{ASR}} A_s' |u| (y^* = d_1) \quad (8.83)$$

where $b$ is the width of the beam, $y_1$ is the depth of the compression zone in Crack I (position of rotation point $R_{1I}$), $h_1$ if the height of Crack I, $d_1$ is the effective depth of Crack I, $A_s'$ is the cross sectional area of the reinforcement in the top of the beam and $|u|$ is the length of the displacement vector, which is determined as:

$$|u| = \begin{cases} \alpha_1 (y_1 - y^*) , & y^* \leq y_1 \\ \alpha_1 (y^* - y_1) , & y^* \geq y_1 \end{cases} \quad (8.84)$$

By inserting the right hand side of Equation 8.74, $|u|$ can be by $\theta_1$:

$$|u| = \frac{\theta_1 x_0}{x_2} \begin{cases} y_1 - y^* , & y^* \leq y_1 \\ y^* - y_1 , & y^* \geq y_1 \end{cases} \quad (8.85)$$

Thereby, the internal work for Crack I may be expressed by $\theta_1$ as:

$$W_{i,1} = \theta_1 \frac{x_0}{x_2} \left( \frac{1}{2} f_{c||,ef} b y_1^2 + \frac{1}{2} f_{t||,ef} b (h_1 - y_1)^2 + \sigma_{\text{ASR}} A_s' |d_1 - y_1| \right) \quad (8.86)$$

In a similar way the internal work for Crack III can be derived:

$$W_{i,III} = \theta_1 \frac{c_0 x_0}{x_1} \left( \frac{1}{2} f_{c||,ef} b y_{III}^2 + \frac{1}{2} f_{t||,ef} b (h_{III} - y_{III})^2 + \sigma_{\text{ASR}} A_s |d_{III} - y_{III}| \right) \quad (8.87)$$

where $y_{III}$, $h_{III}$ and $d_{III}$ are shown on Figure 8.12b whilst $A_s$ is the cross sectional area of the reinforcement in the bottom of the beam.

**Internal work for Crack II and IIII**

Figure 8.13 shows the assumed normal stress distribution in Crack II and IIII. As for Crack I and IIII, $f_{c||,ef}$ is adopted for the compression zone and the effectiveness factor for bending (Equation 8.79) is adopted. The rotation points, $R_{II}$ and $R_{III}$, are placed in the centerline of the reinforcement. The reinforcement can therefore only contribute here by its dowel action. Since dowel action requires rather large deformations and since it is the crack formation that is under consideration, where the deformations are small, the dowel effect in the reinforcement is neglected.
The internal work for Crack II can be found as:

$$W_{i,II} = \int_{0}^{h_{II}} f_{c|\sigma} b |u| dy^*$$  \hspace{1cm} (8.88)

where $h_{II}$ is the height of Crack II and $|u|$ is the length of the displacement vector which can be determined as follows when Equation 8.75 is substituted:

$$|u| = \alpha_{II} y^* = c_{\theta} + \frac{x_0}{x_2} - 1 \hspace{1cm} (8.89)$$

Thereby, the internal work for Crack II may be expressed in terms of $\theta_1$ as:

$$W_{i,II} = \frac{1}{2} \theta_1 \left( c_{\theta} + \frac{x_0}{x_2} - 1 \right) h_{II}^2 f_{c|\sigma} b$$  \hspace{1cm} (8.90)

The internal work for Crack IIII be derived similar way:

$$W_{i,III} = \frac{1}{2} \theta_1 \left( 1 + \frac{c_{\theta} x_0}{x_1} - c_{\theta} \right) h_{III}^2 f_{c|\sigma} b$$  \hspace{1cm} (8.91)

where $h_{III}$ is the height of Crack III.

**Internal work for the diagonal crack**

Figure 8.14 shows the considered mechanism and the relative displacement vectors (fields) in the diagonal crack for two scenarios: (i) $x_0 - x_2 \leq x_1$ and (ii) $x_0 - x_2 \geq x_1$. As described earlier, the relative displacement between parts $1_\circ$ and $4_\circ$, $u_{1/4}$, and between parts $2_\circ$ and $3_\circ$, $u_{2/3}$, are perpendicular to the diagonal crack while the displacement vectors on the remaining part ($u_{3/4}$ for Scenario 1 and $u_{1/2}$ for Scenario 2) are not perpendicular to the diagonal crack. The internal work for the three segments of the diagonal crack is therefore determined individually. To distinguish between the contribution to the internal work for Scenario 1 and Scenario 2, respectively, "$sc1"$ and "$sc2"$ are added to the index of their name.
Due to the low inclination of the diagonal crack it tends to follow the ASR-induced macro cracks. Conservatively, the contribution to the internal work from the diagonal crack is based on the concrete strength measured perpendicular to the ASR-induced macro cracks, $f_{\perp\perp}$ and $f_{\perp\perp}$, which were shown to be highly affected by the macro cracks, see Chapter 5 and 6. The effective tensile strength, $f_{\perp\perp,ef}$, can then be found as:

$$f_{\perp\perp,ef} = \nu_t f_{\perp\perp} s(h_{\text{crack}})$$  \hspace{1cm} (8.92)$$

where $\nu_t = 0.6$, as for the vertical cracks and the size effect ($s(h_{\text{crack}})$) can be determined by Equation 8.81 where $h_{\text{crack}}$ is the length of the diagonal crack. In the second segment of the diagonal crack where the relative displacement vectors may not be perpendicular to the diagonal crack, sliding might occur. In establishing the model for the shear capacity for simply supported beams with ASR-damages, an effectiveness factor for the compressive strength for beams without shear reinforcement and without ASR-damages was adopted (Equation 8.17). Since this effectiveness factor is based on a total shear failure of a beam and it thereby includes effects such as dowel actions, it is not
representative for the considered cracking mechanism. Therefore, the effectiveness factor for shear, which only accounts for softening is adopted to determined the effective compressive strength in the crack, see (Nielsen and Hoang, 2010):

\[ f_{c,\perp,ef} = f_{c,\perp} \nu_{c,shear} = f_{c,\perp} \left(0.7 - \frac{f_{c,\perp,ef}}{200}\right) \]  

(8.93)

Later, it will however be shown that the contribution from the compressive strength in segment 2 of the diagonal crack is zero (see Section 8.3.1.2.4). The effectiveness factor for the compressive strength is thus irrelevant and is therefore not investigated further. Since the relative displacement vectors are expressed according to the \((x, y)\)-coordinate system and the diagonal crack is inclined, the internal work is found by means of line integrals where the coordinates of the diagonal crack are expressed by a simple parametrisation.

**Parametrisation**

The coordinates of the diagonal crack are parametrized by \(t\) as follows:

\[ x = t \]

\[ y = t \tan \alpha_{dia} \]  

(8.95)

For the use in the line integrals, the derivatives of the coordinates are determined:

\[ \frac{dx}{dt} = 1 \]  

\[ \frac{dy}{dt} = \tan \alpha_{dia} \]  

(8.97)

Internal work for the diagonal crack - Scenario 1

The diagonal crack is divided into segments: (i) \(x < x_0 - x_2\); (ii) \(x_0 - x_2 \leq x < x_1\) and; (iii) \(x \geq x_1\). The internal work for the three segments is determined individually. For the first segment of the diagonal crack \((x < x_0 - x_2)\), the length of the relative displacement vectors may be expressed by the parameter \(t\) by means of Equation 8.94 and 8.58:

\[ |u_{1/4}| = \theta_1 \left(1 + \frac{c_0 x_0}{x_1} - c_\theta \right) \frac{t}{\cos \alpha_{dia}} \]  

(8.98)

The internal work for the first segment can thereby be found as:

\[ W_{i,\text{dia},1/4,sc1} = b f_{t,\perp,ef} \int_{0}^{x_0-x_2} |u_{1/4}| \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt \]  

(8.99)

\[ W_{i,\text{dia},1/4,sc1} = \frac{1}{2} b f_{t,\perp,ef} \theta_1 \left(1 + \frac{c_0 x_0}{x_1} - c_\theta \right) \frac{(x_0 - x_2)^2}{\cos \alpha_{dia}^2} \]  

(8.100)
For the third segment of the diagonal crack \((x > x_1)\), the length of the relative displacement vectors may be expressed by the parameter \(t\) by means of Equation 8.94 and 8.59:

\[
\mathbf{u}_{2/3} = \theta_1 \left( c_0 + \frac{x_0 - x_2}{x_2} \right) \frac{x_0 - t}{\cos \alpha_{\text{dia}}} \tag{8.101}
\]

By means of a line integral the internal work can be found as:

\[
W_{i,\text{dia},2/3,sc1} = \frac{1}{2} bf_{\perp,ef} \theta_1 \left( c_0 + \frac{x_0 - x_2}{x_2} \right) \frac{(x_0 - x_1)^2}{\cos \alpha_{\text{dia}}} \tag{8.102}
\]

Since the relative displacement vectors in the second segment of the diagonal crack \((x_0 - x_2 \leq x \leq x_1)\) may not be perpendicular to the diagonal crack, the internal work cannot be determined by the concrete tensile strength, exclusively, as for the first and third segment.

By employing Equation 8.9, the internal work for the second segment can be found as:

\[
W_{i,\text{dia},3/4,sc1} = \int_{x_0 - x_2}^{x_2} \frac{1}{2} bf_{\perp,ef} (l - m \sin(\alpha_{3/4,t})) |\mathbf{u}_{3/4,t}| \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} \, dt, \quad \varphi \leq \alpha_{3/4} \leq \pi - \varphi \tag{8.103}
\]

where \(\alpha_{3/4,t}\) and \(|\mathbf{u}_{3/4,t}|\) are the inclination and the length of the relative displacement vector, respectively, parametrized with the parameter \(t\) while \(l\) and \(m\) can be found as:

\[
l = 1 - 2 \frac{f_{\perp,ef}}{f_{c,\perp,ef}} \frac{\sin \varphi}{1 - \sin \varphi} \tag{8.104}
\]

\[
m = 1 - 2 \frac{f_{\perp,ef}}{f_{c,\perp,ef}} \frac{1}{1 - \sin \varphi} \tag{8.105}
\]

where \(\varphi\) is the angle of friction for the concrete, which is a material parameter that can be tested. To the author’s knowledge, no studies regarding \(\varphi\) for ASR-damaged 2D-restrained slabs have been published. Consequently, it is assumed that \(\varphi\) is unchanged, i.e. \(\tan \varphi = 0.75\).

Due to the complexity of the expression for \(W_{i,\text{dia},3/4,sc1}\), it is solved numerically, where the second segment of the diagonal crack is discretized into 100 subsegments.

Internal work for the diagonal crack - Scenario 2

The internal work in Scenario 2 can be determined by the same procedure as for Scenario 1. Hence:

\[
W_{i,\text{dia},1/4,sc2} = \frac{1}{2} bf_{\perp,ef} \theta_1 \left( 1 + \frac{c_0 x_0}{x_1} - c_0 \right) \frac{x_1^2}{\cos \alpha_{\text{dia}}^2} \tag{8.106}
\]

\[
W_{i,\text{dia},2/3,sc2} = \frac{1}{2} bf_{\perp,ef} \theta_1 \left( c_0 + \frac{x_0 - x_2}{x_2} \right) \frac{x_2^2}{\cos \alpha_{\text{dia}}^2} \tag{8.107}
\]
\[ W_{i,\text{dia,1/2,sc2}} = \int_{x_1}^{x_0-x_2} \frac{1}{2} h f_{c,\perp,\text{et}} (l - m \sin(\alpha_{1/2,t})) |u_{1/2,t}| \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt, \quad \varphi \leq \alpha_{1/2} \leq \pi - \varphi \]  

(8.108)

where \( \alpha_{1/2,t} \) and \( |u_{1/2,t}| \) are the inclination and the length of the relative displacement vector, respectively, parametrized with the parameter \( t \).

As for Scenario 1, the internal work for the second segment \((x_1 \leq x \leq x_0 - x_2)\) is determined numerically.

The total internal work, \( W_1 \), is the sum of the contribution from the vertical cracks and the contribution from the diagonal crack:

\[ W_1 = \sum_{n=1}^{\text{III}} W_{i,n} + W_{i,\text{dia}} \]  

(8.109)

### 8.3.1.2.2 External work

Since \( R_{\text{II}} \) and \( R_{\text{III}} \) are fixed against vertical displacements, only \( P_1 \) and \( R_2 \) contribute to the external work. Since structural calculations are often conducted on the level of cross-sectional forces, the external work is expressed by the sectional moments. \( M_1 \) is the hogging moment at reaction \( R_1 \) and \( M_2 \) is the sagging moment at load \( P_1 \), see Figure 7.9 as well as Figure 8.7 and 8.8.

The external work can be found as:

\[ W_E = M_2 \theta_2 + |M_1| \theta_1 \]  

(8.110)

By introducing the ratio between the moments, \( c_M \), the external work can be found as:

\[ c_M = \frac{|M_1|}{M_2} \]  

(8.111)

\[ W_E = \theta_1 |M_1| \left(1 + \frac{c_\theta}{c_M}\right) \]  

(8.112)

where \( c_\theta \) is defined in Equation 8.49.

### 8.3.1.2.3 Work equation

The moment, \( M_{1,\text{crack}} \), and thereby the set of loads \( P_1 \) and \( P_2 \) that causes the formation of the diagonal crack can be found by equating the internal and external work \((W_1 = W_E)\):

\[ M_{1,\text{crack}}(y_1, y_{\text{III}}, x_1, x_2) = \frac{W_1}{\theta_1 \left(1 + \frac{c_\theta}{c_M}\right)} \]  

(8.113)
where $M_{1,\text{crack}}$ depends on four unknown variables: $y_I$, $y_{III}$, $x_1$ and $x_2$. Since the expression is based on an upper bound solution, the unknown variables can be determined by an optimization (minimization in this case).

### 8.3.1.2.4 Optimization

In the optimization, the material parameters and the geometry for Beam 5.1, 5.2 and 5.3 from Lindenborg are employed, since the tensile strength was only measured for these three (continuous) beams. A numerical minimization of $M_{1,\text{crack}}(y_I, y_{III}, x_1, x_2)$ shows that $y_I \to 0$ or $y_{III} \to 0$, which means that parts 1 and 2 rotate independently, and that the relative displacement vectors in the diagonal crack are perpendicular to the diagonal crack. The equations can consequently be simplified dramatically.

Figure 8.15 shows the two independent cracking mechanisms associated with the independent rotation of parts 1 and 2, called Cracking mechanism 1 and Cracking mechanism 2, respectively. In Cracking mechanism 1, it is a rotation of parts 1 and 3 that leads to the formation of the diagonal crack whilst it is a rotation of parts 2 and 4 that causes the formation of the diagonal crack in Cracking mechanism 2.

