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I.M. Akhmedzhanov, S.I. BozhevolnyI

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Scanning Differential Microscopy for Characterization of Reflecting Phase-Gradient Metasurfaces

I. M. Akhmedzhanov\textsuperscript{a,}* S. I. Bozhevolnyi\textsuperscript{b}

\textsuperscript{a} Prokhorov General Physics Institute of Russian Academy of Sciences, Vavilov Str., 38, Moscow, 119991, Russia
\textsuperscript{b} Centre for Nano Optics, University of Southern Denmark, Campusvej 55, DK-5230, Odense M, Denmark

*Corresponding author. E-mail address: eldar@kapella.gpi.ru

Abstract
Gap-plasmon based phase-gradient metasurfaces operating in reflection are widely used for the realization of diverse flat optical components, ranging from spectropolarimeters to efficient couplers for surface waves. Successful implementation of carefully designed metasurfaces is however often hampered by technological imperfections that could be related to deviations of geometrical parameters of fabricated nanostructures from the designed ones or material properties, such as the metal and/or dielectric susceptibilities, from the handbook data. While the overall performance of fabricated components might indicate the existence of a potential problem, it is very difficult to identify its origin, which, for example, can simply be related to the deviation in only one cell of the metasurface supercell. We suggest exploiting well-developed experimental techniques of scanning differential heterodyne microscopy (SDHM) to characterize fabricated phase-gradient metasurfaces designed to operate in reflection. We further establish that, by carefully measuring the SDHM response of a gradient metasurface, one should be able of detecting small (~ 5%) amplitude and phase deviations (with respect to the design values) in the optical field reflected by an individual subwavelength-sized cell of the metasurface supercell.

Research highlights
Complex scanning differential heterodyne microscope (SDHM) response of phase-gradient metasurfaces operating in reflection is numerically investigated, revealing a direct relationship between the SDHM phase response and the average phase gradient of the metasurface. Furthermore, we establish that small (~ 5%) amplitude and phase deviations (from the design values) in the complex reflection of an individual subwavelength-sized cell of the metasurface supercell can be detected by analysing the SDHM response.

Abbreviations
SDHM: Scanning differential heterodyne microscope
SOM: Scanning optical microscope

Keywords
Heterodyne microscopy
Gradient metasurface
Phase response
Amplitude response
1. Introduction

Metasurfaces have experienced enormous progress over the last years and attracted a great deal of attention due to unprecedented control over optical fields that can be exercised, resulting in extremely diverse functionalities demonstrated already along with several technologically appealing features, such as planar thin-film design [1-9]. Gap-plasmon based phase-gradient metasurfaces operating in reflection represent an important sub-class of metasurfaces, and are widely used for the realization of diverse flat optical components, ranging from spectropolarimeters to efficient couplers for surface waves [10]. Successful implementation of carefully designed metasurfaces is however often hampered by technological imperfections that could be related to deviations of geometrical parameters of fabricated nanostructures from the designed ones or material properties, such as the metal and/or dielectric susceptibilities, from the handbook data [11]. While the overall performance of fabricated components might indicate the existence of a potential problem, it is very difficult to identify its origin, which, for example, can simply be related to the deviation in only one cell of the metasurface supercell. Characterization techniques that would enable differentiating the performances of individual metasurface cells are crucial for further progress in this field, especially towards the implementation of practical flat optical components that have to compete in quality with conventional (and very well developed) optical components.

Several methods for optical characterization of metasurfaces were recently proposed and experimentally tested [12,13], utilizing different physical principles and revealing different limitations in their performance. Spectrally and spatially resolved interferometry using a Mach-Zehnder interferometer with the imaging spectrometer and supercontinuum laser [12] allows one to reach the accuracy of phase characterization of ~ 0.02 rad within +/- 0.5 rad region. At the same time, the reported spatial resolution was only ~ 50 μm, which is not enough for the performance assessment of individual cells that should be of subwavelength sizes. Another approach, the vortex based interferometric method [13], exploits a specially structured light beam at the wavelength of 633 nm and exhibit the spatial resolution determined by the focused laser beam size of ~ 6 μm (at the 1/e² intensity level). Both methods [12, 13], apart from featuring insufficient spatial resolution for the individual cell assessment, are not stable versus microphonic noises, requiring thereby proper acoustic and vibration isolation.

