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Reciprocity Analysis of Electronic Stopping of Slow Ions

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Abstract

Reciprocity denotes the (approximate) equality of the electronic stopping cross section of element A in element B with the one of element B in element A. Reciprocity is expected in the energy range below $\sim 0.1$ MeV/u. For ions heavier than protons, most measurements are performed by the transmission technique. In order to minimize nuclear energy loss, only a narrow beam of penetrating ions is allowed to enter the detector. This implies that also the electronic energy loss is underestimated. We have estimated the magnitude of the necessary correction on the basis of the PASS code and its influence on the expected reciprocity and find that reciprocity is not necessarily violated, even if the reported cross sections differ significantly. However, we also find differences between reported stopping cross sections and computed reduced stopping cross sections that we cannot reconcile on the basis of the PASS code. These differences are found to reflect the well-known $Z_1$ structure, which is not inherent in the present version of PASS.

Keywords: Stopping power, Reciprocity, Transmission measurement, Threshold effects, PASS code

1. Introduction

The slowing down of energetic ions in matter is characterized primarily by the loss of kinetic energy $E$ per travelled path length $R$, which, for a monoatomic stopping medium, can be expressed by

$$\frac{dE}{dR} = -NS(E),$$

(1)
where $N$ is the mean number of atoms per volume and

$$S(E) = \int T d\sigma(E, T)$$  \hfill (2)$$

the stopping cross section. $d\sigma(E, T)$ denotes the differential cross section for kinetic-energy loss $(T, dT)$ by the projectile in a collision with an atom of the medium. The stopping cross section is an atomic parameter depending on the atomic numbers, masses and charges of the collision partners as well as the energy. It is customary [1] to separate $S(E)$ into an electronic $S_e(E)$ and a nuclear component $S_n(E)$,

$$S(E) = S_e(E) + S_n(E) + S_{ne}(E),$$  \hfill (3)$$

where $S_{ne}(E)$ is a coupling term that is usually neglected [2].

The reciprocity principle [3] claims that

$$S_e(Z_1, Z_2, v) = S_e(Z_2, Z_1, v),$$  \hfill (4)$$

where $S_e(Z_1, Z_2, v)$ denotes the electronic stopping cross section of an ion with atomic number $Z_1$ at speed $v$ in a monoatomic target with atomic number $Z_2$.

Reciprocity is expected to hold rigorously in binary, non-ionizing collisions between neutral gas atoms. It is important that stopping cross sections are compared and not stopping forces, and that the comparison is made at the same speed (or energy per nucleon) and not at the same beam energy. In the present work we deal exclusively with ‘cold’, i.e., non-ionized targets. Therefore, reciprocity can only be expected when projectiles are close to neutral, i.e., at low beam speed.

There is also a reciprocity relation for nuclear stopping which reads [3]

$$M_2 S_n(Z_1, Z_2, v) = M_1 S_n(Z_2, Z_1, v).$$  \hfill (5)$$

The occurrence of $M_1$ and $M_2$, which denote the atomic masses of the collision partners, indicates that studying the validity of (4) requires careful separation of nuclear from electronic stopping. This is one of several hurdles in testing reciprocity.

Somewhat surprisingly, experimental evidence for reciprocity was not only found in gas targets but also in gas-solid and solid-solid [3]. It is of interest, therefore, to explore the range of validity of the principle in $(Z_1, Z_2, E)$-space, not the least to what degree reciprocity can be found when stopping in one of the elements is primarily due to free electrons.
Even though a large amount of stopping data have been collected over more than 100 years, coverage with data is sparse in the low-velocity region. In the IAEA database [4] one may find systematic studies of a series of elements bombarded by a certain ion, or a certain target species bombarded by a series of ions, but a systematic attempt to study reciprocity experimentally has not yet been made.

As a matter of fact, very few experimental data obey the reciprocity principle on first sight. For the majority of atom pairs (A,B), stopping data in the energy range well below the stopping maximum either do not exist at all or are available only for one of the two species, either as a target or a projectile. Amongst the rest, the scatter between different data for a given ion-target combination may be drastic, and even where there are well-defined data for \( Z_1 \) in \( Z_2 \) and vice versa, the two may differ by a factor of two or more. As a result, the number of atom pairs for which published stopping cross sections clearly obey the reciprocity principle can be counted on two hands.