The internal work for Cracking mechanism 1 has four contributors: Crack I, Crack II, Crack III and the diagonal crack. The internal work for Crack I can be found as:
\[ W_{i,1} = \theta_1 \frac{x_0}{x_2} \left( \frac{1}{2} f_{\parallel,ef} b h_1^2 + \sigma_{ASR} A'_d t \right) \]  

(8.114)

The internal work for Crack II can be found as:

\[ W_{i,II} = \frac{1}{2} \theta_1 \frac{x_0 - x_2}{x_2} f_{\parallel,ef} b h_2^2 \]  

(8.115)

The internal work for Crack IIII can be found as:

\[ W_{i,III} = \frac{1}{2} \theta_1 f_{\parallel,ef} b h_3^2 \]  

(8.116)

As previous, the internal work for the diagonal crack can be determined by means of a line integral and a parametrisation of the coordinates \((x = t)\).

\[ W_{i,\text{dia}} = b f_{\perp,ef} \int_0^{x_0} |u_{\text{dia}}| \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} dt \]  

(8.117)

where \(\frac{dx}{dt}\) and \(\frac{dy}{dt}\) are given in Equation 8.96 and 8.97, respectively and the length of the relative displacement vector is:

\[ |u_{\text{dia}}| = \begin{cases} \theta_1 \frac{t}{\cos \alpha_{\text{dia}}}, & t \leq x_0 - x_2 \\ \theta_1 \frac{x_0 - x_2}{x_2} \frac{x_0 - t}{\cos \alpha_{\text{dia}}}, & t \geq x_0 - x_2 \end{cases} \]  

(8.118)

The internal work can thereby be found as:

\[ W_{i,\text{dia}} = b f_{\perp,ef} \theta_1 \frac{1}{2} \left( \int_0^{x_0 - x_2} t dt + \frac{x_0 - x_2}{x_2} \int_{x_0 - x_2}^{x_0} x_0 - t \ dt \right) \]  

(8.119)

\[ W_{i,\text{dia}} = b f_{\perp,ef} \theta_1 \frac{(x_0 - x_2)(x_0)}{2 \cos \alpha_{\text{dia}}^2} \]  

(8.120)

The external work for Cracking mechanism 1 can be expressed by the moment \(M_1\), over the reaction \(R_1\):

\[ W_E = |M_1| \theta_1 \]  

(8.121)

The internal work for Cracking mechanism 2 has four contributors; Crack II, Crack III, Crack IIII and the diagonal crack.

The internal work for Crack II can be found as:

\[ W_{i,II} = \frac{1}{2} \theta_2 f_{\parallel,ef} b h_2^2 \]  

(8.122)
The internal work for Crack III can be found as:

\[ W_{i,III} = \frac{x_0}{x_1} \left( \frac{1}{2} f_{t,ef} b h_{III}^2 + \sigma_{ASR} A_s d_{III} \right) \]  

(8.123)

The internal work for Crack IIII can be found as:

\[ W_{i,IIII} = \frac{1}{2} \frac{x_0 - x_1}{x_1} f_{c,ef} b h_{IIII}^2 \]  

(8.124)

To determine the internal work for the diagonal crack, the relative displacement vectors are needed:

\[ |u_{dia}| = \begin{cases} \frac{\theta_2}{x_1} (x_0 - x_1) \frac{t}{\cos \alpha_{dia}}, & t \leq x_1 \\ \frac{\theta_2}{x_0 - t} \frac{t}{\cos \alpha_{dia}}, & t \geq x_1 \end{cases} \]  

(8.125)

The internal work can thereby be found be the line integral in Equation 8.117:

\[ W_{i,\text{dia}} = b f_{t,ef} \frac{\theta_2}{\cos \alpha_{dia}} \left( \frac{x_0 - x_1}{x_1} \int_0^{x_1} t \, dt + \int_{x_1}^{x_0} x_0 - t \, dt \right) \]  

(8.126)

\[ W_{i,\text{dia}} = \frac{1}{2} f_{t,ef} b \theta_2 \left( \frac{x_0 - x_1}{x_1} x_0 \right) \frac{1}{\cos \alpha_{dia}^2} \]  

(8.127)

The external work for Cracking mechanism 2 can be expressed by the moment \( M_2 \), under the load \( P_2 \):

\[ W_E = M_2 \theta_2 \]  

(8.128)

Figure 8.16 shows a graphical example of the calculated crack formation moment for Beam 5.1, expressed by the load \( P \). It appears that the crack formation curve for Cracking mechanism 2 (dashed line) is lower than the crack formation curve for Cracking mechanism 1. Both curves have minimum where \( x_1 \) or \( x_2 \) are largest (= \( x_0 = 1502 \) mm). The cases where \( x_1 = x_0 \) and \( x_2 = x_0 \) correspond to formation of traditional flexural crack under load \( P_2 \) and over reaction \( R_1 \), respectively. These cases do not include formation of the diagonal crack, which is of interest. However, all cases where \( x_1 \) or \( x_2 \) are lower than \( x_0 \) include formation of the diagonal crack. It can be seen that the crack formation moment is decreasing with an increase of \( x_1 \) and \( x_2 \). Therefore, it is of interest how close \( x_1 \) and \( x_2 \) can come to \( x_0 \), which will be decisive for the moment (and thereby the load level \( P \)) where the diagonal crack is formed. In the figure, two red dots depict the calculated cracking moment for Cracking mechanism 1 and 2, respectively. To find these values, the crack spacing is needed.
Crack spacing

Since this PhD project has not focused on the crack spacing or bonding between ASR-damaged concrete and reinforcement, the model from Model Code 2010 (referred to as MC10) is adopted (Beverly, 2013). Here, the minimum spacing between two cracks, $l_{s,\text{max}}$, is found by:

$$l_{s,\text{max}} = 1.0c + \frac{f_{ct}}{4\tau_{\text{bms}}} \varphi_{s} $$

(8.129)

where $c$ is the concrete cover of the reinforcement in tension, $f_{ct}$ is the tensile strength of the concrete, $\tau_{\text{bms}}$ is the bond strength between the reinforcement and the concrete, $\varphi_{s}$ is the diameter of the reinforcement bars in tension and $\rho_{s,\text{ef}}$ is the effective reinforcement degree:

$$\rho_{s,\text{ef}} = \frac{A_{s}}{A_{c,\text{ef}}} $$

(8.130)

where $A_{s}$ is the reinforcement in tension and $A_{c,\text{ef}}$ is the effective area of concrete in tension:
$A_{c,ef} = b \cdot \min \left\{ \frac{2.5(c + \varphi_s/2)}{(h - x)/3} \right\}$

where $x$ is the depth of the compression zone in the elastic regime. In order to find the depth of the compression zone the Young’s modulus is needed. However, in the Lindenborg test programme, the Young’s modulus has not been measured. The lower requirement for finding $A_{c,ef}$ is consequently ignored in the later comparison. The bond strength, $\tau_{bms}$, for sound concrete can be found as follows, according to MC10:

$$\tau_{bms} = 1.8f_{ct}$$

MC10 states that the bond strength in ASR-damaged structures may be reduced by up to 50% for ribbed reinforcement bars without transverse reinforcement (stirrups). However, it is unknown which tensile strength to adopt in the calculation. The statement in MC10 is based on a study by Chana (1989). Chana (1989) conducted an experimental investigation of the influence of ASR on the bond strength. He found that the bond strength for ribbed bars without links (stirrups) and the tensile strength were both halved due to ASR, i.e. the ratio between the bond strength and the tensile strength was the same as for undamaged concrete. By substituting the expression for the bond strength (Equation 8.132) in the expression for the crack spacing (Equation 8.129), it appears that the crack spacing is independent of the tensile strength of the concrete, i.e. the crack spacing is not affected by ASR. It is remarked that the restraining conditions during the ASR acceleration in experiments by Chana (1989) were not representative for 2D-restrained slabs. When the crack spacing ($l_{s,max}$) is known, it is possible to determine the critical values for $x_1$ and $x_2$ ($x_0 - l_{s,max}$), which are decisive for the crack formation moment. The crack formation moment for the two mechanisms ($M_{1,crack}$ and $M_{2,crack}$) are shown as red dots in Figure 8.16. The moment that is governing for the formation of the diagonal crack is the minimum of $M_{1,crack}$ and $M_{2,crack}$ while the corresponding load is denoted $P_{crack}$.

### 8.3.2 Criterion for collapse

After the formation of the diagonal crack, the load-carrying mechanism of the beam changes. In this section, models for a flexural collapse and a shear collapse after formation of the diagonal crack are derived.

#### 8.3.2.1 Flexural collapse

To derive the model for a flexural collapse, the same mechanism as for formation of the diagonal crack is considered, see Figure 8.7. However, the experiments show that the crack width of the diagonal crack is very large before the failure occurs. Consequently, the contribution of the diagonal crack to the internal work is neglected. Therefore, the internal work can be determined based on the four vertical cracks, exclusively.
Internal work for Crack I and III
Figure 8.17 shows the assumed normal stress distribution in Crack I and III. Due to the
direction of the compressive stresses, $f_{c\parallel,\text{ef}}$ is adopted and effectiveness factor for bending
can be applied, see Equation 8.78 and 8.79. Since it is the ultimate moment capacity that
is under consideration, it is assumed that the reinforcement is yielding (reinforcement stresses = $f_y$) and there is no tensile strength left in the concrete.

The internal work for Crack I can be found as:

$$W_{i,1} = \int_{y_0}^{y_1} f_{c\parallel,\text{ef}} b y' dy' + f_y A'_s (y' = d_1) \quad (8.133)$$

By employing the displacement vectors from Equation 8.85, the internal work can be found as:

$$W_{i,1} = \theta_1 \frac{x_0}{x_2} \left( \frac{1}{2} f_{c\parallel,\text{ef}} b y_1^2 + f_y A'_s |d_1 - y_1| \right) \quad (8.134)$$

Similarly, the internal work for Crack III can be derived as:

$$W_{i,\text{III}} = \theta_1 \frac{c_b x_0}{x_1} \left( \frac{1}{2} f_{c\parallel,\text{ef}} b y_{\text{III}}^2 + f_y A_s |d_{\text{III}} - y_{\text{III}}| \right) \quad (8.135)$$

Internal work for Crack II and IIII
Figure 8.18 shows the assumed normal stress distribution in Crack II and IIII. As for
Crack I and III, $f_{c\parallel,\text{ef}}$ is adopted for the compression zone, and the tensile strength for the
concrete is neglected. The contribution from the reinforcement is determined as dowel action where it is assumed that the reinforcement is yielding (reinforcement stresses = $f_y$).
The internal work for crack II can be found as:

\[ W_{i,II} = \int_0^{h_{II}} f_{c,\parallel,et} b \mid u \mid dy + M'_{dowel} \alpha_{II} \]  

(8.136)

where:

\[ M'_{dowel} = f_y \frac{\varphi'^3}{6} n'_{\text{bars}} \]  

(8.137)

where \( \varphi' \) is the diameter of the reinforcement bars in the top of the beam and \( n'_{\text{bars}} \) is the number of these bars.

By employing the displacement vectors from Equation 8.89, the internal work can be found as:

\[ W_{i,II} = \theta_1 \left( c_\theta + \frac{x_0}{x_2} - 1 \right) \left( \frac{1}{2} f_{c,\parallel,et} b h_{II}^2 + M'_{dowel} \right) \]  

(8.138)

Similarly, the internal work for Crack IIII can be derived as:

\[ W_{i,III} = \theta_1 \left( 1 + \frac{c_\varphi x_0}{x_1} - c_\theta \right) \left( \frac{1}{2} f_{c,\parallel,et} b h_{III}^2 + M_{dowel} \right) \]  

(8.139)

where:

\[ M_{dowel} = f_y \frac{\varphi^3}{6} n_{\text{bars}} \]  

(8.140)

where \( \varphi \) is the diameter of the reinforcement bars in the bottom of the beam and \( n_{\text{bars}} \) is the number of these bars.

The total internal work is the sum of the internal work for the four vertical cracks:

\[ W_I = \sum_{n=1}^{III} W_{i,n} \]  

(8.141)

External work

The external work can be found as for the case of formation of the diagonal crack, see Equation 8.110. An expression for the load that causes flexural collapse, \( P_{\text{mom}} \), can be found from the work equation \( (W_E = W_I) \). Written in terms of the moment over reaction \( R_1, M_{1,\text{mom}} \), the load that causes flexural collapse can be expressed as:

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The expression for the moment that causes flexural collapse \( M_{1,\text{mom}} \) depends on four unknown variables. Since the expression is based on an upper bound solution, the four unknown variables can be determined by an optimization (minimization).

**Optimization**

A numerical minimization of \( M_{1,\text{mom}} \) for the continuous beams from Lindenborg shows that \( x_1 \to 0 \) and \( x_2 \to 0 \) which means the failure will occur as a normal moment failure in a vertical flexural crack, either over reaction \( R_1 \) or under load \( P_2 \), i.e. the failure is not influence by the diagonal crack and the moment capacity can be determined by adopting \( f_{\text{cij}} \) and the ASR-induced prestressing in a traditional calculation of the moment capacity for concrete beams.

The load corresponding to flexural failure is denoted \( P_{\text{mom}} \).

### 8.3.2.2 Shear collapse

After the diagonal crack is formed, the shear force is mainly carried by the compression zones. The considered failure mechanism for deriving the model for the shear capacity is shown in Figure 8.19. The failure mechanism consists of sliding failure in a yield line in the main shear span. The yield line consists of three parts: (i) a yield line formed in compression zone near Reaction \( R_1 \) (marked with \( \text{A} \) on the figure, mentioned as yield line \( \text{A} \)); (ii) a yield line that follows the diagonal crack and; (iii) a yield line formed in compression zone near Load \( P_1 \) (marked with \( \text{B} \) on the figure, mentioned as yield line \( \text{B} \)). It is assumed that the displacement vector, \( \mathbf{u} \), is vertical.
8.3.2.2.1 Internal work

As mentioned earlier, the experiments showed that the crack width of the diagonal crack was very large before the failure occurred. Therefore, the contribution to the internal work from the yield line that follows the diagonal crack is neglected. The internal work can therefore be determined based on the two yield lines in the compression zones, exclusively.

**Internal work for yield line A**

Yield line A is straight and runs from the reaction $R_1$ and through the compression zone to a point where it merges with the diagonal crack. The shear is transferred as sliding in the yield line. The internal work can be found by the dissipation formula, where the tensile strength of the concrete is neglected, see Equation 8.12. As concluded in Section 8.2 $f_{c\parallel}$ should be employed in load scenarios where sliding occurs independently of the ASR-induced macro-cracks. The effectiveness factor is discussed later. By introducing $h_A$ and $x_A$ as the height (vertical projection) and the length (horizontal direction) of the yield line, respectively, the internal work can be found as:

$$W_{I,A} = \frac{1}{2} \psi_{c,\text{shear}} f_{c\parallel} b |u| \left( \sqrt{x_A^2 + h_A^2} - x_A \right)$$  \hspace{1cm} (8.143)

The height of the yield line depends on where the yield line merges with the diagonal crack. The height can be found by simple geometry:

$$h_A = h_{\text{III}} + \frac{h - h_{\text{III}} - h_{\text{II}}}{x_0} x_A$$  \hspace{1cm} (8.144)
Internal work for yield line B
Yield line B is straight and runs from the load $P_2$ and through the compression zone to a point where it merges with the diagonal crack. The internal work can be determined as for yield line A.

$$W_{I,B} = \frac{1}{2} f_{c,\text{shear}} \nu_{c,\parallel} [u] \left( \sqrt{x_B^2 + h_B^2} - x_B \right)$$

(8.145)

where the height for the yield line can be found as:

$$h_B = h_{II} + \frac{h - h_{III} - h_{II}}{x_0} x_B$$

(8.146)

8.3.2.2.2 External work

The external work can be found as:

$$W_E = (R_1 - P_1) |u| = V_2 |u|$$

(8.147)

where $V_2$ is the shear in the main shear span, see Figure 7.9.