Scanning differential heterodyne microscopy (SDHM) allows one to accurately compare the phases of two reflected laser beams that are frequency shifted (typically, by an acousto-optic deflector) and focused on a sample surface at close locations (Fig.1). This characterization technique is capable of detecting sub-nanometer steps in the surface reflection phase and reaching the spatial resolution close to the operating wavelength λ, while also being inherently stable with respect to microphonic noises due to the common-path optical scheme [14]. The SDHM seems therefore very well suited for the characterization of reflective phase-gradient metasurfaces, in general, and gap-plasmon based gradient metasurfaces, in particular. Generally speaking, the SDHM can be considered as the modification of conventional far-field scanning optical microscopy (SOM) with the additional possibility of accurately determining spatial gradients in the phase of reflected optical beams [15-17].

The typical SDHM configuration contains a modified Mach-Zehnder common-path interferometer with an optical frequency shift introduced in the interferometric arms, and exploits coherent registration of the two reflected (frequency-shifted) optical beams at the Fourier plane (Fig. 1). The SDHM has already been successfully applied for the characterization of integrated optical channel waveguides [17], channel plasmon waveguides [18] and diffraction gratings [19], but its use and potential as an experimental tool for the characterization of phase-gradient metasurfaces has so far not been considered. While the idea of using the SDHM for the metasurface characterization is straightforward, one should first properly analyse the SDHM response in the case of reflective phase-gradient metasurfaces before embarking on the corresponding experimental investigations. In this work, we discuss in detail the possibility of SDHM application for the
gradient metasurface characterization. The analysis is made on the basis of analytical considerations and numerical calculations of the SDHM optical responses for different parameters of probing laser beams and the reflecting phase-gradient metasurface configuration. The paper is organized as follows. Section 2 is devoted to establishing the basics of the SDHM response that makes it easier to understand the SDHM response when characterizing phase-gradient metasurfaces, which is calculated and presented in Section 3. In Section 4, we summarize the results obtained and offer our conclusions.

2. SDHM response

The basics of the SDHM operation (Fig1) has already been discussed at length in previous publications [15, 17], so that we can simply reiterate the main SDHM features: (i) the common-path optical interferometric microscopy with coherent optical detection, which involves two probe beams with the intermediate (heterodyne) frequency shift: \( f_i \sim 0.1 \text{ - } 4 \text{ MHz} \), (ii) the distance \( \delta \) between the probe beams in the object plane (typically \( \sim 0.1 \text{ - } 4 \mu m \)) can conveniently be adjusted by varying the frequency shift, (iii) the object is located at the front focal plane of the objective with the point-like photodetector positioned at the Fourier plane of the objective, and (iv) the phase and amplitude response determined by the sample reflectivity is detected at the intermediate frequency \( f_i \). Overall, these features ensure rather reliable and robust optical characterisation of the sample reflectivity via registration of both phase and amplitude components of the differential optical response.

Within the thin phase-amplitude screen approximation [15,16], the output current of the point photodetector (coherent registration scheme) at the center of the objective Fourier plane contains both amplitude and phase information directly related to the amplitude and phase of a complex response function [15, 17]:

\[
D(x, \delta) = L(x, -\delta / 2) \cdot L'(x, \delta / 2), \quad \text{with} \quad L(x, \delta) = B \cdot \int_{-\infty}^{\infty} h(x) \cdot r(x - \delta - x)dx,
\]

where \( x \) is the scanning coordinate, \( h(x) \) is the optical field amplitude distribution of a probe beam at the object (sample) plane, \( r(x) = R(x) \exp[i \phi(x)] \) is the complex reflection coefficient of the sample, and \( B \) is the normalization constant. The phase of the SDHM response is determined by the phase difference between the two probe (frequency-shifted) beams, which is in turn related to the difference in the reflection phase of two adjacent sample areas illuminated by the focused probe beams (Fig.1). The distance between the two adjacent probed areas producing the phase response is thereby set by the distance \( \delta \) between the two (focused) probe beams, which can be characterized at the sample surface by Gaussian distributions (Fig. 2a) with a half width \( w \) at 1/e² intensity:

\[
h(x) = E(x) = E_0 \exp \left[ -\frac{(x \pm 0.5 \delta)^2}{w^2} \right].
\]

The SDHM phase response, \( \phi(x, \delta) = \arg(D(x, \delta)) \), is therefore related to the gradient of the sample phase reflectivity, generally increasing with the probe beam separation \( \delta \). The SDHM amplitude response, \( |D(x, \delta)| = \text{abs}(D(x, \delta)) \), also depends on the parameter \( \delta \), but its nature is rather similar to that of the conventional SOM, at least for small values of \( \delta \).