On the other hand, reciprocity is well-founded in existing theory [5, 6, 3]. The purpose of this study was originally to use reciprocity as a tool to distinguish reliable from less reliable experimental data. However, we recently noticed that numerous published values of stopping cross sections measured in transmission need to be corrected, either because of a questionable correction for nuclear stopping or, more important, because of a neglect of the impact-parameter dependence of electronic stopping in the analysis [7]. This added significantly to the scope of the present study, since it greatly enhanced the number of atom pairs obeying approximately the reciprocity principle.

Tracking experimental data suitable for studying reciprocity is facilitated by the REST code [8] which reads data from [4] and, in graphical form, provides information on the existence of pertinent data. The code also facilitates the drawing of graphs utilizing GNUPLOT and, in its latest version [9], allows for inclusion of theoretical estimates, in particular Lindhard & Scharff [6], Firsov [5, 10] and PASS [11, 12], as well as the SRIM tabulation [13].

2. Characteristic Examples

Figure 1, generated by REST [9] like most of the other graphs in this paper, shows stopping cross sections for two atom pairs which belong to a rather rare category: Not only are experimental data available in an energy range where reciprocity can be expected to be obeyed, i.e., well below the stopping maximum, but where reciprocity actually appears to be obeyed between pertinent data in the
IAEA database [4]. At the same time the two graphs confirm that reciprocity is not obeyed in the energy range around and above the stopping maximum, where the heavier atom has a higher stopping cross section in view of its higher equilibrium charge. Finally, conflicting data sets are seen for C-Ti. Here the reciprocity principle suggests that the difference is primarily due to the Ve79 data [14].

Figure 2 shows two typical graphs generated by the new version of REST [9]. It is seen that experimental data for (N,He) obey the reciprocity principle. Also included are theoretical predictions from refs. [6, 5, 10, 12].

The Firsov prediction [5, 10] (upper graph), is based on

$$T(v, p) = 0.35 \frac{\hbar v}{a_0} \frac{(Z_1 + Z_2)^{5/3}}{[1 + 0.16(Z_1 + Z_2)^{1/3}r_{\text{min}}/a_0]^5},$$

where $T(v, p)$ is the mean electronic energy transfer in a collision at impact parameter $p$, and $r_{\text{min}}$ is the distance of closest approach between the colliding nuclei. Both $T(p)$ and $r_{\text{min}}$ are invariant against interchange between $Z_1$ and $Z_2$. Therefore, the resulting stopping cross section obeys reciprocity rigorously. This reflects the fact that in a free binary nonionizing collision, the transfer of electronic energy must be independent of the reference frame from which it is observed [3]. However, the Firsov formula appears to overestimate the reported stopping cross section significantly, as does the PASS prediction [12].

The PASS prediction shows a slight departure from reciprocity. This is frequently found and primarily due to the treatment of projectile excitation which is difficult to handle in a strictly symmetric way [15].

The Lindhard-Scharff formula [6], when applied with the recommended factor $\xi_e \approx Z_1^{1/6}$, yields good agreement with the data for He-N but overestimates the reported stopping cross section for N-He. If the factor $\xi_e$ were dropped, the Lindhard-Scharff formula would not only obey the reciprocity principle but also describe both data sets within experimental scatter. This factor has no theoretical foundation to our knowledge but was inserted to fit ranges of fission fragments [16] in a velocity regime where ions are not electrically neutral in charge equilibrium.

Finally, the SRIM tabulation follows the experimental data closely, although the indication of an oscillatory behavior is likely to be an artifact caused by the interpolation procedure.

However, Figure 13 below will show that Figure 2 is not the last word on the N-He atom pair.

Figure 3 shows electronic stopping cross sections for two atom pairs where pronounced deviations from reciprocity are found. In both cases the deviating data
Figure 1: Electronic stopping cross sections for Ne in He and He in Ne (top) and for Ti in C and C in Ti (bottom). Data compiled in [4]. Labels in this and most of the following graphs refer to codes in [4].
Figure 3: Electronic stopping cross sections for Ar in C and C in Ar (top) and for Ne in N and N in Ne. Data compiled in [4].
show a steeper slope than the commonly-found linear dependence on velocity. This type of behavior has been analyzed recently [7] with the C-Ar pair as the main example, as will be summarized below.