An expression for the shear capacity for the main shear span after the diagonal crack is formed can be found from the work equation ($W_E = W_1$). Since it is now the shear capacity that is considered (not the applied shear force), $V_2$ is renamed $V_{2,R}$:

$$V_{2,R}(h_A, h_B) = \frac{W_{I,A} + W_{I,B}}{|u|}$$

(8.148)

It appears that the shear capacity depends on two unknown variables, namely $h_A$ and $h_B$. Since the expression is based on an upper bound solution, the unknown variables can be determined by optimization (minimization in this case).

Effectiveness factor

For shear in beams without shear reinforcement, the effectiveness factor accounts for a number of effects that are not included (sufficiently) in a given model. Zhang (1997) and later Fisker and Hagsten (2016) suggest that if the crack width is included in the model, an effectiveness factor that only depends on the compressive strength may be adopted. As the sliding occurs in the compression zones in the considered failure mechanism (see Figure 8.19), no load-induced cracks will affect the shear capacity of the yield lines. Consequently, an effectiveness factor for shear that only accounts for the compressive strength is adopted (Nielsen and Hoang, 2010):

$$f_{c,\parallel, \text{ef}} = \frac{f_{c,\parallel} \nu_{c,\text{shear}}}{f_{c,\parallel}} = f_{c,\parallel} \left( 0.7 - \frac{f_{c,\parallel, \text{ef}}}{200} \right)$$

(8.149)

The load corresponding to a shear failure is denoted $P_{\text{shear}}$. 171
8.3.3 The load-carrying capacity

Since the derived model for the shear capacity is only valid if the diagonal crack is formed, it requires more than a minimization to find the load-carrying capacity. If the diagonal crack forms at a lower load level than the load for the shear failure \( P_{\text{shear}} \), then the shear capacity is governing for the load-carrying capacity of the beam. On the other hand, if the diagonal crack forms at a higher load level than the load for the calculated shear failure, then the formation of the diagonal crack \( P_{\text{crack}} \) is governing for the load-carrying capacity of the beam. Good examples on this are Beam 4.2 and 5.3 from Lindenborg. Figure 8.20 shows the load-deflection curves for the two beams, whilst Figure 8.21 and 8.22 show major strain plots for the two beams for two load stages: (i) the load stage where the diagonal crack forms and; (ii) the load stage for the finale collapse. These two load stages are marked with red dots on the load-deflection curves. The load-deflection curve for Beam 4.2 (Figure 8.20a) has a peak at \( P = 346.5 \) kN and the load drops hereafter to a post-peak plateau at \( P \approx 305 \) kN. Figure 8.21a shows that the diagonal crack (it is not perfectly diagonal) is formed at the first peak and Figure 8.21b shows that a shear collapse occurs at the post-peak plateau. The load-carrying capacity for Beam 4.2 is therefore governed by the formation the diagonal crack \( P_{\text{crack}} \). The load-deflection curve for Beam 5.3 (Figure 8.20b) looks different. It has a small drop at \( P \approx 230 \) kN but the load increases subsequently to \( P = 301.9 \) kN. Figure 8.22a shows that the small drop on the load-deflection curve was caused by the formation of the diagonal crack and Figure 8.22b shows that a shear collapse occurs when \( P = 301.9 \) kN. Hence, the load-carrying capacity for Beam 5.2 is governed by the shear collapse \( P_{\text{shear}} \).

Finally, it remarked that if the load for a flexural collapse is smaller than the largest of \( P_{\text{crack}} \) and \( P_{\text{shear}} \), it the flexural collapse that is governing for the load-carrying capacity.

![Load-deflection curves for two continuous beams from Lindenborg.](image-url)
(a) Formation of the diagonal crack, $P = 346.5\, \text{kN}$.

(b) Final collapse, $P = 303.6\, \text{kN}$.

Figure 8.21: Major strain plot for Beam 4.2 from Lindenborg.

$\text{(a)}$ Formation of the diagonal crack, $P = 230.4\, \text{kN}$.

(b) Final collapse, $P = 301.9\, \text{kN}$.

Figure 8.22: Major strain plot for Beam 5.3 from Lindenborg.
8.3.4 Discussion of proposed model

8.3.4.1 Limitations

The proposed model for continuous beams is based on an assumption that the diagonal crack forms and the load-carrying capacity is governed by either the formation of the diagonal crack or a subsequently shear or flexural collapse. This diagonal crack is a characteristic for the continuous beams with ASR-damages. A kind of diagonal crack was observed in all the tested beams with ASR-damages, whilst it was not observed in any of the continuous reference beams. By studying the expression for formation of the diagonal crack (see Section 8.3.1.2.4), it appears that the tensile strength perpendicular to the ASR-induced crack \( f_{t \perp} \) plays an important role for the load level where the diagonal crack forms. \( f_{t \perp} \) is greatly reduced by the ASR damages. In the beams from Gammelrand and Lindenborg, \( f_{t \perp} \) was reduced to 0.4 MPa. However, both the beams from Gammelrand and Lindenborg were severely damaged due to ASR. One could imagine that if a slab is less damaged, \( f_{t \perp} \) would be less reduced as well and the load for formation of the diagonal crack would be higher. At a certain point, another failure mechanism without the diagonal crack will be governing for the load-carrying capacity. This scenario is not covered by the proposed model.

The models for formation of the diagonal crack and for the flexural collapse are based on results of an optimization of the established upper bound solutions. These optimizations are based on material parameters and geometry for the beams from Lindenborg. These beams have the same cross section and reinforcement. If the design of a given slab deviates significantly from this (e.g. contains curtailed reinforcement), the results of the optimizations may be different. This is not covered by the simplified models (the reduced expression after optimization).

Finally, in the model for the shear collapse, the displacement vector is assumed to be vertical, in contrast to the shear model for the simply supported beams where the displacement vector can be inclined if the mechanical reinforcement degree is small \((< \frac{1}{2} \nu)\).

For continuous beams, this is not covered by the proposed models.

8.3.4.2 Influencing parameters

The proposed model consists of three submodels: (i) a model for formation of the diagonal crack; (ii) a model for the load of a shear collapse and; (iii) a model for the load of a flexural collapse. In this section these submodels are treated individually.

8.3.4.2.1 Formation of the diagonal crack

The model is described in Section 8.3.1.2.4. Table 8.1 shows an overview of the influencing parameters.
Beam dimensions \( b, h \)
Concrete cover \( c, c' \)
Shear span \( x_0 \)
Concrete compressive strength parallel to the ASR-induced cracks \( f_{c\parallel} \)
Concrete tensile strength \( f_{t\parallel}, f_{t\perp} \)
Prestressing \( \sigma_{ASR} \)
Reinforcement \( A_s, A_s', \phi_s, \phi_s' \)

1. \( A_s' \) and \( \phi_s' \) for Cracking mechanism 1 and \( A_s \) and \( \phi_s \) for Cracking mechanism 2.

<table>
<thead>
<tr>
<th>Beam dimensions</th>
<th>( b, h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete cover</td>
<td>( c, c' )</td>
</tr>
<tr>
<td>Shear span</td>
<td>( x_0 )</td>
</tr>
<tr>
<td>Concrete compressive strength parallel to the ASR-induced cracks</td>
<td>( f_{c\parallel} )</td>
</tr>
</tbody>
</table>

Table 8.1: Influencing parameters for the model for formation of the diagonal crack.

8.3.4.2.2 Shear collapse

The model is described in Section 8.3.2.2. Table 8.2 shows an overview of the influencing parameters.

<table>
<thead>
<tr>
<th>Beam dimensions</th>
<th>( b, h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete cover</td>
<td>( c, c' )</td>
</tr>
<tr>
<td>Shear span</td>
<td>( x_0 )</td>
</tr>
<tr>
<td>Concrete compressive strength parallel to the ASR-induced cracks</td>
<td>( f_{c\parallel} )</td>
</tr>
</tbody>
</table>

Table 8.2: Influencing parameters for the model for the shear collapse.

8.3.4.2.3 Flexural collapse

In Section 8.3.2.1 it is found that the moment that is required to induce a flexural collapse can be found by means of a traditional calculation of the moment capacity for concrete beams where \( f_{c\parallel} \) and the ASR-induced prestressing is adopted. The influencing parameters are therefore as shown in Table 8.3.

<table>
<thead>
<tr>
<th>Beam dimensions</th>
<th>( b, h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete cover</td>
<td>( c, c' )</td>
</tr>
<tr>
<td>Reinforcement(^1)</td>
<td>( A_s, A_s', A_y, \phi_s, \phi_s' )</td>
</tr>
<tr>
<td>Prestressing</td>
<td>( \sigma_{ASR} )</td>
</tr>
<tr>
<td>Concrete compressive strength parallel to the ASR-induced cracks</td>
<td>( f_{c\parallel} )</td>
</tr>
<tr>
<td>Young's modulus for the concrete, parallel to the ASR-induced cracks</td>
<td>( E_{c\parallel} )</td>
</tr>
</tbody>
</table>

\(^1\) \( c \) for the positive moment capacity and \( c' \) for the negative moment capacity.
\(^2\) \( A_s \) for the positive moment capacity and \( A_s' \) for the negative moment capacity.

Table 8.3: Influencing parameters for the model for the flexural collapse.
It is remarked that the concrete tensile strength only has an influence on the formation of the diagonal crack, whilst the remaining parameters are influencing both the formation of the diagonal crack and the subsequently shear or flexural collapse.

8.4 Comparison of calculations with test results

This section provides a comparison between the proposed models and the experimental results for the simply supported and continuous ASR-damaged beams.

8.4.1 Simply supported beams

Table 8.4 and 8.5 show the test results and the model predictions for the simply supported beams from Gammelrand and Lindenborg, respectively. The comparison between the proposed shear model and the tested shear capacity for the three beams originating from Gammelrand is presented graphically in Figure 8.23 as well. From the graphical presentation, it appears that the shear model predicts the shear capacity of the beams from Gammelrand very well.

Since the simply supported beams from Lindenborg failed in bending (see Section 7.1.1), the test results cannot be used for direct comparison to the model. However, the experiments can be used to make the model more probable. From Table 8.5 it appears that the predicted moment capacity is lower than the predicted shear capacity, i.e. a moment failure is expected. This is in alignment with the observed moment failure (see Section 7.1.3). It appears furthermore that the predicted shear capacity is higher than the failure loads. All in all, it can be concluded that the test results from the simply supported beams from Lindenborg are consistent with the proposed shear model.

<table>
<thead>
<tr>
<th>Beam</th>
<th>$P_{mom}$ [kN]</th>
<th>$P_{shear}$ [kN]</th>
<th>$P_{test}$ [kN]</th>
<th>$P_{test}/P_{shear}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>476.6</td>
<td>460.1</td>
<td>458.9</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>488.4</td>
<td>422.1</td>
<td>421.8</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>485.1</td>
<td>361.2</td>
<td>366.0</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Table 8.4: Comparison between the tested shear capacity of the beams from Gammelrand and the proposed shear model. The mean value and the standard deviation of $P_{test}/P_{shear}$ are 1.00 and 0.01, respectively.
<table>
<thead>
<tr>
<th>Beam</th>
<th>$P_{\text{mom}}$ [kN]</th>
<th>$P_{\text{shear}}$ [kN]</th>
<th>$P_{\text{test}}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>190.4</td>
<td>297.9</td>
<td>223.8</td>
</tr>
<tr>
<td>2.2</td>
<td>215.3</td>
<td>349.6</td>
<td>262.4</td>
</tr>
<tr>
<td>2.3</td>
<td>269.7</td>
<td>403.2</td>
<td>323.9</td>
</tr>
<tr>
<td>3.1</td>
<td>175.2</td>
<td>280.2</td>
<td>204.0</td>
</tr>
<tr>
<td>3.2</td>
<td>210.1</td>
<td>332.9</td>
<td>234.9</td>
</tr>
<tr>
<td>3.3</td>
<td>254.6</td>
<td>409.6</td>
<td>270.2</td>
</tr>
<tr>
<td>6.1</td>
<td>128.9</td>
<td>206.1</td>
<td>120.1</td>
</tr>
<tr>
<td>6.2</td>
<td>188.0</td>
<td>304.9</td>
<td>196.0</td>
</tr>
<tr>
<td>6.3</td>
<td>234.7</td>
<td>372.1</td>
<td>228.9</td>
</tr>
</tbody>
</table>

Table 8.5: Comparison between the tested shear capacity of the simply supported beams from Lindenborg and the proposed shear model.

![Shear capacity comparison](image)

Figure 8.23: Comparison between the tested shear capacity of the beams from Gammelrand and the proposed model.

### 8.4.2 Continuous beams

For the continuous beams, the model for formation of the diagonal crack as well as the model for shear collapse are compared with the experimental results for the beams from Lindenborg.

#### 8.4.2.1 Formation of diagonal crack

Since the concrete tensile strength was only determined for Slab segment 5, it is only possible to compare the proposed model for formation of the diagonal crack to three
beams. Figure 8.24 shows the load-displacement diagrams for the three beams that originate from Slab segment 5. On the diagrams, the observed crack formation load is shown as a red dot and the predicted crack formation load is shown as a horizontal blue line. It appears that the model predicts the crack formation load for Beam 5.1 and Beam 5.2 very well while the model overestimates the crack formation load by 30% for Beam 5.3.

Figure 8.24: Load-displacement diagrams with indication of the observed and theoretical crack formation load.

8.4.2.2 Load-carrying capacity

In the comparison of the measured and predicted load-carrying capacity, the predicted capacity is based on the proposed shear model, where it is decisive whether the diagonal
crack is formed or not. Since the tensile strength has only been measured for the beams that originate from Slab segment 5, the formation of the diagonal crack has been investigated by means of DIC for the remaining beams. It is found that $P_{\text{crack}}$ was higher than $P_{\text{shear}}$ for Beam 4.1, and 4.2, i.e. $P_{\text{crack}}$ is governing for the load-carrying capacity. For the remaining beams, $P_{\text{crack}}$ was smaller than $P_{\text{shear}}$. Table 8.6 shows the test results and the model predictions, whilst they are shown graphically in Figure 8.25. The predicted capacities are shown as columns and the tested capacities are shown as red dots. It is seen that the model predicts the tested capacity very well. Despite that the nine beams are cut from the same bridge (Lindenborg), they differ in a number of parameters that (according to the proposed model) play an important role for the shear capacity. These parameters are: (i) the slenderness ($x_0/h$); (ii) the concrete cover ($c$ and $c'$) and; (iii) the degree of ASR damages, which affects the strength of the concrete ($f_{c,\perp}$, $f_{c,\parallel}$, $f_{t,\perp}$ and $f_{t,\parallel}$) and the ASR-induced prestressing ($\sigma_{\text{ASR}}$).