These features of the SDHM response are illustrated with the phase and amplitude dependencies calculated using the above formulae for two basic phase objects – a purely phase step (Fig. 2b) and ridge (Fig. 2c) with the phase increment \( \Delta \phi = 60^\circ \). For the phase step, it is seen that, when the distance \( \delta \) between two probe beams is larger than the beam width \( 2w \), the phase response maximum \( \phi(0) \) is equal to the interrogated phase step \( \Delta \phi = 60^\circ \) (Fig. 2b). When the distance \( \delta \) becomes smaller than the beam width \( 2w \), the phase response maximum \( \phi(0) \) becomes progressively smaller, depending non-linearly on the ratio \( \delta/2w \). As expected, the amplitude response resembles the SOM response, while being influenced by the parameter \( \delta \). Both amplitude and phase
responses, when considered as spatial amplitude and phase dependencies, take up the width of $2w + \delta$, so that it is approximately equal to parameter $\delta$ for large beam separations and to the beam width $2w$ in case of smaller separations. The phase ridge (Fig. 2c) can be treated as a sum of two opposite phase steps, whose distance should be compared to the width of $2w + \delta$, separating thereby the regime of pure superposition of step responses for wide (width $> 2w + \delta$) ridges from the regime of destructive interference of step responses for narrow ridges. It should be noted that the beam separation $\delta$ provides an additional (as compared with the SOM response) degree of freedom for the SDHM response optimization in the cases of known a priori information about the object. For example, for the most interesting in the current context case of surfaces with constant reflection phase gradients, the phase response is directly proportional to the product of the phase gradient value and the parameter $\delta$.

The main subject of our analysis is the SDHM application for the characterization of reflective phase-gradient metasurfaces, consisting of periodically repeated supercells producing phase gradients for reflected optical fields [10]. Each supercell consists of individual cells (of the same subwavelength size) producing different phases in reflected optical fields. In the case of gap-plasmon based metasurfaces, specific reflection phases are ensured by properly choosing the dimensions of metal nanobricks of individual cells [20]. The simplest phase-gradient metasurfaces are used for beam steering that is achieved by producing a constant (average) reflection phase gradient of $2\pi/\Lambda$ within the supercell of the length $\Lambda$, so that the beam steering angle is determined by the ratio between the light wavelength $\lambda$ and the metasurface period $\Lambda$. These gradient metasurfaces are therefore equivalent in their operation to blazed diffraction gratings [21].

Generally speaking, the phase and amplitude reflection characteristics of the sample surface are encoded in the SDHM phase and amplitude responses in a rather complicated manner that does not allow them to be separately retrieved. Even the direct problem of finding the SDHM response for a given reflectivity profile within the thin phase-amplitude screen approximation requires numerical simulations using the above formulae [15]. Nevertheless, there is one particularly important configuration, a sample surface with a purely phase profile featuring a constant reflection phase gradient $G$ (i.e., the case of an ideal phase-gradient reflective metasurface designed for the beam steering [21]), that can analytically be dealt with. It is seen [Eq. (1)] that, in this case, the complex SDHM response reads:

$$D(x, \delta) = B \cdot \exp \left[ -i \cdot G \cdot \delta \right] \int_{-\infty}^{\infty} h(x) \cdot R \cdot \exp \left[ i \cdot G \cdot (x - x) \right] dx.$$  \hspace{1cm} (3)

The SDHM phase response can then be expressed as follows:

$$\phi(x, \delta) = \arg(D(x, \delta)) = G \cdot \delta.$$  \hspace{1cm} (4)

The phase response is thereby proportional to both phase gradient and probe beam separation, providing a direct way of adjusting the SDHM sensitivity by simply changing the frequency shift between probe laser beams. Once the SDHM phase response $\phi$ and probe beam separation $\delta$ are known, the constant phase gradient $G$ can straightforwardly be determined. It should be emphasized, that such a simple procedure can only be applied to metasurfaces with a constant (spatially independent) amplitude reflection coefficient. Only in this case, does the phase response depend only on the phase reflection coefficient while the amplitude response depends only on the constant amplitude coefficient $R$ of the complex reflection coefficient $r(x)$. If the amplitude reflection coefficient is a function of the $x$-coordinate, a simple analytical solution does not exist, a direct way of finding the phase gradient cannot be devised, and the SDHM response can only be calculated numerically.