3. Data Analysis

Stopping cross sections have been determined experimentally mostly in transmission measurements or, less directly, via ion ranges [17]. For specific ions, especially Lithium, measurements have been performed by inverted Doppler shift attenuation, and measurements with protons have also been performed in reflection. For all ions heavier than protons, separation into electronic and nuclear stopping is a central aspect of data analysis.

For the most frequently used transmission technique pertinent features have been discussed in [7], based on earlier work by Fastrup et al. [18] and ref. [19]. The essential point is that the contribution from nuclear stopping is minimized by analyzing transmitted ions only within a narrow cone around the incident-beam direction. In this way, ions that suffer large nuclear energy losses are scattered out of the beam. However, particles suffering large-angle deflection also experience relatively high electronic energy losses. Thus, while reduced nuclear energy loss is minimized, electronic energy loss is reduced. In other words, the actual electronic stopping cross section will be higher than what is measured with a small detector opening angle.

In the conventional picture [6, 16, 18, 13, 20, 21], electronic stopping is treated as a friction force and its dependence on scattering angle or impact parameter is neglected.

3.1. Nuclear Stopping

According to the Bohr-Williams model of multiple scattering [22], the angular distribution of a penetrating beam can be described by a gaussian profile centered around the initial direction of motion and a single-scattering tail at larger angles. The analysis developed in refs. [19] and [7] is an extension of a scheme proposed by Fastrup et al. [18]. It is assumed that particles emerging at an angle $\alpha$ less than the multiple-scattering angle $\alpha_1$ experience a nuclear energy loss given by

$$T_1 = N_x \int_0^{T_1} T d\sigma(E,T),$$

whereas particles emitted at an angle $\alpha > \alpha_1$ experience single-scattering loss $T_n(\alpha)$ given by energy-momentum conservation laws. In contrast to the treatment
in ref. [18] we distinguish between the case where the detector opening angle \( \phi \) is greater or less than the multiple-scattering angle. This implies that the reduced nuclear stopping cross section changes slope as a function of energy at the cross-over between the multiple-scattering regime at low energy and the single-scattering regime at high energy, where the multiple-scattering angle is smaller than the detector opening angle.

3.2. Electronic Stopping

Reduced electronic stopping cross sections are evaluated in a very similar way, the main difference being that the scattering angle is replaced by the impact parameter as the independent variable. The crucial quantity is the mean electronic energy loss \( T_e(E, p) \) versus impact parameter \( p \). We have performed computations both with the Firsov formula [5] and with numerical results computed by PASS [23]. As in the case of nuclear stopping, the reduced electronic energy loss versus beam energy changes slope at the point where the multiple-scattering angle equals the detector opening angle. Above that energy, an increasing fraction of the emitted beam enters the detector. However, at least for \( M_1 < M_2 \), where scattering angles up to 180\( ^\circ \) occur, the reduced electronic stopping cross section will not reach the unrestricted value at any energy.

In the following we study reciprocity within the energy interval 0.001-0.1 MeV/u: Above 0.1 MeV/u ions are no longer close to neutral. Below 0.001 MeV/u, angular deflection causes deviations from the expected linear velocity dependence of the stopping cross section [12]. Computations are performed with the ASCS code [19], which determines reduced stopping cross sections for a given projectile-target combination as a function of beam energy \( E \), detector opening angle \( \phi \) and a dimensionless target thickness

\[
\xi = N x a_0^2, \tag{8}
\]

where \( x \) is the layer thickness, \( N \) the number of target atoms per volume and \( a_0 \) the Bohr radius.

3.3. Cautional Remarks

Figure 4 illustrates the correction for nuclear stopping for the (Al,C) pair. Graphs of this kind go back to Ormrod et al. [24]. It is seen that for Al in C, the reduced nuclear stopping cross section amounts to 10\% at \( E = 0.01 \) MeV of the reduced electronic stopping cross section and increases with decreasing energy with a cross-over at about 0.002 MeV/u. For C in Al these energies become only slightly smaller.
Figure 4: Full and reduced nuclear stopping cross sections compared with reduced electronic stopping cross sections for Al-C and C-Al. Calculations following [19] and PASS [23].
Clearly, at energies below \( \simeq 0.01 \text{ MeV} \), the magnitude of extracted stopping cross sections depends increasingly sensitively on details of the adopted model for single and multiple scattering. Here we note that results of the Bohr-Williams-Fastrup model have been confirmed by comparison with exact calculations based on the Bothe-Landau model [20]. We have also checked that results of computations with a straight Thomas-Fermi interaction potential differ by less than 1% from what is found for Molière interaction.