<table>
<thead>
<tr>
<th>Beam</th>
<th>$P_{\text{crack}}$ [kN]</th>
<th>$P_{\text{shear}}$ [kN]</th>
<th>$P_{\text{model}}$ (max[$P_{\text{crack}}$;$P_{\text{shear}}$]) [kN]</th>
<th>$P_{\text{test}}$ [kN]</th>
<th>$P_{\text{test}}/P_{\text{model}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>$&lt; P_{\text{shear}}$</td>
<td>327.5</td>
<td>327.5</td>
<td>340.7</td>
<td>1.04</td>
</tr>
<tr>
<td>1.2</td>
<td>$&lt; P_{\text{shear}}$</td>
<td>371.1</td>
<td>371.1</td>
<td>333.1</td>
<td>0.90</td>
</tr>
<tr>
<td>1.3</td>
<td>$&lt; P_{\text{shear}}$</td>
<td>382.6</td>
<td>382.6</td>
<td>393.5</td>
<td>1.03</td>
</tr>
<tr>
<td>4.1</td>
<td>408.7$^A$</td>
<td>303.1</td>
<td>408.7</td>
<td>408.7</td>
<td>1.00</td>
</tr>
<tr>
<td>4.2</td>
<td>346.5$^A$</td>
<td>259.7</td>
<td>346.5</td>
<td>346.5</td>
<td>1.00</td>
</tr>
<tr>
<td>4.3</td>
<td>$&lt; P_{\text{shear}}$</td>
<td>302.0</td>
<td>302.0</td>
<td>325.0</td>
<td>1.08</td>
</tr>
<tr>
<td>5.1</td>
<td>301.9$^B$</td>
<td>297.3</td>
<td>301.9</td>
<td>301.9</td>
<td>1.00</td>
</tr>
<tr>
<td>5.2</td>
<td>300.5$^B$</td>
<td>308.6</td>
<td>308.6</td>
<td>344.2</td>
<td>1.12</td>
</tr>
<tr>
<td>5.3</td>
<td>302.0$^B$</td>
<td>302.0</td>
<td>302.0</td>
<td>304.4</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Note:
$^A$ determined by DIC.
$^B$ determined by the model for formation of the diagonal crack.

Table 8.6: Comparison between the tested shear capacity of the continuous beams from Lindenborg and the proposed shear model. The mean value and the standard deviation of $P_{\text{test}}/P_{\text{model}}$ are 1.02 and 0.06, respectively.
Figure 8.25: Comparison of tested and predicted load-carrying capacities for the continuous beams from Lindenborg.

8.5 Practical applications

This section provides some recommendations on how the load-carrying capacity of ASR-damaged slabs without shear reinforcement can be determined in practice. The shear and moment capacity are treated individually. The last section provides some few recommendations on health monitoring.

8.5.1 Shear capacity

8.5.1.1 Simply supported beams

The model for the shear capacity of simply supported ASR-damaged beams is described in Section 8.2. Here, it is found that the shear capacity can be determined as:

\[ V_R = \begin{cases} \frac{1}{2} f_{c\parallel} \nu b h \left( \sqrt{1 + \left( \frac{a}{h} \right)^2 - \frac{a}{h}} \right), & \Phi \geq \frac{1}{2} \nu \\ \frac{1}{2} f_{c\parallel} \nu b h \left( \sqrt{\left( \frac{a}{h} \right)^2 + \frac{4 \Phi}{\nu} \left( 1 - \frac{\Phi}{\nu} \right) - \frac{a}{h}} \right), & \Phi \leq \frac{1}{2} \nu \end{cases} \]  

(8.150)

where \( \nu \) is the effectiveness factor, \( b \) and \( h \) are the average width and the height of the beam, respectively, \( a \) is the shear span while \( \Phi \) is the mechanical reinforcement degree:

\[ \Phi = \frac{f_y A_s}{f_{c\parallel} bh} \]  

(8.151)

The effectiveness factor, \( \nu \), can be found as:
where \( f_{c\parallel} \) (in MPa) is obtained by cores that are drilled and loaded parallel to the ASR-induced cracks, i.e. drilled horizontally in a slab without shear reinforcement.

Equation 8.150 applies directly to simply supported members with a point load acting at a distance \( a \) from the support. Since this is an upper bound solution, it is in fact possible to use the solution to make approximate verifications of members subjected to distributed loads as well as members subjected to arbitrary loading. The procedure has been described by Nielsen and Hoang (2010) (Figure 5.16, page 335 in this ref.).

Basically, when replacing the parameter \( a \) in Equation 8.150 with \( x \), which denotes the position of a given cross section in the member, then Equation 8.150 can be interpreted as the cross-sectional shear capacity. Then graphically, this shear capacity should just lie above the actual shear force diagram, \( V_E \), in order to ensure sufficient shear capacity.

The approach is illustrated in Figure 8.26 for the case with a uniformly distributed load, \( p \).

\[
\nu = \frac{0.88}{\sqrt{f_{c\parallel}}} \left( 1 + \frac{1}{\sqrt{h}} \right) (1 + 26\rho) \gg 1.0, h \text{ in meters} \quad (8.152)
\]

8.5.1.2 Continuous beams

The model for the shear capacity of continuous beams consists of two criteria: (i) a criterion for formation of the diagonal crack and; (ii) a criterion for a shear collapse in the case where the diagonal crack has been formed.

8.5.1.2.1 Criterion for formation of the diagonal crack

The criterion for formation of the diagonal crack is described in Section 8.3.1. Due to the complexity of the model, it is not shown in this section. It appears that besides the
compressive strength parallel to the ASR-induced cracks, the tensile strength parallel and perpendicuar to the ASR-induced cracks ($f_{t\parallel}$ and $f_{t\perp}$) and the ASR-induced prestressing ($\sigma_{ASR}$) are needed to determine the load for formation of the diagonal crack. $f_{t\parallel}$ and $f_{t\perp}$ shall be determined by means of wedge splitting tests. Figure 8.27 shows how the ASR-induced cracks shall be orientation to determine $f_{t\parallel}$ and $f_{t\perp}$, respectively.

![Figure 8.27: The orientation of the ASR-induced cracks in wedge splitting tests.](image)

Since wedge splitting tests can be conducted with 100 $\times$ 100 $\times$ 100 mm cubes, both crack orientations can be obtained by cubes cut from a $\varnothing$ 150 mm core drilled vertically from the slab. The prestressing can be determined by measuring it directly on the reinforcement, by removing the concrete locally around a reinforcement bar, mounting a strain gauge and subsequently cutting the reinforcement bar.

8.5.1.2.2 Criterion for shear collapse
The criterion for a shear collapse is described in Section 8.3.2.2. Since the model is not expressed as a closed-form solution, it is not shown in this section. It appears that only the compressive strength parallel to the ASR-induced cracks ($f_{c\parallel}$) are needed to calculate the load for a shear collapse.

8.5.1.2.3 Conservative estimate
The shear capacity is governed by the maximum of two load levels, namely the load related to formation of the diagonal crack and the load related to the shear collapse. As mentioned above, wedge splitting tests and destructive testing of the reinforcement are needed in order to determine the load level that enables formation of the diagonal crack. Both tests are troublesome and rather expensive. A conservative estimate for the shear capacity can be obtained by assuming that the diagonal crack is formed. Thereby, only the compressive strength parallel to the ASR-induced cracks ($f_{c\parallel}$) is needed. Considering the nine continuous beams from Lindenborg. For three of the beams, formation of the diagonal crack was governing for the shear capacity. This means that the "conservative" approach corresponds to the full model for the remaining six beams. The conservative approach underestimates the shear capacity of the three above-mentioned beams by up to 25.8%. The conservative estimate can be extended to continuous beams subjected
to distributed loads as well. Assuming the existence of a diagonal crack that runs from reaction $R_1$ to the point with maximum moment at the onset of a shear failure, the procedure described in Section 8.3.2.2 may be applied. For illustration, Figure 8.28 shows a continuous beam with a distributed load. The distance from reaction $R_1$ to the point with maximum moment is denoted $x_{0,\text{max}}$. Then, by determining the shear capacity, $V_{2,R}$ (see Equation 8.148) for any given value of $x_0$ between, say, $h$ and $x_{0,\text{max}}$, a shear capacity diagram may be generated as described above for the simply supported beams.
8.5.2 Moment capacity

The moment capacity can be determined by adopting the compressive strength parallel to the ASR-induced cracks ($f_{c\parallel}$) and the ASR-induced prestressing ($\sigma_{ASR}$) in a traditional calculation of the moment capacity of concrete beams. It is remarked that prestressing often only has a minor influence on the moment capacity.
8.5.3 Health monitoring

The best way to monitor the health (structural integrity) of an ASR-damaged slab is to measure the development of the strength parameters and the prestressing. If the conservative estimation approach is employed, only $f_{c∥}$ shall be monitored. If the prestressing is crucial for the moment capacity, the change of prestressing can be monitored by mounting strain gauge on the reinforcement without cutting the reinforcement bars subsequently.
Chapter 9

Conclusions

The purpose of this thesis has been to study the influence of alkali-silica reactions on the shear capacity of reinforced concrete slabs without shear reinforcement. The research presented in the thesis stands on the shoulders of decades of important research. The first chapter of the thesis is therefore a literature study where the following topics are studied: (i) cracking and expansion behaviour of ASR-damaged reinforced concrete structures; (ii) the effects of ASR on parameters that are important for the shear capacity and; (iii) existing experimental studies on the shear capacity of ASR-damaged slabs.

Literature study

The literature study showed:

- The restraining condition during the development of ASR has a large influence on the crack pattern. Slabs without shear reinforcement (2D-restrained), which are of interest in this thesis, have two types of ASR-induced cracks: (i) randomly orientated micro-cracks and; (ii) horizontal macro-cracks. Furthermore, it is concluded that experimental results obtained from specimens, which are not restrained as slabs cannot be generalised for slabs.

- ASR has a large influence on the concrete compressive strength. The compressive strength is reduced, and it becomes an anisotropic property when the concrete is restrained during the development of ASR. However, the compressive strength and its anisotropy had never been investigated for 2D-restrained slabs.

- ASR has a large influence on the concrete tensile strength. For 2D-restrained slabs, the tensile strength is reduced and becomes an anisotropic property. It is furthermore found that the tensile strength is strongly dependent on the employed test method. However, it was unknown how to test the tensile strength for the use in structural calculations.

- ASR induces an expansion that leads to a significant prestressing of the reinforcement.
ASR has an influence on the shape of the critical shear crack, and thereby the failure mechanism in a shear failure. However, the shape of the critical crack had never been investigated for 2D-restrained slabs.

There existed no experimental studies on shear where the results can be generalised to the shear capacity for 2D-restrained slabs.

Due to these insufficiencies, the research work of this PhD project has focused on investigations of the compressive strength, the tensile strength and the shear capacity.

Material parameters
The most important findings of the investigation on the compressive strength are:

- The compressive strength is anisotropic in 2D-restrained ASR-damaged slabs. The compressive strength perpendicular to the ASR-induced (macro-) cracks ($f_{c \perp}$) is significantly lower than the compressive strength parallel to the ASR-induced cracks ($f_{c \parallel}$). The difference was up to 11.0 MPa.
- The anisotropy found cannot be explained by the anisotropy in sound concrete, i.e. the anisotropy is caused by the ASR-induced cracks.
- The compressive strength and stiffness perpendicular to the ASR-induced cracks ($f_{c \perp}$ and $E_{\perp}$) are highly influenced by closure of the ASR-induced macro-cracks, and are only applicable for load scenarios where the compression is perpendicular to the ASR-induced cracks.
- The compressive strength parallel to the ASR-induced cracks ($f_{c \parallel}$) is mainly influenced by the reduced strength of the cement paste due to the randomly orientated ASR-induced micro-cracks.
- Concrete with ASR-induced cracks parallel to the loading direction behaves like a down-scale of sound concrete under compression.

In the investigation on the tensile strength, it is found that the most common ways of testing the tensile strength of concrete (uniaxial tensile tests and Brazilian split tests) are not applicable for concrete from a 2D-restrained ASR-damaged slab. It is found that the wedge splitting test can be used to determine the tensile strength of ASR-damaged concrete; both parallel and perpendicular to the ASR-induced cracks ($f_{t \parallel}$ and $f_{t \perp}$).

Shear capacity
The shear capacity has been investigated by a large experimental programme that consists of two test series with beams cut from ASR-damaged slab bridges and one reference test series with beams made of sound concrete. The investigation shows:

- ASR has an influence on the formation of the load-induced cracks. The flexural cracks in the ASR-damaged beams form at a higher load level, and the height and
the width of the flexural crack are smaller than in the sound beams. Flat inclined cracks are formed instead of the flexural-shear cracks that are typically formed in similar beams without ASR damages.

- ASR has an influence on the shape of the critical shear crack and thereby the shear failure mechanism. The critical shear crack follows the flat inclined cracks instead of the (non-existing) flexural-shear cracks.

In Chapter 8, models for the shear capacity of simply supported and continuous beams (2D-restrained slabs) are developed. The models include the findings from the previous chapters. The models are based on rigid-plastic upper bound solutions, where the considered cracking and failure mechanisms are inspired by the crack formation and failure mechanisms observed during the experiments.

The model for simply supported beams predicts the tested shear capacity very well. The mean value and the standard deviation of the test to calculated ratio are \( \mu = 1.00 \) and \( \sigma = 0.01 \), respectively. However, it is remarked that the number of experiments for comparison is low. Only three of the simply supported beams failed in shear and these beams are identical, both regarding geometry, degree of ASR damages and slenderness. The model for the continuous beams predicts the tested shear capacity very well. The mean value and the standard deviation of the test to calculated ratio are \( \mu = 1.02 \) and \( \sigma = 0.06 \), respectively. All nine beams failed in shear. Despite that the nine beams originate from the same bridge, they differ in a number of important design parameters (according the the proposed model). These parameters are: (i) the slenderness \( (x_0/h) \); (ii) the concrete cover \( (c \) and \( c') \) and; (iii) the degree of ASR damages, which affects the strength of the concrete \( (f_{c\perp}, f_{c\parallel}, f_{l\perp} \) and \( f_{l\parallel} \) and the ASR-induced prestressing \( (\sigma_{ASR}) \). Additionally, it is found that the moment capacity can be determined by adopting the compressive strength parallel to the ASR-induced cracks \( (f_{c\parallel}) \) and the ASR-induced prestressing \( (\sigma_{ASR}) \) in a traditional calculation of the moment capacity for concrete beams.

Finally, a simple model is proposed for use in practice. The simple model gives conservative estimates of the shear capacity and is simple to use in practice.

### 9.1 Recommendations for future research

#### 9.1.1 Experiments

In the experiments with the simply supported beams, it was observed that a small part of the yield line (the critical crack) was formed in an existing flat inclined crack. In the derived model, the length of this part is neglected. The proposed shear model for the simply supported beams is therefore based on the assumption that the yield line (critical crack) runs from the reaction to the applied load independent of the existing flat inclined crack. Since this is based on observations from experiments with three almost identical beams in the same experimental setup, it is unknown whether the path of the yield
line will change if the slenderness increases. Consequently, more experiments should be conducted with beams with a slenderness larger than 2.5.

The proposed shear model for continuous beams includes two criteria - a criterion for formation of a diagonal crack and a criterion for a shear failure. The criterion for diagonal cracking depends, among other strength parameters, on the concrete tensile strength. Since the tensile strength was only measured for three of the continuous beams, the cracking criterion is only validated by these three beams. More experiments should be conducted with continuous beams, where the tensile strength parallel and perpendicular to the ASR-induced cracks is measured. The ASR-damaged beams presented in this thesis are cut from existing slab bridges. It is therefore not possible to freely design a test series where a number of important design parameters are varied systematically, e.g. the degree of ASR-damages, the reinforcement ratio, or the height of the beams. This could be done with laboratory experiments where ASR is accelerated in a climate chamber under restraining and exposure conditions that represent a bridge slab.

The beams presented in this thesis originate from bridges where only the sand fraction is reactive. This is often the case for ASR-damaged structures in Denmark. To generalise the model to ASR-damaged beams where the reactive particles are larger, more experiments are needed.