Another important reflective sample configuration, a purely phase object featuring one step-like phase change $\Delta \phi$, can only be treated approximately with asymptotically accurate formulae obtained for two limiting cases:

1. If a distance between probe beams is significantly larger than the beam widths: $\delta >> w$, the phase response maximum is simply equal to the phase step:

$$\phi(\text{max}) = \Delta \phi.$$  \hspace{1cm} (5)

2. If a distance between probe beams is significantly smaller than the beam widths: $\delta < w$, the phase response is more complicated and can be calculated numerically.
2. In the opposite limit $\delta \ll w$, the phase response maximum can be evaluated as follows:

$$\phi(\text{max}) = \frac{[\Delta \phi]}{2w} \delta.$$  \hfill (6)

The above relations can be considered as consequences of Eq. (4) when assuming that a sharp phase step can be perceived (by the SDHM) as a phase gradient $G \sim (\Delta \phi) / \delta$, in the first case, and $G \sim (\Delta \phi) / 2w$, in the second case.

The aforementioned relations [Eqs. (3)- (6)] are important for the understanding of general SDHM characteristics, but not sufficient for the characterization of phase-gradient metasurfaces with the help of the SDHM response. In the numerical calculations presented in the next section, we take into account the fact that the phase reflectivity of practical phase-gradient metasurfaces consists of a discretized phase levels repeated within the metasurface supercell [1, 10]. In this case, detailed numerical calculations based on Eqs. (1) and (2) are needed.

3. Numerical modelling

For modelling purposes, the phase-gradient metasurface can be represented as a reflecting surface, whose phase reflection coefficient varies periodically (period $\Lambda$) featuring six discretization levels that cover the $2\pi$ phase range (Fig. 3a). It means that the object reflectivity depends on only one surface coordinate, $x$, and we can use the thin phase-screen model neglecting effects caused by defocusing. The SDHM response characteristics are not changed, if the geometrical configuration parameters are scaled with the light wavelength, and these parameters ($w$, $\delta$, and $\Lambda$) will hereafter be given normalized by the wavelength along with the scanning coordinate. Furthermore, since one would like to achieve the best SDHM spatial resolution, we consider in the following only one value of the probe beam half-width: $w = 1$, which can be realized in practice with a microscope objective having a sufficiently large numerical aperture. For simplicity, we have also use the same scan interval of [-15,15] containing the number of supercells corresponding to the considered metasurface period or, in other words, the considered supercell width $\Lambda$.

Our analysis of the SDHM response to a constant average phase-gradient profile we commence with the consideration of the influence of supercell width or, more precisely, the width of an individual cell ($\Lambda/6$). The beam separation $\delta = 1$ is chosen as a good compromise between the phase sensitivity, increasing for larger $\delta$, and the spatial resolution, which is close to $2w + \delta$ and thus decreasing for larger $\delta$. It is seen (Fig. 3b) that, for sufficiently small supercell widths ($\Lambda = 4$ and 8) so that the cell widths are close to subwavelength ($\Lambda/6 = 2/3$ and 4/3) and thereby not optically resolved ($2w + \delta = 3$), the SDHM phase response is essentially flat featuring the phase level given by Eq. (4), in which the phase gradient should be considered as being averaged over the supercell: $G = 2\pi/\Lambda$. For sufficiently large supercell widths ($\Lambda = 14$ and 30), the individual cells ($\Lambda/6 = 7/3$ and 5) become optically resolved, and the SDHM phase response transforms gradually into a series of phase-step responses akin those shown in Fig. 2b. These two extreme cases, $\Lambda = 4$ and 30, are considered further to highlight the influence of the beam separation $\delta$ on the SDHM phase response. It is seen that, for unresolved cells ($\Lambda/6 = 2/3 \ll 2 + \delta$), the SDHM response is flat and, in accordance with above discussion, equal to $G\delta = 2\pi \delta / \Lambda$ (Fig. 3c). For well or partially resolved cells ($\Lambda/6 = 5 \geq 2 + \delta$), the SDHM response transforms from well resolved peaks to oscillations around the average level that is again is equal to $G\delta = 2\pi \delta / \Lambda$ (Fig. 3d).