While we thus have reasons to trust our nuclear-stopping correction, the error margin is greater for electronic stopping, both with regard to the magnitude of the stopping cross section and the dependence of the energy loss on impact parameter. Despite the strength of the Lindhard-Scharff formula indicated by Figure 2 we do not apply it here because there is no impact-parameter dependence assigned to it. Computations based on the Firanov formula have been very useful in the development of our scheme [19], but in the present work we mainly work with PASS, which covers a wider range of atomic numbers and beam energies.

4. Reciprocity Plots

Stopping measurements in transmission are typically done with significantly thinner layers in gases than in solids. For measurements on carbon by Ormrod et al. [25] and Fastrup et al. [18], minimum thicknesses were equivalent to \( \xi = 4.2 \) and 9.3, respectively, whereas \( \xi = 0.38 \) in measurements on gases by Ormrod [26], as was found in our analysis [7].

Figures 5 and 6 show that the difference is dramatic. For SiC-CSi, the upper graph in Figure 5 shows that, within the considered energy interval, there is practically no dependence of the reduced stopping cross section on the detector opening angle \( \phi \) at a fixed layer thickness \( \xi = 4 \). Likewise, the lower graph shows that the variation of the reduced electronic stopping cross section with \( \xi \) at a fixed opening angle \( \phi = 0.01 \text{ rad} \) is much smaller than the difference between the full and the reduced stopping cross section.

There is near-perfect reciprocity between Si-C and C-Si, both in the full and the reduced stopping cross section, but at the same time there is a significant difference between full and reduced. To explain this behavior we recall that the theory assumes that the energy loss is constant for that part of the beam which emerges within the multiple-scattering angle. Since the multiple-scattering angle increases with increasing target thickness, there must be a critical thickness, above which the recorded energy loss must be independent of the detector opening. This is the case for most measurements on solid foils.
Figure 5: Full and reduced stopping cross sections for SiC-CSi. Upper graph: Varying detector opening angle $\phi = 0.001-1.0$ radians at fixed thickness, $\xi = N x a_0^2 = 4$. Lower graph: Varying thickness at fixed detector opening angle $\phi = 0.001, 0.01$ and 0.1. Dashed lines: Reduced stopping cross sections for C in Si. Dotted lines: Reduced stopping cross sections for Si in C.
Figure 6: Same as upper graph in Figure 5 for Ar-N.
Conversely, for ArN-NAr shown in Figure 6 we are in the transition region between single and multiple scattering, as is seen from the change in slope for N-Ar ($\phi = 0.01$) at $E = 0.005$ MeV/u. Here we find a significant dependence of the reduced stopping cross section on the detector opening angle and deviations from reciprocity by up to a factor of two.

We also note that while for $\phi = 0.001$ the reduced stopping cross section for N-Ar is consistently smaller than the one for Ar-N, a cross-over is seen for $\phi = 0.01$ at $E = 0.07$ MeV/u and for $\phi = 0.1$ at $E = 0.002$ MeV/u.

4.1. Carbon, Aluminium and Silicon

Figure 7 shows reciprocity plots for Si-Al, Al-C and Si-C, three atom pairs formed by three elements that are relatively well supported by experimental data. Plots for Si-Si, Al-Al and C-C have been added in Figure 8 to complete the picture. Full and reduced electronic stopping cross sections are based on PASS.

All reduced stopping cross sections have been computed with parameters $\phi = 0.02$ rad and $\xi = 10$. However, in aluminium, only the stopping cross sections for the K and L shell depend on the scattering angle, whereas the M electrons are treated as a free electron gas [19]. Since, at low energies, stopping is mainly due to outer-shell electrons, the difference between reduced and full stopping cross section is seen to be very small for Si-Al, C-Al and Al-Al. For the same reason, full stopping cross sections for the (Si,Al) and (Al,C) pairs show significant deviations from reciprocity. On the other hand, reduced stopping cross sections appear to obey the reciprocity principle in all three atom pairs.