9.1.2 Model

The two criteria in the proposed shear model for continuous beams are too complex in their present form. By means of a systematic parameter study, it may be possible to establish a closed-form approximation that is more applicable for use in practice. Both proposed shear models are developed for slabs that are 2D-restrained during the development of ASR. By studying structures with other restraining conditions, it may be possible to generalise the models to 1D- and 3D-restrained structures.
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Nomenclature

Abbreviations

ASR  Alkali silica reaction
BS   Brazilian split test
CMOD Crack mouth opening displacement
Core\textsubscript{0} Core drilled parallel to the casting direction
Core\textsubscript{∥} Drilled core with ASR-induced macrocracks parallel to the loading direction
Core\textsubscript{⊥} Drilled core with ASR-induced macrocracks perpendicular to the loading direction
Core\textsubscript{90} Core drilled perpendicular to the casting direction
DIC  Digital Image Correlation
FEM  Finite Element Method
HS   Heemraadsingel (Dutch bridge)
Prism\textsubscript{∥} Prism with ASR-induced macrocracks parallel to the loading direction
Prism\textsubscript{⊥} Prism with ASR-induced macrocracks perpendicular to the loading direction
RQ   Research question
WS   Wedge splitting test
ZB   Zaltbommel (Dutch bridge)

Symbols

2a\textsubscript{strip} Width of the loading in Brazilian split test
\( \alpha \) Angle between displacement vector and yield line
\( \alpha\textsubscript{dia} \) Inclination of diagonal crack
$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ Rotation in crack/yield line

$\Delta f_c$ Anisotropy of the compressive strength measured as an absolute strength difference, i.e. $f_{c,0} - f_{c,90}$

$\Delta y$ Deformation in the $y$-direction

$\delta$ Displacement vector

$\delta_i$ The $i^{th}$ component of displacement vector $\delta$

$\phi_s, \phi'_s$ Diameter of reinforcement bars (bottom/top)

$\nu$ Effectiveness factor

$\Phi$ Mechanical reinforcement degree

$\rho$ Reinforcement degree

$\rho_{s,ef}$ Effective reinforcement degree

$\sigma$ Normal stress

$\sigma_{bmd}$ Average prestress-induced compressive stress

$\sigma_1$ First principal stress

$\sigma_3$ Third principal stress

$\sigma_{ASR}$ ASR-induced prestressing in the reinforcement

$\sigma_{comp}$ Compressive stress transferred perpendicular to ASR-induced macro-crack

$\sigma_p$ ASR-induced prestress

$\sigma_{split}$ Tensile stress induced by ASR-induced splitting

$\sigma_x$ Normal stress in the $x$-direction

$\sigma_y$ Normal stress in the $y$-direction

$\sigma_i$ $i^{th}$ component of normal and shear stress, in Section 8.1.4.1

$\tau$ Shear stress

$\tau_{1,m}$ Average shear stress

$\tau_{bms}$ Bond strength

$\mathbf{u}$ Relative displacement vector

$\mathbf{u}_i$ Component $i$ to relative displacement vector $\mathbf{u}$

$\theta$ Angle of rotation of rotation parts
\( \varepsilon \) Normal strain
\( \varepsilon'_{c,\text{ASR}} \) ASR-induced compressive strains in concrete in the bottom of the beam
\( \varepsilon_{\text{axial}} \) Axial strain
\( \varepsilon_{c,\text{ASR}} \) ASR-induced compressive strains in concrete
\( \varepsilon_{c,m}, \varepsilon_{s,m}, \varepsilon'_{s,m} \) Moment-induced strains
\( \varepsilon_{c,\text{tot}}, \varepsilon'_{s,\text{tot}}, \varepsilon'_{s,\text{tot}} \) Total strains
\( \varepsilon_{\text{lat}} \) Lateral strain
\( \varepsilon_{\text{restrained},x,y} \) The mean value of ASR-induced tensile strains in the reinforcement in \( x \)- and \( y \)-direction
\( \varepsilon_{s,\text{ASR}} \) ASR-induced tensile strains in reinforcement
\( \varepsilon_v \) Volumetric strain
\( \varepsilon_x \) Horizontal normal strain
\( \varepsilon_y \) Vertical normal strain
\( \varepsilon_i \) Strain component \( i \)
\( \varphi \) Angle of friction
\( A_{c,\text{ef}} \) Effective area of concrete in tension
\( a_L \) Length of left shear zone for the simply supported beams
\( a_R \) Length of right shear zone for the simply supported beams
\( A_s, A'_s \) Cross-sectional area of reinforcement (bottom/top)
\( b \) Beam width
\( c \) Cohesion in Section 8.1.4.1
\( c \) Concrete cover for the reinforcement in the bottom of the beam
\( c' \) Concrete cover for the reinforcement in the top of the beam
\( D \) Dissipated energy
\( d \) Effective depth
\( d_{\text{core}} \) Diameter of drilled core
\( d_{\text{cylin}} \) Diameter of cylinder
\( E \) Young’s modulus
\( E_{\parallel} \) Young’s modulus parallel to the ASR-induced macrocracks (concrete)
$E_\perp$  Young’s modulus perpendicular to the ASR-induced macrocracks (concrete)

$E_{\text{m}}$  Mean value of measured Young’s modulus

$E_s$  Young’s modulus for reinforcement steel

$F$  Load

$f_{c,\text{measured}}$  Measured concrete compressive strength

$f_{c,\text{std}}$  Concrete compressive strength for a standard cylinder

$f_{c,\text{und}}$  Undamaged concrete compressive strength

$f_{\text{ct,0,ref}}$  Uniaxial tensile strength of the concrete perpendicular to the axis of the structural component

$f_{\text{ct,90}}$  Uniaxial tensile strength of the concrete parallel to the axis of the structural component

$F_c$  Resultant of the compressive forces

$f_c$  Concrete compressive strength

$F_s,F_s'$  Resultant of the tensile forces

$f_{\text{t,ef}}$  Effective tensile strength

$f_t$  Concrete tensile strength

$f_{u\text{m}}$  Mean value of measured ultimate stress

$F_u$  Ultimate applied load

$f_{y\text{m}}$  Mean value of measured yield stress

$f_y$  Yield stress

$f_{c,0}$  Core compressive strength parallel to the casting direction

$f_{c,90}$  Core compressive strength perpendicular to the casting direction

$f_{c,\text{cylin}}$  Reference cylinder strength

$f_{c||,\text{ef}}$  Effective concrete compressive strength parallel to the ASR-induced macrocracks

$f_{c||}$  Concrete compressive strength parallel to the ASR-induced macrocracks

$f_{c\perp,\text{ef}}$  Effective concrete compressive strength perpendicular to the ASR-induced macrocracks
$f_{c\perp}$  Concrete compressive strength perpendicular to the ASR-induced macrocracks

$f_{t\parallel,ef}$  Effective concrete tensile strength parallel to the ASR-induced macrocracks

$f_{t\parallel}$  Concrete tensile strength parallel to the ASR-induced macrocracks

$f_{t\perp,ef}$  Effective concrete tensile strength perpendicular to the ASR-induced macrocracks

$f_{t\perp}$  Concrete tensile strength perpendicular to the ASR-induced macrocracks

$h$  Beam height

$h_A, h_B$  Height of yield line

$h_{core}$  Height of drilled core

$h_{cylin}$  Height of cylinder

$h_I, h_{III}, h_{III}$  Height of crack/yield line

$I$  Moment of inertia - Second moment of area

$L_1$  Length of left shear zone for the continuous beams

$L_2$  Length of right shear zone for the continuous beams

$L_L$  Length of left anchorage zone

$L_R$  Length of right anchorage zone

$l_{s,max}$  Minimum crack spacing

$M_1, M_2$  Applied moment

$M_{1,crack}, M_{2,crack}$  Cracking moment

$M_{R,mean}$  Theoretical moment capacity

$M_u$  Ultimate applied moment

$P, P_1, P_2$  Load

$P_{BS,II}$  Failure load for Brazilian split test in direction II

$P_{BS,I}$  Failure load for Brazilian split test in direction I

$P_{fail}$  Failure load

$P_{max}$  Maximum applied load

$P_{mom}$  Theoretical load-carrying capacity governed by the moment capacity
$P_{\text{shear}}$ Theoretical load-carrying capacity governed by the shear capacity

$P_{\text{test}}$ Tested load-carrying capacity

$P_a$ Ultimate load

$P_i$ External load in point $i$

$R_1, R_2$ Reaction force

$S$ First moment of area

$s(h)$ Factor accounting for size effect

$u$ Deflection, in Chapter 7

$u_{2,\text{max}}$ Maximum deformation, in Chapter 7

$u_{\text{max}}$ Maximum deflection measured at the position of load $P_2$, in Chapter 7

$u_i$ Displacement in point $i$

$V$ Shear force

$V_1$ Shear force in left shear zone for the continuous beams

$V_2$ Shear force in main shear zone for the continuous beams

$V_3$ Shear force in right shear zone for the continuous beams

$V_{2,\text{max}}$ Maximum applied shear force in the main shear zone for the continuous beams

$V_{\text{agg}}$ Contribution to the shear capacity from aggregate interlock

$V_a$ Absolute volumetric proportions of air

$V_{\text{compr}}$ Contribution to the shear capacity from compression zone

$V_c$ Absolute volumetric proportions of cement

$V_{\text{dow}}$ Contribution to the shear capacity from dowel action

$V_{\text{max}}$ Maximum applied shear force for the simply supported beams

$V_{\text{res}}$ Contribution to the shear capacity from residual tensile strength

$V_R$ Shear capacity

$V_w$ Absolute volumetric proportions of water

$W$ Work

$w/c$ Water/cement ratio
\begin{itemize}
  \item \( W_E \) \quad External work
  \item \( W_{IA}, W_{IB} \) \quad Contribution from the crack/yield line A and B to the internal work
  \item \( W_{i,\text{compressive}} \) \quad Dissipation for pure compression
  \item \( W_{i,\text{tensile}} \) \quad Dissipation for pure tension
  \item \( W_{i} \) \quad Internal work
  \item \( W_i \) \quad Contribution from the cracks/yield lines to the internal work
  \item \( x_1, x_2 \) \quad Position of rotation points
  \item \( x_{0,\text{max}} \) \quad Distance from the reaction to the point with maximum moment
  \item \( x_0 \) \quad Length of main shear zone for the continuous beams
  \item \( x_A, x_B \) \quad Length of yield line
  \item \( y_I, y_{III} \) \quad Position of rotation points
  \item \( u \) \quad Displacement vector
\end{itemize}
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B.1 Concrete compressive strengths perpendicular to the ASR-induced cracks for cores from Gammelrand bridge. 

B.2 Concrete compressive strengths parallel to the ASR-induced cracks for cores from Gammelrand bridge. 

C.1 Concrete compressive strengths perpendicular to the ASR-induced cracks for cores from Slab segment 1 to 3 from Lindenborg bridge. 

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D.2 Concrete compressive strengths for Mix B from Test series 1. The height of the cores is provided in brackets. 

D.3 Concrete compressive strengths from Test series 2. 


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D.6 TS2-RG1: Estimates of regression coefficients. 

D.7 TS2-RM2: Estimates of regression coefficient. 

D.8 TS1-RM1: $p$-values for regression coefficients and $R^2$. 

D.9 $p$-values for regression coefficients and $R^2$ for the first regression model for TS1 without outliers. 

D.10 TS1-RM2: $p$-values for regression coefficients and $R^2$. 

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D.12 TS2-RM1: $p$-values for regression coefficients and $R^2$. 

D.13 TS2-RM1: $p$-values for regression coefficients and $R^2$ for the first regression model for TS2 where the outlier is excluded. 

D.14 TS2-RM2: $p$-values for regression coefficients and $R^2$. 

D.15 TS2-RM2: $p$-values for the regression coefficients and $R^2$ for the second regression model for TS2 where the outlier is excluded.
Appendices
Appendix A

Published research and cooperations

A.1 Papers

Parts of this PhD thesis are already published in scientific papers. Due to the copyrights of the published papers, the papers and the associated co-author statements are appended in a separated document. This appendix provides a list over the published papers and which parts of the thesis that are published. As the papers are prepared with colleagues, the list includes information on the contribution of the author of this thesis.

Paper 1

Title: Prestressing of reinforcing bars in concrete slabs due to concrete expansion induced by Alkali-Silica Reaction,
Publication details: In the proceedings to fib Symposium 2016: Performance-based approaches for concrete structures.,
Authors: Søren Gustenhoff Hansen, Ricardo Antonio Barbosa, Linh Cao Hoang,

Involved sections:
This paper (Hansen et al., 2016a) presents a part of the literature study that regards the ASR-induced cracks and expansion (Section 2.1), a part of the phenomenological explanation of ASR-induced crack growth in 2D-restrained slabs (Chapter 3) and the quantification of the prestressing for Lindeborg bridge (shown in Section 7.1.1) and Gammelrand bridge shown in Section 7.1.3).

Author contributions:
I (the author of this PhD thesis) am the main author of this paper. The literature study, the establishment of the phenomenological explanation and the discussion of the test results are conducted by me. The quantification of the prestressing are planned and conducted by former PhD student Ricardo Antonio Barbosa.
Paper 2
Title: Influence of alkali-silica reaction and crack orientation on the uniaxial compressive strength of concrete cores from slab bridges,
Publication details: Construction and Building materials, Volume 176, July 2018, Pages 440-451,
Authors: Ricardo Antonio Barbosa, Søren Gustenhoff Hansen, Kurt Kielsgaard Hansen, Linh Cao Hoang, Bent Grek

Involved sections:
This paper (Barbosa et al., 2018a) presents the quantification of the anisotropy of the compressive strength in Lindenborg bridge and Gammelrand bridge (Section 5.1) .

Author contributions:
I am co-author of this paper. The experiments are conducted and planned by former PhD student Ricardo Antonio Barbosa. Ricardo Antonio Barbosa and me contributed equally in the scientific interpretations and discussions of the experimental results.

Paper 3
Title: Residual shear strength of a severely ASR-damaged flat slab bridge,
Publication details: Engineering Structures, Volume 161, April 2018, Pages 82-95, Authors: Ricardo Antonio Barbosa, Søren Gustenhoff Hansen, Linh Cao Hoang, Kurt Kielsgaard Hansen,

Involved sections:
This paper (Barbosa et al., 2018b) presents the shear experiments with the beams from Lindenborg bridge (Section 7.1.1.2 and 7.1.1.3).

Author contributions:
I am co-author of this paper. The experiments are designed by me and conducted in cooperation with Ricardo Antonio Barbosa. I did all the DIC measurements and analysis.

Paper 4
Title: Experimental and statistical investigation of the compressive strength anisotropy in structural concrete,
Publication details: Cement and Concrete Research, Volume 107, May 2018, Pages 304-316,
Authors: Søren Gustenhoff Hansen, Jørgen Trankjæer Lauridsen, Linh Cao Hoang,

Involved sections:
This paper (Hansen et al., 2018) presents the experimental and statistical investigation of the compressive strength anisotropy in sound concrete (Subsection 5.2.2).
Author contributions:
I am the main of this paper. I have planned and conducted all the experiments and conducted the statistical analysis of the results.

A.2 Cooperations

Parts of the research presented in this PhD thesis are conducted in cooperation with colleagues. The list below shows an overview of the sections where others have contributed to. The sections that do not appears on the list, is conducted by me, exclusively.

- The experiments presented in Section 5.1 are conducted in cooperation with Ricardo Antonio Barbosa, see Paper 2.
- The experiments presented in Section 5.4.2 are conducted by Alberg and Petersen (2014). However, all DIC analysis and interpretations of the test results are conducted by me.
- The shear experiments presented in Section 7.1.1.2 are conducted in cooperation with Ricardo Antonio Barbosa, see Paper 3.
- The shear experiments presented in Section 7.1.3 are planed and conducted by Jensen and Rask (2014). However, all DIC analysis and post-processing of the test results are conducted by me.
Appendix B

Appendix, Gammelrand

This appendix provides the more details on the experiments from Gammelrand presented in Section 7.1.3. Section B.1 presents the measured compressive strengths, which are only presented graphically in Section 5.1.5.1. Section B.2 presents the outcome of an investigation of the exact reinforcement configuration in the 4 beams.

B.1 Compressive strength

<table>
<thead>
<tr>
<th>Height [mm]</th>
<th>Diameter [mm]</th>
<th>Measured $f_{c\perp}$ [MPa]</th>
<th>Converted $f_{c\perp}$ [MPa]</th>
</tr>
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<tr>
<td>200</td>
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</tr>
<tr>
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<td>15.5</td>
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<td>200</td>
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<tr>
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<td>15.3</td>
<td>16.7</td>
</tr>
<tr>
<td>Std. dev.</td>
<td></td>
<td>1.4</td>
<td>1.5</td>
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</table>

*Table B.1: Concrete compressive strengths perpendicular to the ASR-induced cracks for cores from Gammelrand bridge.*
<table>
<thead>
<tr>
<th>Height [mm]</th>
<th>Diameter [mm]</th>
<th>Measured $f_{c\parallel}$ [MPa]</th>
<th>Converted $f_{c\parallel}$ [MPa]</th>
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<td>17.1</td>
<td>18.7</td>
</tr>
</tbody>
</table>

| Mean       | 19.3         | 21.0                          |
| Std. dev.  | 3.6          | 3.9                           |

*Table B.2: Concrete compressive strengths parallel to the ASR-induced cracks for cores from Gammelbrand bridge.*
B.2 Actual reinforcement

Figure B.1: Reinforcement in the critical shear zone in Beam 1 from Gammelrand bridge.
Figure B.2: Reinforcement in the critical shear zone in Beam 2 from Gammelrand bridge.
Figure B.3: Reinforcement in the critical shear zone in Beam 3 from Gammelrand bridge.
Figure B.4: Reinforcement in the critical shear zone in Beam 4 from Gammelrand bridge.
Appendix C

Lindenborg

This appendix provides the more details on the experiments from Lindenborg presented in Section 7.1.1. Section C.1 presents the measured compressive strengths, which are also presented graphically in Section 5.1.5.2. Section C.2 presents the outcome of an investigation of the exact reinforcement configuration in the 18 beams. Section C.3 presents the crack propagation for the 18 beams by means of DIC-generated plots for important load stages.
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<thead>
<tr>
<th>Slab segment</th>
<th>Height [mm]</th>
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*Table C.2: Concrete compressive strengths perpendicular to the ASR-induced cracks for cores from Slab segment 4 to 6 from Lindenborg bridge.*
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*Table C.3: Concrete compressive strengths parallel to the ASR-induced cracks for cores from Slab segment 1 to 3 from Lindenborg bridge.*
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Table C.4: Concrete compressive strengths parallel to the ASR-induced cracks for cores from Slab segment 4 to 6 from Lindenborg bridge.
C.2 Actual reinforcement

Figure C.1: Reinforcement in Beam 1.1 from Lindenborg bridge.

Figure C.2: Reinforcement in Beam 1.2 from Lindenborg bridge.
Figure C.3: Reinforcement in Beam 1.3 from Lindenborg bridge.

Figure C.4: Reinforcement in Beam 2.1 from Lindenborg bridge.
Figure C.5: Reinforcement in Beam 2.2 from Lindenborg bridge.

Figure C.6: Reinforcement in Beam 2.3 from Lindenborg bridge.
Figure C.7: Reinforcement in Beam 3.1 from Lindenborg bridge.

Figure C.8: Reinforcement in Beam 3.2 from Lindenborg bridge.
Figure C.9: Reinforcement in Beam 3.3 from Lindenborg bridge.

(a) Top reinforcement.

(b) Bottom reinforcement.

Figure C.10: Reinforcement in Beam 4.1 from Lindenborg bridge.

(a) Top reinforcement.

(b) Bottom reinforcement.
Figure C.11: Reinforcement in Beam 4.2 from Lindenborg bridge.

Figure C.12: Reinforcement in Beam 4.3 from Lindenborg bridge.
Figure C.13: Reinforcement in Beam 5.1 from Lindenborg bridge.

Figure C.14: Reinforcement in Beam 5.2 from Lindenborg bridge.
Figure C.15: Reinforcement in Beam 5.3 from Lindenborg bridge.

Figure C.16: Reinforcement in Beam 6.1 from Lindenborg bridge.
Figure C.17: Reinforcement in Beam 6.2 from Lindenborg bridge.

Figure C.18: Reinforcement in Beam 6.3 from Lindenborg bridge.
C.3 Crack propagation by DIC

Figure C.19: Shear-deflection curves for the simple supported beams
1) Inclined crack ($V=80.1$ kN)

2) Failure ($V=111.9$ kN)

Figure C.20: Major strain plot from a DIC analysis: Crack propagation for Beam 2.1 during the test. $x/h = 3.3$
Figure C.21: Major strain plot from a DIC analysis: Crack propagation for Beam 3.1 during the test. $x/h = 3.3$
1) Bending crack ($V=58.7$ kN)

2) Failure ($V=75.0$ kN)

---

1) Bending crack ($V=60.3$ kN)

2) Inclined crack ($V=108.0$ kN)

3) Failure ($V=131.2$ kN)

---

Figure C.22: Major strain plot from a DIC analysis: Crack propagation for Beam 6.1 during the test. $x/h = 3.3$

Figure C.23: Major strain plot from a DIC analysis: Crack propagation for Beam 2.2 during the test. $x/h = 2.8$
1) Bending crack ($V=86.3$ kN)

2) Inclined crack ($V=97.7$ kN)

3) Failure ($V=117.5$ kN)

*Figure C.24: Major strain plot from a DIC analysis: Crack propagation for Beam 3.2 during the test. $x/h = 2.8*$
1) Bending and inclined crack ($V=67.7$ kN)

2) Abrupt increase of crack width in the inclined crack ($V=80.6$ kN)

3) Failure ($V=98.0$ kN)

Figure C.25: Major strain plot from a DIC analysis: Crack propagation for Beam 6.2 during the test. $x/h = 2.8$
1) Bending crack ($V=105.6 \text{ kN}$)

2) More cracks ($V=128.8 \text{ kN}$)

3) Failure ($V=162.0 \text{ kN}$)

Figure C.26: Major strain plot from a DIC analysis: Crack propagation for Beam 2.3 during the test. $x/h = 2.3$
Figure C.27: Major strain plot from a DIC analysis: Crack propagation for Beam 3.3 during the test. $x/h = 2.3$

1) Bending crack ($V = 93.6$ kN)

2) Inclined crack, left ($V = 112.9$ kN)

3) Inclined crack, right ($V = 127.2$ kN)

4) Failure ($V = 130.2$ kN)
1) Horizontal cracks ($V=94.9$ kN)

2) Failure ($V=114.4$ kN)

Figure C.28: Crack plot: Crack propagation for Beam 6.3 during the test. $x/h = 2.3$
Figure C.29: Shear-deflection curves for the continuous beams
1) Bending and Inclined cracks ($V=110.9$ kN)

2) Inclined crack opens ($V=111.0$ kN)

3) Inclined crack opens and formation of midspan bending cracks ($V=144.6$ kN)

4) Failure ($V=146.5$ kN)

Figure C.30: Major strain plot from a DIC analysis: Crack propagation for Beam 1.3 during the test. $x/h = 3.0$
1) Inclined crack ($V=71.8$ kN)

2) Inclined crack extended and crack in compression zone (top) ($V=88.3$ kN)

3) Crack opens and formation of inclined crack (right side) ($V=110.6$ kN)

4) Failure ($V=115.7$ kN)

Figure C.31: Major strain plot from a DIC analysis: Crack propagation for Beam 5.3 during the test. $x/h = 3.0$
1) Bending cracks ($V=93.5$ kN)

2) Diagonal cracks ($V=90.4$ kN)

3) Inclined crack opens and formation of midspan bending cracks ($V=101.2$ kN)

4) Failure ($V=102.3$ kN)

*Figure C.32: Major strain plot from a DIC analysis: Crack propagation for Beam 1.2 during the test. $x/h = 4.0$*
1) Bending and inclined cracks ($V=81.1$ kN)

2) Inclined crack (right shear zone) ($V=90.3$ kN)

3) Inclined crack opens abrupt (main shear zone) ($V=79.8$ kN)

4) Inclined crack opens and formation of midspan bending cracks ($V=97.6$ kN)

5) Failure in compression zone (top) ($V=100.1$ kN)

Figure C.33: Major strain plot from a DIC analysis: Crack propagation for Beam 4.3 during the test. $x/h = 4.0$
1) Bending cracks ($V=76.4$ kN)

2) Inclined cracks ($V=93.1$ kN)

3) Extension of inclined crack and formation of midspan bending cracks ($V=102.2$ kN)

4) Failure, cracks between the two inclined cracks ($V=94.5$ kN)

*Figure C.34: Major strain plot from a DIC analysis: Crack propagation for Beam 5.2 during the test. $x/h = 4.0$*
1) Inclined crack ($V=79.1 \text{ kN}$)

2) Inclined crack opens and formation of midspan bending cracks ($V=88.8 \text{ kN}$)

3) Failure ($V=68.5 \text{ kN}$)

\textit{Figure C.35: Major strain plot from a DIC analysis: Crack propagation for Beam 1.1 during the test. $x/h = 5.0$}
1) Bending and horizontal crack ($V=65.4$ kN)

2) Cracks open ($V=92.7$ kN)

3) Inclined crack and crack in compression zone (top) ($V=83.3$ kN)

4) Failure in compression zone (top) ($V=85.0$ kN)

*Figure C.36: Major strain plot from a DIC analysis: Crack propagation for Beam 4.2 during the test. $x/h = 5.0$*
1) Inclined crack ($V=70.2$ kN)

2) Inclined crack opens and formation of midspan bending cracks ($V=65.6$ kN)

3) Failure in compression zone (top) ($V=81.7$ kN)

*Figure C.37: Major strain plot from a DIC analysis: Crack propagation for Beam 5.1 during the test. $x/h = 5.0$*
1) Bending crack (${V=57.2 \text{ kN}}$)

2) Bending and inclined cracks (${V=83.7 \text{ kN}}$)

3) Failure (${V=83.4 \text{ kN}}$)

*Figure C.38: Major strain plot from a DIC analysis: Crack propagation for Beam 4.1 during the test. $x/h = 6.0$*
Appendix D

Sound concrete

This appendix provides the background for the experimental and statistical investigation of anisotropy in sound concrete presented in Section 5.2. Section D.1 presents the measured compressive strength from Test series 1 and 2. Section D.2 presents approach for the statistical analysis. Section D.3 and D.4 present the results of the statistical analyses whilst Section D.5 presents the diagnostics for the statistical analyses.
### D.1 Compressive strength

| Unreinforced | | Reinforced | | |
|--------------|--------------|--------------|--------------|--------------|--------------|
| $f_{c,0}$ [MPa] | $f_{c,90}$ [MPa] | $f_{c,0}$ [MPa] | $f_{c,90}$ [MPa] | $f_{c,\text{cylin}}$ [MPa] | |
| #1 | 33.5 (195.4) | 29.4 (195.5) | 39.9 (194.4) | 31.8 (195.2) | 44.5 | |
| #2 | 37.4 (195.4) | 26.7 (194.9) | 37.8 (194.8) | 39.6 (194.8) | 41.9 | 43.5 |
| #3 | 39.1 (195.5) | 31.8 (195.2) | 37.7 (194.9) | 23.5 (195.2) | 44.1 | |
| #4 | 39.0 (195.3) | 29.0 (194.8) | 36.0 (195.2) | 32.2 (195.0) | 44.6 | 42.5 |
| #5 | 38.1 (195.5) | 30.4 (195.3) | 41.2 (194.0) | 34.1 (195.4) | | |
| #6 | 34.5 (195.2) | 28.7 (194.5) | 41.5 (194.4) | 41.6 (194.6) | | |
| #7 | 36.2 (195.4) | 35.4 (194.6) | 36.9 (193.6) | 38.9 (193.8) | | |
| #8 | 35.9 (193.2) | 33.6 (195.4) | 37.2 (194.8) | 33.1 (195.1) | | |
| #9 | 33.8 (195.4) | 31.3 (195.5) | 40.0 (195.4) | 34.2 (195.4) | | |
| #10 | 35.6 (195.7) | 27.2 (195.3) | 38.7 (195.2) | 36.5 (195.2) | | |
| #11 | 37.3 (194.9) | 29.3 (195.5) | 36.8 (195.2) | 31.2 (195.3) | | |
| #12 | 31.7 (195.7) | 31.9 (195.6) | 39.4 (195.1) | 35.5 (195.1) | | |
| #13 | 33.2 (195.6) | 30.5 (193.4) | | | | |
| #14 | 35.2 (195.5) | | | | | |
| #15 | 31.1 (195.7) | | | | | |
| #16 | 36.8 (195.2) | | | | | |
| #17 | 28.5 (195.5) | | | | | |
| #18 | 32.4 (195.6) | | | | | |
| #19 | 33.3 (195.7) | | | | | |

Table D.1: Concrete compressive strengths for Mix A from Test series 1. The height of the cores is provided in brackets.
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<td>51.6 (199.2)</td>
<td>45.1 (200.7)</td>
<td>63.3 (196.5)</td>
<td>49.3 (192.0)</td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>56.6 (198.3)</td>
<td>54.1 (198.4)</td>
<td>62.9 (197.7)</td>
<td>48.9 (195.1)</td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>50.5 (198.4)</td>
<td>57.6 (197.8)</td>
<td>57.6 (197.7)</td>
<td>55.7 (198.8)</td>
<td></td>
</tr>
<tr>
<td>#12</td>
<td>52.9 (198.5)</td>
<td>42.5 (197.9)</td>
<td>59.7 (197.0)</td>
<td>54.6 (197.5)</td>
<td></td>
</tr>
<tr>
<td>#13</td>
<td>44.0 (200.2)</td>
<td>46.7 (197.7)</td>
<td>60.7 (197.5)</td>
<td>50.8 (197.5)</td>
<td></td>
</tr>
<tr>
<td>#14</td>
<td>51.6 (196.0)</td>
<td>46.7 (197.7)</td>
<td>58.8 (197.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#15</td>
<td>52.0 (196.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#16</td>
<td>48.0 (199.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#17</td>
<td>48.3 (199.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table D.2: Concrete compressive strengths for Mix B from Test series 1. The height of the cores is provided in brackets.