These and subsequent plots extend over an energy interval including all available experimental data, even though the focus is on energies up to $\sim 0.1$ MeV/u.

All calculated stopping cross sections are supported by data. In particular, data from the comprehensive study by Ward et al. [27] for Si-C, Al-C, C-Al, Si-Al, C-C and Al-Al all lie close to the respective reduced stopping cross section. However, significant deviations are found for some data:

4.1.1. Si-Al

For Si in Al, data by Lennard et al. [28] and Ward et al. [27] lie only slightly below the calculated reduced stopping cross section, whereas data from Arstila et al. [29, 30] drop off rapidly below 0.1 MeV/u. The latter data were found by the inverted Doppler-shift attenuation technique and were analyzed with the help of Monte Carlo simulations ignoring, as is common, the impact-parameter dependence of electronic stopping. The error made is not known at this point.
Figure 7: Full and reduced electronic stopping cross sections for SiAl-Al, AlC-CAl and SiSiC-CSi, top to bottom. Data compiled in [4].
Figure 8: Full and reduced electronic stopping cross sections for Al-Al, C-C and Si-Si, top to bottom. Data compiled in [4].
For Al in Si experimental data are consistent with the calculated reduced stopping cross section but do not reach down into the critical energy region below 0.1 MeV/u.

### 4.1.2. Al–C

Similarly, for C–Al data from Ormrod et al. [25] lie close to the calculated stopping cross section, as do data from Santry et al. [31], whereas Ward et al. [27] lies slightly lower.

For Al in C, data from Fastrup et al. [18] and Ward et al. [27] fall right on the reduced stopping cross section, whereas [25], and to a lesser extent, [28], lie significantly below that curve.

We note that despite predicted reciprocity between the two reduced stopping cross sections, measured stopping cross sections from Ormrod [26] differ by about a factor of two. This is remarkable, especially because it is found in data originating in the same apparatus. We come back to this point below.

### 4.1.3. Si–C

For Si in C we find near-perfect agreement between the calculated reduced stopping cross section and measurements from [25, 26, 28, 27].

For C in Si, data from Grahmann et al. [32] and Santry et al. [31] lie close to the calculated reduced stopping cross section, while a single point from Hoffmann et al. [33] falls on the full instead of the reduced stopping cross section. A correction for nuclear stopping has not been reported for those measurements.

### 4.2. $Z_1$ Structure

Stopping of slow ions is known to exhibit $Z_1$ Structure, i.e., an oscillatory dependence of the stopping cross section at a given velocity as a function of the atomic number $Z_1$ of the penetrating ion. This effect has been studied for several materials, in particular for carbon and aluminium [24, 25, 18, 34, 27].

In Figure 9 we have plotted the ratio between measured stopping cross sections in carbon and aluminium [24, 25, 27] and reduced electronic stopping cross sections for $\phi = 0.02$ rad and $\xi = N a_0^2 x = 10$. In the energy range covered by the above data, this ratio turns out to be almost independent of the beam energy. We have, therefore, plotted the mean value and the RMS, expressed by error bars. We emphasize that these error bars do not represent experimental errors, but only the deviation of the ratio $S_{\text{exp}}/S_{\text{red}}$ from a horizontal straight line.

Figure 9 shows that for $Z_1$ in C the plotted ratio goes through a minimum for Al ions and should be lower there by $\sim 25\%$ than the reduced stopping cross section.

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Figure 9: The ratio of measured electronic stopping cross sections in carbon and aluminium and the calculated reduced electronic stopping cross section as a function of the atomic number $Z_1$ of the ion. See text.
section predicted on the basis of the PASS code. This is consistent with the measurements of Ormrod et al. and Lennard et al., contrary to what would be expected by mere inspection of Figure 7.

Figure 9 also reveals that for \( Z_1 \) in Al the plotted ratio goes through a maximum for N ions and, for neighboring carbon ions should lie \( \sim 35\% \) above the calculated reduced stopping cross section, as is seen in the data from Ormrod et al. in Figure 7.

Figure 8 reflects the fact that data for C in C, data from [24, 18, 33] all lie significantly above the reduced stopping cross section, as seen in Figure 9, which predicts a \( \sim 40\% \) enhancement. For Al in Al experimental data do not show a clear message, but data from [28] are consistent with a value smaller by \( \sim 15\% \) following from Figure 9.

Finally, Figure 9 shows that S-C and Si-Al are close to a minimum, where both ratios are \( 10\% \) below 1 in both cases, as is seen in Figure 7.

We do not show a graph for \( Z_1 \) structure in silicon. We have no reason to question the existence of such structure, but available data do not show a clear picture.

### 4.3. Carbon, Neon and Argon

Figure 10 shows reciprocity plots for Ar-N, Ar-C and N-C. Reduced stopping cross sections for Ar-C and N-C have been evaluated assuming standard parameters for solids, \( \phi = 0.02 \) rad and \( \xi = 10 \). For the gas targets Ar and N we adopted \( \phi = 0.02 \) and \( \xi = 0.38 \) in accordance with ref. [7]. Figure 11 shows \( Z_1 \) structure for Ar and N. \( Z_1 \) structure for C is shown in Figure 9.

#### 4.3.1. Ar-N

Data for Ar in N from [4] do not extend into the critical energy region, where reciprocity can be tested. However, excellent agreement is found for N in Ar between measurements from Ormrod [26] and reduced stopping cross sections. Figure 11 shows that nitrogen lies in the maximum of \( Z_1 \) oscillations for both argon and nitrogen targets. Data from Fukuda [35] do not match in either magnitude or slope. It has been argued in ref. [7] that these data have been found by subtracting the full nuclear stopping cross section from the raw data. Elimination of this error led to a result that almost coincides with the measurements from [26].

#### 4.3.2. Ar-C

The case of C in Ar has been the test case in ref. [7]. The parameter \( \xi = 0.38 \) underlying data for argon gas has been extracted from the nuclear stopping
Figure 10: Full and reduced electronic stopping cross sections for ArN-NAr, ArC-CAr and NC-CN, top to bottom. Data compiled in [4].
Figure 11: Same as Figure 9 for nitrogen and argon.
Figure 12: Reduced electronic stopping cross section for Ne in Ar compared to data from [26].
\( \xi = 0.38 \), \( \phi = 0.002 \) and 0.02. See text.
cross section for that system and been confirmed for other ions up to neon. The parameter $\phi$ was found by matching the reduced electronic stopping cross section for C in Ar to the data from ref. [26]. Since C lies close to the maximum in the $Z_1$ structure for argon, the ratio $S_{\text{exp}}/S_{\text{red}}$ is smaller than unity for the neighboring elements. If $\phi$ had been chosen to match the experimental data for Ne in Ar, Figure 11 would change.

Figure 12 shows that unlike for C-Ar, where variation of the opening angle $\phi$ resulted in an accurate fit, this is not possible for Ne in Ar. We conclude, therefore, that the low value of the ratio $S_{\text{exp}}/S_{\text{red}}$ for Ne is real and reflects a $Z_1$ structure which is not described by PASS.

For Ar in C, Figure 10 shows a major enhancement of the plotted ratio. Figure 9 shows that Ar constitutes the maximum in the $Z_1$ structure for C.

4.3.3. N-C

For C in N, Figure 10 shows perfect agreement between measurements and the reduced stopping cross section. Although C lies in the maximum of the $Z_1$ structure for nitrogen, as is seen in Figure 11, that maximum lies close to unity here.

Conversely, for N in C the data rather follow the full than the reduced stopping cross section. However, as Figure 9 shows, N lies in the maximum of the $Z_1$ structure for C, which in this case lies at a value 1.43.

4.4. Neon, nitrogen and helium

Figure 13 shows reciprocity plots for combinations between Ne, N and He. Total stopping cross sections for gases are expected to obey the reciprocity principle. The three graphs show near-perfect reciprocity between the respective experimental data, with the exception of data by Fukuda [35] for neon in nitrogen.

4.4.1. N-He

For the N-He pair reciprocity appears also to be obeyed by the predicted reduced stopping cross sections, and good agreement with measured values is found for data from Fukuda [35], Weyl [36] and Hvelplund [37] for N in He and data from Ormrod [26] and Baumgart et al. [38] for He in N, whereas data from Fukuda [39] are noticeably lower.