<table>
<thead>
<tr>
<th>Curing time [weeks]</th>
<th>$f_{c,0}$ [MPa]</th>
<th>$f_{c,90}$ [MPa]</th>
<th>$f_{c,\text{cyl}}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>45.3</td>
<td>49.5</td>
<td>49.8</td>
</tr>
<tr>
<td>#2</td>
<td>41.3*</td>
<td>49.9*</td>
<td>49.7</td>
</tr>
<tr>
<td>#3</td>
<td>42.3*</td>
<td>42.4*</td>
<td>46.2</td>
</tr>
<tr>
<td>#4</td>
<td>48.1</td>
<td>51.0*</td>
<td>46.2</td>
</tr>
<tr>
<td>#5</td>
<td>52.1*</td>
<td>56.2</td>
<td>49.8</td>
</tr>
<tr>
<td>#6</td>
<td></td>
<td></td>
<td>58.8</td>
</tr>
</tbody>
</table>

* Grinded in both ends

Table D.3: Concrete compressive strengths from Test series 2.
D.2 Approach for the statistical analysis

D.2.1 Regression models

Regression models are used to predict the value of a certain response variable (here, core compressive strength) based on the values of certain explanatory variables (here, the drilling direction and the design parameters). Linear regression models assume that the influences, or effects, of these explanatory variables can be added according to a linear model. Regression models may account for both main effects and interaction effects. The main effects regard the direct influence of the explanatory variables on the response variable (e.g. the direct increase of the core compressive strength with the reference cylinder strength), while the interaction effects account for the case where the effect of a variable is depending on the level of another variable.

D.2.2 General form

The general form of the regression model adopted in this research is provided in Equation D.1. The model predicts the core compressive strength ($f_c$) on the basis of the drilling direction ($x_{dir}$) and the design parameters of interest ($x_1, ..., x_n$). The regression model accounts for all main effects. As the main interest goes to the dependency of the core compressive strength as a function of drilling direction, the model additionally accounts for the interaction effects between the drilling direction ($x_{dir}$) and the design parameters of interest ($x_1, ..., x_n$). The general form of the regression model can be described by:

$$f_c = \beta_0 + \beta_{dir} x_{dir} + \sum_{i=1}^{n} \beta_i x_i + \theta_i x_i x_{dir} \pm \varepsilon$$ (D.1)

The parameters $\beta_0$ to $\beta_n$ are the regression coefficients for the main effects and the parameters $\theta_1$ to $\theta_n$ are the regression coefficients for the interaction effects. The term $\varepsilon$ represents the random error. Depending on the performed statistical analysis, the regression model may consist of both quantitative and qualitative explanatory variables. In this research, the drilling direction ($x_{dir}$), the presence of reinforcement ($x_\rho$) and the reference cylinder strength, represented by the type of concrete mix, ($x_{mix}$) are taken as qualitative explanatory variables, where the curing time ($x_T$) is taken as a quantitative explanatory variable. The program used for analysis (JMP) uses effect coding to include qualitative explanatory variables, where it typifies binary qualitative variables with states -1 and 1. For example, the explanatory variable $x_{dir}$ takes the following values:

$$x_{dir} = \begin{cases} 
1, & \text{in case of core}_{90} \\
-1, & \text{in case of core}_0 
\end{cases}$$ (D.2)

The effect coding is used instead of dummy (0-1) coding, because in that case the coefficient provides the difference from overall mean for the two cores, rather than the difference between them.
D.2.3 Quantification of anisotropy

To quantify the anisotropy, $f_c$ needs to be predicted for both drilling directions, i.e. $f_{c,90}$ and $f_{c,0}$. By substituting Equation D.2 in the regression model (Equation D.1), this results in:

$$f_{c,90} = \beta_0 - \beta_{\text{dir}} + \sum_{i=1}^{n} \beta_i x_i - \theta_i x_i \pm \varepsilon$$

$$f_{c,0} = \beta_0 + \beta_{\text{dir}} + \sum_{i=1}^{n} \beta_i x_i + \theta_i x_i \pm \varepsilon$$

When implemented in the definition of anisotropy ($\Delta f_c = f_{c,0} - f_{c,90}$), this results in the following prediction formula for the anisotropy:

$$\Delta f_c = 2 \beta_0 + 2 \sum_{i=1}^{n} \theta_i x_i$$

This means that the anisotropy, $\Delta f_c$, can be predicted as a function of the obtained regression coefficients $\beta_{\text{dir}}$ and $\{\theta_i\}$ combined with the values for the design parameters $\{x_i\}$. This means that the regression coefficients $\{\theta_i\}$ provide an estimate of the influence of their corresponding design parameters $\{x_i\}$ on the anisotropy. The total influence of the design parameters on the anisotropy is estimated by:

$$\Delta f_{c,0} = 2 \sum_{i=1}^{n} \theta_i x_i$$

D.2.4 Statistical inference

In the first analysis of each test series, the influence of the design parameters on the anisotropy is investigated. For this, the full regression model as provided in Equation D.1 is evaluated. The influence of the design parameters is assessed through the (i) significance and (ii) the magnitude of the regression coefficients of the interaction effects $\{\theta_i\}$. In the second analysis, the anisotropy is investigated. This is done on the basis of an updated regression model, where the insignificant effects of the design parameters found from the first analysis are excluded. The anisotropy is assessed on its (i) significance and (ii) its magnitude.

D.2.5 Diagnostics

For each regression analysis, diagnostics are performed and only regression models where all regression assumptions are valid are used, including assessment of normal distribution of the residual, which is a condition for valid statistical conclusions. Section D.3 and D.4 provide overview of the statistical calculations and the results. The actual calculations and diagnostics are shown in Section D.5.
D.3 Statistical analysis, Test series 1

D.3.1 Influence of the design parameters on the anisotropy

The following regression model is adopted for the first data analysis:

\[
f_c = \beta_0 + \beta_{\text{dir}}x_{\text{dir}} + \beta_{\text{mix}}x_{\text{mix}} + \beta_{\rho}x_{\rho} + \theta_{\text{mix}}x_{\text{mix}}x_{\text{dir}} + \theta_{\rho}x_{\rho}x_{\text{dir}} \pm \varepsilon \quad (D.6)
\]

Interest goes to the regression coefficients \(\theta_{\text{mix}}\) and \(\theta_{\rho}\), which provide an estimate on the influence of the reference cylinder strength \((f_{c,\text{cylin}})\), represented by the type of concrete mix, and the presence of reinforcement on the anisotropy respectively. Their estimates and corresponding \(p\)-values are provided in Table D.4, for all other variables see extended analysis in Section D.5.1.

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>Estimate</th>
<th>(p)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_{\text{mix}})</td>
<td>0.29 MPa</td>
<td>0.38</td>
</tr>
<tr>
<td>(\theta_{\rho})</td>
<td>0.61 MPa</td>
<td>0.065</td>
</tr>
</tbody>
</table>

Table D.4: TS1-RM1: Estimates of regression coefficients.

The influence of \(f_{c,\text{cylin}}\) and the presence of reinforcement is assessed through the (i) significance and (ii) the magnitude of the regression coefficients \(\theta_{\text{mix}}\) and \(\theta_{\rho}\):

(i) Both regression coefficients have a \(p\)-value above 5%. This means that there is not sufficient evidence to state that \(f_{c,\text{cylin}}\) or the presence of reinforcement have a significant influence on the anisotropy.

(ii) The total influence of the design parameters on the anisotropy is quantified by (see Equation D.5):

\[
\Delta f_c = 2(\theta_{\text{mix}}x_{\text{mix}} + \theta_{\rho}x_{\rho}) \quad (D.7)
\]

where \(x_{\text{mix}}\) takes the value 1 in the case of Mix A and the value -1 in the case of Mix B and \(x_{\rho}\) takes the value 1 in case of reinforced concrete and the value -1 in the case of unreinforced concrete. A visualisation of Equation D.7 is displayed in Figure D.1. The anisotropy is slightly larger for Mix A than for Mix B (difference of 1.2 MPa). The anisotropy is slightly larger in the case of reinforced concrete than for unreinforced concrete (difference of 2.4 MPa).
Figure D.1: TS1-RM1: Estimated influence of the reference cylinder strength and the presence of reinforcement on the anisotropy.

Based on the evaluation of the significance as well as the magnitude, it can be concluded that the influence of $f_{c,cylin}$ and the presence of reinforcement on the anisotropy is small. The regression analysis identifies 4 possible outliers, see Section D.5.1. A regression analysis where the possible outliers are excluded shows the same as the shown analysis. The $p$-value for $\theta_{mix}$ and $\theta_{\rho}$ is found even higher. The outliers are excluded in the further analysis.

### D.3.2 Significance and magnitude of the anisotropy

Since the first regression analysis shows that the influence of the $f_{c,cylin}$ and the presence of reinforcement on the anisotropy is insignificant, the anisotropy is further studied by a regression model without the interaction effects. The following regression model is adopted:

$$f_c = \beta_0 + \beta_{dir}x_{dir} + \beta_{mix}x_{mix} + \beta_{\rho}x_{\rho} \pm \varepsilon$$  \hspace{1cm} (D.8)

Interest goes to the regression coefficient $\beta_{dir}$, which provides an estimate of the anisotropy. The estimate and corresponding $p$-value of the regression coefficient are provided in Table D.5.
The anisotropy is assessed through the (\(i\)) significance and (\(ii\)) the magnitude of the regression coefficient \(\beta_{\text{dir}}\):

\(i\) The regression coefficient shows an extremely low \(p\)-value. This means that there is an extremely strong evidence that the concrete core compressive strength is anisotropic.

\(ii\) The magnitude of the anisotropy is quantified by Equation D.4 without the interaction effects:

\[ \Delta f_c = 2\beta_{\text{dir}} \]  \hspace{1cm} (D.9)

The mean of the anisotropy of the concrete core compressive strength is 4.5 MPa and the corresponding 95\% confidence interval is [3.4 to 5.7 MPa].

If the outliers are included in the analysis the mean anisotropy is found to 4.3 MPa, which is close to 4.5 MPa.

### D.4 Statistical analysis, Test series 2

#### D.4.1 Influence of the design parameters on the anisotropy

The following regression model is adopted for the first data analysis:

\[ f_c = \beta_0 + \beta_{\text{dir}} x_{\text{dir}} + \beta_{\text{time}} x_{\text{time}} + \theta_{\text{time}} x_{\text{time}} x_{\text{dir}} \pm \varepsilon \]  \hspace{1cm} (D.10)

Interest goes to the regression coefficients \(\theta_{\text{time}}\), which provide an estimate on the influence of the curing time on the anisotropy. The estimate and corresponding \(p\)-values are provided in Table D.4.

<table>
<thead>
<tr>
<th>(\theta_{\text{time}})</th>
<th>Estimate</th>
<th>(p)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_{\text{time}})</td>
<td>0.05</td>
<td>0.652</td>
</tr>
</tbody>
</table>

Table D.6: TS2-RG1: Estimates of regression coefficients.

The influence of the curing time is assessed through the (\(i\)) significance and (\(ii\)) the magnitude of the regression coefficients \(\theta_{\text{time}}\):

\(i\) The regression coefficient has a \(p\)-value above 5\%. This means that there is not sufficient evidence to state that the curing time has a significant influence on the anisotropy.

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The total influence of the curing time on the anisotropy is quantified by (see Equation D.5):

$$\Delta f_{c\theta} = 2\theta_{\text{time}}x_{\text{time}}$$  \hspace{1cm} (D.11)

where $x_{\text{time}}$ is the curing time in weeks. A visualisation of Equation D.11 is displayed in Figure D.2. The anisotropy is slightly larger after 12 weeks than after 2 weeks (difference of 0.9 MPa).

![Figure D.2: TS2-RM1: Estimated influence of the curing time on the anisotropy.](image)

Based on the evaluation of the significance as well as the magnitude, it can be concluded that the influence of the curing time on the anisotropy is small.

### D.4.2 Significance and magnitude of the anisotropy

Since the first regression analysis shows that the influence of the curing time on the anisotropy is insignificant, the anisotropy is further studied by a regression model without the interaction effects. The following regression model is adopted:

$$f_c = \beta_0 + \beta_{\text{dir}}x_{\text{dir}} + \beta_{\text{time}}x_{\text{time}} \pm \varepsilon$$  \hspace{1cm} (D.12)

Interest goes to the regression coefficient $\beta_{\text{dir}}$, which provides an estimate on the anisotropy. The estimate and corresponding $p$-value of the regression coefficient are provided in Table D.7.
<table>
<thead>
<tr>
<th>Estimate</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{dir}$</td>
<td>-1.26</td>
</tr>
</tbody>
</table>

*Table D.7: TS2-RM2: Estimates of regression coefficient.*

The anisotropy is assessed through the $(i)$ significance and $(ii)$ the magnitude of the regression coefficients $\beta_{dir}$:

1. The regression coefficient has a $p$-value lower than 5%. This means that there is strong evidence that the concrete core compressive strength is anisotropic.

2. The anisotropy is quantified by Equation D.4 without the interaction effects:

   $$\Delta f_c = 2\beta_{dir} \quad (D.13)$$

   The mean of the anisotropy of the compressive strength is -2.5 MPa and the corresponding 95% confidence interval is [-0.9 to -4.1 MPa]. It is noted that the anisotropy is negative, contrary to Test series 1. This means that $f_{c,0}$ is smaller than $f_{c,90}$.

### D.5 Statistical analysis, diagnostics

This section contains the background calculations for the statistical analysis provided in Section D.3 and D.4. For the statistical analysis, multiple linear regression models are employed. These analyses are based on following four (regression) assumptions:

1. The mean of the error, $\varepsilon$, is 0.
2. The variance of $\varepsilon$ is constant and independent of the value of the independent variables.
3. $\varepsilon$ has a normal distribution.
4. $\varepsilon$ is statistically independent.

$\varepsilon$ is the difference between an observed value and a true value. Since the true value is unknown, $\varepsilon$ may be estimated by residuals that is the difference an observed value and a predicted values. The residuals are employed to check the validity of the regression assumptions. The assumptions will be verified under the each statistical analysis.
D.5.1 Test series 1

This section contains a statistical analysis of the data obtained in Test series 1. Figure D.3 shows a box plot of the measured compressive strengths \( f_c \). The blue boxes are for \( \text{core}_\parallel \) and the red boxes are for \( \text{core}_\perp \). From the box plot it can be seen that:

- \( f_c \) of the cores made of concrete Mix A is lower than \( f_c \) of the cores made of concrete Mix B.
- \( f_c \) of the cores drilled from slabs with reinforcement seems to be slightly higher than \( f_c \) of the cores drilled from the slabs without reinforcement.
- \( f_{c\parallel} \) is slightly higher than \( f_{c\perp} \).
- One result is indicated as a possible outlier.

![Box plot of test results \( f_c \)](image)

*Figure D.3: TS1-RM1: Box plot of test results \( f_c \), all test results are included.*
More detailed statistical analyses are conducted to determine the anisotropy and whether it is influenced by the presence of reinforcement or the reference cylinder strength ($f_{c,cylin}$). As described in Section D.3.1, the first analysis conducted on the test results is based on the following regression model:

\[ f_c = \beta_0 + \beta_{\text{dir}}x_{\text{dir}} + \beta_{\text{mix}}x_{\text{mix}} + \beta_{\rho}x_{\rho} + \theta_{\text{mix}}x_{\text{mix}}x_{\text{dir}} + \theta_{\rho}x_{\rho}x_{\text{dir}} \pm \varepsilon \]

Table D.8 shows the $p$-values for regression coefficients and $R^2$ for the first employed regression model for Test series 1. The $p$-values show that:

- There is extremely strong evidence that the drilling direction ($\beta_{\text{dir}}$) influences $f_c$ ($p < 0.0001$).
- There is extremely strong evidence that $f_c$ ($\beta_{\text{mix}}$) influences $f_c$ ($p < 0.0001$), as expected.
- There is extremely strong evidence that the presence of reinforcement ($\beta_{\rho}$) influences $f_c$ ($p < 0.0001$).
- There is no evidence that either the concrete reference strength, $f_{c,cylin}$ ($\theta_{\text{Mix}}$), or the presence of the reinforcement ($\theta_{\rho}$) influences the anisotropy ($p = 0.065$ and $p = 0.376$, respectively).