4.4.2. Ne-He

For the Ne-He pair we find a different scenery. First of all, the stopping maximum has the same height as the one for the N-He system, whereas theory suggests
Figure 13: Same as Figure 7 for Ne-H, N-Hr and Ne-N.
Figure 14: Same as Figure 9 for Ne and He.
Figure 15: Explaining pronounced deviation from reciprocity for Ne in nitrogen due to overestimated nuclear-stopping correction.

a significantly higher value for Ne-He. This is another manifestation of $Z_1$ structure: Figure 14 shows that in the $Z_1$-He plot Ne lies about a factor of 3 below the maximum, whereas He lies high in the $Z_1$-Ne plot. The large difference between the reduced cross sections is caused by the difference in the respective scattering angles, which appears to be correctly described in view of the good agreement between theory and measurement for He in Ne.

4.4.3. Ne-N

For Ne in N the difference between $S_{\text{red}}$ and $S_{\text{exp}}$ is very similar to the one found for Ne in He, again a manifestation of $Z_1$ structure. For N in Ne, data from Hvelplund [37] noticeably below the reduced stopping cross section, whereas data from Fukuda [35] show an abrupt change in slope, very much like what has been observed for C in Ar in Figure 3. In Figure 15 we show that adding the full nuclear stopping cross section to the electronic stopping cross section reported in [35] leads to a result close to the calculated reduced stopping cross section.
5. Discussion

The present analysis has focused on stopping measurements in the transmission geometry, in particular systematic studies of the penetration of a series of ions through solid (C, Al) and gaseous (He, N, Ne, Ar) materials by Ormrod et al. [24, 25, 26] and Hvelplund [37]. Also data by Fastrup et al. [18], Ward et al. [27] and Fukuda [39, 35] were considered.

Our analysis shows that all considered data need to be corrected for electronic energy lost by particles that do not hit the detector. This correction, which is related to the nuclear-stopping correction, is significant from $\sim 0.1\,\text{MeV/u}$ down and may well amount to a factor of two or more at $0.001\,\text{MeV/u}$.

From a quantitative point of view, we have to keep in mind that complete information on the important experimental parameters, the acceptance angle $\phi$ of the detector and the parameter $\xi$ characterizing the layer thickness, is not available to us for any of the considered systems. This is less of a handicap for measurements on solid targets, where the target thickness is most often mentioned and where measurements are mostly performed in the multiple-scattering regime, where the dependence on $\phi$ is weak. For measurements in gases, indirect information on the target thickness could be extracted via the adopted nuclear-stopping corrections in measurements by Ormrod [26], and information on $\phi$ was received for Hvelplund’s apparatus [37] from the author.

This implied that one parameter is more or less free, but once determined, the adopted $(\phi, \xi)$ pair has been applied to all $(Z_1, Z_2)$ combinations for a fixed target $Z_2$.

When initiating this and our previous study [7] we intended to find a measure of the relative significance of impact-parameter-dependent stopping on the velocity dependence of the stopping cross section and on reciprocity. It was, therefore, a pleasant surprise that results from PASS, which was developed as a high-energy theory, were in agreement with experimental data for several ion-target combinations [7].

It came as a second surprise that a systematic ordering of deviations from reciprocity reproduced the well-known $Z_1$ structure. By plotting the relative variation from the ‘expected’ reduced stopping cross section we actually find fairly large amplitudes compared to conventional plots. However, caution is indicated regarding the quantitative content of these plots. They depend on the adopted choice of $\phi$ and/or $\xi$, the nuclear-stopping correction adopted in the original papers, and the impact-parameter dependence of energy losses computed by PASS.

Reciprocity has been suggested to be a tool to estimate stopping cross sec-
tions. From what we have now learned, reciprocity found in published stopping cross sections does not imply that the data are correct, if found in transmission measurements. However, if the reduced stopping cross section agrees with the published data, we may assume that the full stopping cross section calculated from PASS is the best available estimate. An example is the (Si,C) pair in Figure 7, where there is found reciprocity both in the uncorrected and the reduced stopping cross sections as well as agreement between the latter and the measured values. But the actual electronic stopping cross section is claimed to be the unrestricted value calculated by PASS.

We emphasize that the linear velocity dependence of stopping cross sections calculated by PASS does not extend down to zero but is affected by angular deflection for $E < 0.001\ MeV/u$ [12].

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References