The table shows furthermore that $R^2 = 0.90$, which means that this regression model can explain 90% of all the variation of the data, which is very high considering the simplicity of the regression model.

<table>
<thead>
<tr>
<th>$p$-value</th>
<th>$\beta_0$</th>
<th>$\beta_{\text{dir}}$</th>
<th>$\beta_{\text{mix}}$</th>
<th>$\beta_{\rho}$</th>
<th>$\theta_{\text{mix}}$</th>
<th>$\theta_{\rho}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>0.065</td>
<td>0.376</td>
<td></td>
<td>0.90</td>
</tr>
</tbody>
</table>

\textit{Table D.8: TS1-RM1: p-values for regression coefficients and $R^2$.}

Figure D.4 shows a so-called Residual plot where the residuals are plotted against predicted values, $f_c$ and the independent variables (the design parameters: Drilling direction, Concrete mix and Reinforcement). It is seen that the residuals fluctuate around 0 MPa and that the variance of the residuals seems constant in all four subplots (the residuals appear as horizontal band). This confirms that the mean of the residuals is close to 0 (Assumption 1) and the variance is approximately constant and independent of the value of the independent variables (Assumption 2). It is furthermore seen that some residuals are substantially larger (numerically) than the rest. This is an indication of outliers. This is addressed later.

The normality assumption (Assumption 3) is checked by analysing the residuals with a histogram with a fitted normal distribution, see Figure D.5 and a so-called normal plot, see Figure D.6. The histogram for the residuals looks bell-shaped and is approximately
symmetrical around 0. On the normal plot the residuals have almost a perfect straight-line appearance and none of residuals are outside Lilliefors 95% confidence interval (red dashed lines). Consequently, both figures indicate that the normality assumption is not violated. This is confirmed by a "Goodness-of-Fit Test" ($p = 0.195$, the null hypothesis states that the residuals are normal distributed).
To check for statistical independence of $\varepsilon$ (Assumption 4) the residuals shall be checked for e.g. time dependency (e.g. Learning Effect) and any other possible effects (e.g. spatial effects). In this regression model, all known variables/effects are included. Therefore, it is superfluous to check for statistical independence of $\varepsilon$ in this case.

The regression diagnostic shows that all regression assumptions are valid. Figure D.7 shows the studentized residuals with a lower and upper limit for a 95% confidence interval. There is evidence that an observation is an outlier if its studentized residual is outside these limits. It is seen that 6 observations are outside the limits for the 95% confidence interval.
Another regression analysis is conducted with the same regression model but without the 6 outliers. Table D.9 shows the \( p \)-values for the regression coefficients and \( R^2 \) for the first regression model after the outliers are removed from the sample. The table shows that removing the outliers does not change the aforementioned conclusions.

<table>
<thead>
<tr>
<th>( p )-value</th>
<th>( \beta_0 )</th>
<th>( \beta_{\text{dir}} )</th>
<th>( \beta_{\text{mix}} )</th>
<th>( \beta_\rho )</th>
<th>( \theta_{\text{mix}} )</th>
<th>( \theta_{\rho} )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>0.792</td>
<td>0.141</td>
<td>0.93</td>
<td></td>
</tr>
</tbody>
</table>

*Table D.9:* \( p \)-values for regression coefficients and \( R^2 \) for the first regression model for TS1 without outliers.

**Conclusions on the first regression analysis**

The first regression analysis shows that:

- all the main effects of the design parameters (drilling direction, concrete mix and the presence of reinforcement) have a significant influence on \( f_c \).

- none of the interaction effects have a significant influence on the anisotropy (*Direction*). Thus, neither the concrete mix nor the presence of reinforcement affect the anisotropy.
D.5.1.2 Second regression analysis

Since the first regression analysis shows that neither the concrete mix nor the presence of reinforcement affect the anisotropy a new regression analysis is conducted without interaction effects, see Section D.3.2:

\[ f_c = \beta_0 + \beta_{\text{dir}} x_{\text{dir}} + \beta_{\text{mix}} x_{\text{mix}} + \beta_{\rho} x_{\rho} \pm \varepsilon \]  \hspace{1cm} (D.14)

The observations that were categorised as outliers in the first analysis are included this analysis.

Table D.10 shows the \( p \)-values for regression coefficients and \( R^2 \) for the second regression model for Test series 1. The \( p \)-values show that there is extremely strong evidence that the main effects of the design parameters (drilling direction, concrete mix and the presence of reinforcement) have an influence on \( f_c \). The \( R \)-value shows that model can explain 89% of all the variation of the data, which is very high considering the simplicity of the regression model.

<table>
<thead>
<tr>
<th>( \beta_0 )</th>
<th>( \beta_{\text{dir}} )</th>
<th>( \beta_{\text{mix}} )</th>
<th>( \beta_{\rho} )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )-value</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td></td>
</tr>
</tbody>
</table>

Table D.10: TS1-RM2: \( p \)-values for regression coefficients and \( R^2 \).

Figure D.8 shows the residual plot. It is seen the residuals fluctuate around 0 MPa and the variance of the residuals seems constant in all four subplots (the residuals appears as horizontal band). Regression Assumption 1 and 2 seem fulfilled. However, the residual of four observations are substantially larger (numerically) than the rest.

Figure D.9 shows the studentized residuals. It can be seen that there is evidence that the four observations are outliers.
Figure D.8: TS1-RM2: Residual plot.

Figure D.9: TS1-RM2: Plot of the studentized residuals.
Another regression analysis is conducted with the same regression model but without
the 4 outliers. Table D.11 shows the $p$-values for the regression coefficients and $R^2$ for
the second regression model after the outliers are removed from the sample. The table
shows that removing the outliers does not change the extremely strong evidence that the
design parameters affect $f_c$. $R^2$ shows that model now can 92% of the variations.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_{dir}$</th>
<th>$\beta_{mix}$</th>
<th>$\beta_\rho$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$-value</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>0.92</td>
</tr>
</tbody>
</table>

*Table D.11: TS1-RM2: $p$-values for regression coefficients and $R^2$ for the second regression model for TS1 without outliers.*

Figure D.10 shows the residual plot after the outliers have been excluded. The residuals
fluctuate around 0 MPa and the variance of the residuals seems constant in all four
subplots (the residuals appear as horizontal band). Regression Assumption 1 and 2
seem fulfilled.

![Residual plot](image)

*Figure D.10: TS1-RM2: Residual plot for the second regression model where outliers are excluded.*

Figure D.11 and D.12 show a histogram and a normal plot for the residuals. The his-
togram looks bell-shaped and is approximately symmetrical around 0. On the normal
plot the residuals have almost a perfect straight line appearance and none of residuals
are outside Lilliefors 95% confidence interval (red dashed lines). Consequently, both
figures indicate that the normality assumption (Assumption 3) is not violated. This is confirmed by a "Goodness-of-Fit Test" \((p = 0.225)\).

**Figure D.11**: TS1-RM2: Histogram for the residuals for the second regression model where the outliers are excluded and a fitted normal distribution.

**Figure D.12**: TS1-RM2: Normal plot for the residuals for the second regression model where the outliers are excluded with Lilliefors’ 95%-bounds.
D.5.2 Test series 2

This section contains a statistical analysis of the experimental results from Test series 2. The experimental results are the concrete compressive strength of cores drilled from four unreinforced beams. The cores were drilled and tested after 2, 4, 8 and 12 weeks of curing to study how curing time influences the anisotropy. Figure D.13 shows a box plot of $f_c$. The blue boxes are for core $\parallel$ and the red boxes are for core $\perp$. From the box plot it can be seen that:

- $f_c$ increases with increasing curing time, as expected.
- $f_{c\parallel}$ is slightly lower than $f_{c\perp}$ for week 2, 4 and 12.

![Figure D.13: TS2-RM1: Box plot of test results ($f_c$).](image)

D.5.2.1 First regression analysis

More detailed statistical analyses are conducted to determine the anisotropy and whether it is influenced by the curing time. As described in Section D.4.1, the first analysis conducted on the test results is based on following regression model:

$$f_c = \beta_0 + \beta_{dir}x_{dir} + \beta_{time}x_{time} + \theta_{time}x_{time}x_{dir} \pm \varepsilon$$

Table D.12 shows the $p$-values for the regression coefficients and $R^2$ for the first employed regression model. The $p$-values show that:
• There is strong evidence that the drilling direction ($\beta_{\text{dir}}$) influences $f_c$ ($p = 0.0279$).

• There is extremely strong evidence that the curing time influences $f_c$ ($p < 0.0001$), as expected.

• There is not sufficient evidence that the curing time influences the anisotropy ($p = 0.579$).

The table shows furthermore that $R^2 = 0.67$, which is fair considering the simplicity of the regression model.

<table>
<thead>
<tr>
<th>$p$-value</th>
<th>$\beta_0$</th>
<th>$\beta_{\text{dir}}$</th>
<th>$\beta_{\text{time}}$</th>
<th>$\theta_{\text{time}}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.0001</td>
<td>0.0279</td>
<td>&lt;0.0001</td>
<td>0.579</td>
<td></td>
<td>0.67</td>
</tr>
</tbody>
</table>

*Table D.12: TS2-RM1: $p$-values for regression coefficients and $R^2$.*

Figure D.14 shows the residual plot. The residuals fluctuate around 0 MPa and the variance of the residuals seems constant in all three subplots (the residuals appear as horizontal band). Regression Assumption 1 and 2 seem fulfilled. However, the residual of one observation is substantially smaller than the rest. This indicates that the observation is an outlier. This is confirmed by Figure D.15, which shows the studentized residuals. Here it can be seen that there is evidence that the observation is an outlier. Consequently, the same analysis is conducted again where the outlier is excluded from the sample.
Table D.13 shows the $p$-values for regression coefficients and $R^2$ for this analysis. The $p$-values are approximately the same as the analysis where the outlier was included. However, there is even stronger evidence that the strength is anisotropic ($\beta_{\text{dir}}$); the $p$-value for $\theta_{\text{time}}$ is higher and $R^2$ is increased.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_{\text{dir}}$</th>
<th>$\beta_{\text{time}}$</th>
<th>$\theta_{\text{time}}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$-value</td>
<td>&lt;0.0001</td>
<td>0.0037</td>
<td>&lt;0.0001</td>
<td>0.652</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Table D.13: TS2-RM1: $p$-values for regression coefficients and $R^2$ for the first regression model for TS2 where the outlier is excluded.

Figure D.16 shows the residual plot for the new analysis. The residuals fluctuate around 0 MPa and the variance of the residuals seems constant in all three subplots (the residuals appears as horizontal bands). Regression Assumption 1 and 2 seem fulfilled. It is noted that no residuals seem to significantly different from the others.
Figure D.16: TS2-RM1: Residual plot.

Figure D.17 and D.18 show a histogram and a normal plot for the residuals. The histogram is not perfectly bell-shaped but it is approximately symmetrical around 0. Furthermore, the residuals appear as a straight-line on the normal plot and none of the residuals are outside Lilliefors 95% confidence interval (red dashed lines). Therefore it is concluded that the normality assumption (Assumption 3) is not violated. This is confirmed by a "Goodness-of-Fit Test" ($p = 0.354$).
Conclusions on the first regression analysis

The first regression analysis shows that:
• the drilling direction ($x_{\text{dir}}$) has a significant influence on $f_c$.
• the curing time ($x_{\text{time}}$) has a significant influence on $f_c$.
• there is not sufficient evidence that the curing time influences the anisotropy.

The regression diagnostics show that all regression assumptions are valid. However, the regression model includes an interaction that does not have a significant influence on $f_c$.

D.5.2.2 Second regression analysis

Since the first regression analysis shows that the influence of curing time on the anisotropy is insignificant, the anisotropy is further studied by a regression model without this interaction effects. The following regression model is adopted:

$$f_c = \beta_0 + \beta_{\text{dir}} x_{\text{dir}} + \beta_{\text{time}} x_{\text{time}} \pm \varepsilon$$  \hfill (D.15)

Table D.14 shows the $p$-values for the regression coefficients and $R^2$ for the second regression model. The $p$-values show that there is strong evidence that the curing time and drilling direction have an influence on $f_c$. The $R$-value shows that the model can explain 67% of all the variation of the data, which is fair considering the simplicity of the regression model.

<table>
<thead>
<tr>
<th>$p$-value</th>
<th>$\beta_0$</th>
<th>$\beta_{\text{dir}}$</th>
<th>$\beta_{\text{time}}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.0001</td>
<td>0.0263</td>
<td>&lt;0.0001</td>
<td>0.67</td>
<td></td>
</tr>
</tbody>
</table>

Table D.14: TS2-RM2: p-values for regression coefficients and $R^2$.

Figure D.8 shows the residual plot. The residuals fluctuate around 0 MPa and the variance of the residuals seems constant in all four subplots (the residuals appears as horizontal band). Regression Assumption 1 and 2 seem fulfilled. However, the residual of one observation is substantially smaller than the rest. This indicates that the observation is an outlier. This is confirmed by Figure D.20, which shows the studentized residuals. Here it can be seen that there is evidence that the observation is an outlier. Therefore, the same analysis is conducted again where the outlier is excluded from the sample.
Figure D.19: TS2-RM2: Residual plot.

Figure D.20: TS2-RM2: Plot of the studentized residuals
Table D.15 shows the \( p \)-values for the regression coefficients and \( R^2 \) for this analysis. The \( p \)-values are approximately the same as the analysis where the outlier was included. However, there is now much stronger evidence that the strength is anisotropic (\( \beta_{\text{dir}} \)), the \( p \)-value for \( \theta_{\text{time}} \) is even higher and \( R^2 \) is increased to 0.74.

\[
\begin{array}{cccc}
 p \text{-value} & \beta_0 & \beta_{\text{dir}} & \beta_{\text{time}} & R^2 \\
<0.0001 & 0.0033 & <0.0001 & & 0.74 \\
\end{array}
\]

*Table D.15: TS2-RM2: \( p \)-values for the regression coefficients and \( R^2 \) for the second regression model for TS2 where the outlier is excluded.*

Figure D.21 shows the residual plot for the new analysis. The residuals fluctuate around 0 MPa and the variance of the residuals seems constant in all three subplots (the residuals appears as horizontal band). Regression Assumption 1 and 2 seem fulfilled. No residuals seem to significantly different from the others.

\[\text{Figure D.21: TS2-RM2: Residual plot.}\]

Figure D.22 and D.23 show a histogram and a normal plot for the residuals. The histogram is not perfectly bell-shaped but it is approximately symmetrical around 0. On
the normal plot the residuals have a straight-line appearance and none of residuals are outside Lilliefors 95% confidence interval (red dashed lines). Therefore, it is concluded that the normality assumption (Assumption 3) is not violated. This is confirmed by a "Goodness-of-Fit Test" ($p = 0.220$).

Figure D.22: TS2-RM2: Histogram for the residuals for the second regression model where the outlier is excluded and a fitted normal distribution.
Figure D.23: TS2-RM2: Normal plot for the residuals for the second regression model where the outlier is excluded with Lilliefors’ 95%-bounds.