We would like to emphasize that, in practice, gradient metasurfaces feature subwavelength-sized individual cells to avoid diffraction (on individual cells) into free-space propagating waves, so that the number of individual cells in a supercell is typically increasing proportionally for large metasurfaces periods $\Lambda$ [1, 10]. In the following, we consider the supercell width constant: $\Lambda = 3$, so that the individual cells are sufficiently subwavelength ($\Lambda/6 = 0.5$), and investigate the influence of small deviations in the phase or amplitude reflection coefficients introduced into only one (third)
cell in each supercell: $\delta \varphi = \pm m \frac{\pi}{60}, m = 0, \pm 1, \pm 2, \ldots$ (see Fig. 3a) or $\delta R = -m \frac{1}{20}, m = 0, 1, 2, \ldots$. In practice, these deviations can be resulting from fabrication imperfections and/or design inaccuracies due to errors in the material constant used for the design simulations [20]. It is seen that, upon comparison with the SDHM responses from the perfect (undisturbed) metasurface (Fig. 4a), both the phase (Fig. 4b) and amplitude (Fig. 4c) deviations can easily be detected due to ripples in the corresponding SDHM phase responses. Moreover, the presence of a phase deviation can be distinguished from that of an amplitude one due to different symmetry in the SDHM phase responses in these cases for not very large beam separations: $\delta \leq 1$ (cf. Figs. 4b and 4c). The observed symmetry in the response for phase deviations can be understood from the symmetry of the SDHM phase response for a phase ridge (Fig. 2c), since a phase deviation can be viewed as a phase ridge on the averaged linear phase profile of an ideal phase-gradient metasurfaces. We note in passing that the ripple amplitude in the SDHM phase response disturbed by phase or amplitude deviations is as expected found increasing linearly with amplitude of these deviations, which is controlled in our calculations by the choice of the parameter $m$ in the above expressions for phase and amplitude deviations.

We conclude by considering the influence of the supercell width on the ripple amplitude in the SDHM phase response disturbed by phase deviations in one cell of each supercell. The corresponding dependencies for two most practical SDHM configurations (with the beam separations $\delta = 0.5$ and 1) ensuring both relatively high sensitivity and spatial resolution demonstrate that even a rather small phase deviation of $\sim 3^\circ$ can be detected (Fig. 5). The ripple amplitude is found to be practically constant for reasonably large supercell widths ($\Lambda > 4$), while being proportional to the beam separation. Overall, in a simplest case of a periodic defect in only one cell, the SDHM phase response should help in identifying the nature and strength of the defect, especially when comparing the experimental responses (with different beam separations) with numerical simulations carried out for an ideal (designed) metasurface. The general case, of both phase and amplitude deviations in different cells, especially in non-periodic fashion poses many challenges that require very extensive and detailed numerical simulations going beyond the framework of the current work.

4. Conclusions

We have considered the usage of well-developed SDHM experimental techniques for the characterization of reflective phase-gradient metasurfaces designed for the beam steering, i.e., the phase-gradient metasurfaces that introduce linear phase gradients in reflected optical fields. The SDHM complex differential response when imaging the phase-gradient metasurfaces was considered in the framework of the thin-screen model. It was shown, both analytically and numerically, that for the linear phase-gradient metasurfaces with a constant (spatially independent) amplitude reflection coefficient, the SDHM phase response is equal to the product of the phase gradient and probe beam separation. For practical cases of phase-gradient metasurfaces with step-like phase reflectivity profiles, the SDHM response can be adjusted for the characterization of the average phase gradient or, in principle, for the characterization of reflection phases of individual cells if the latter are large enough. It was also found that by carefully measuring the SDHM response of a gradient metasurface, one should be able of detecting small ($\sim 5\%$) amplitude and phase deviations (with respect to the design values) in the optical field reflected by an individual subwavelength-sized cell of the metasurface supercell.

The SDHM technique described can be used not only for investigations of metasurfaces with linear phase gradients, but also for other metasurface types. Considering the results reported here, one can surmise that, for any other (particular) metasurface type, the probe beam parameters (their sizes and separation) should be optimized in order to make the metasurface profile retrieval easier and more reliable. However, the proper modeling of the SDHM response and the inverse
problem solution [22] would be required to understand the characterization results obtained for metasurfaces with very high phase gradients (and therefore very small cell sizes). In the extreme case of metasurfaces designed for the excitation of surface waves, one can also resort to the metasurface characterization at shorter wavelengths, since it is rather straightforward to simulate the “ideal metasurface” response (with known design parameters) for any wavelength. Therefore, while there are certain limitations when the SDHM technique is applied in a simple direct manner, usage of the SDHM response modeling and conducting the characterization with different wavelengths should allow one to deal with any metasurface operating in reflection. Overall, we believe that the SDHM can advantageously be used for the characterization of reflective phase-gradient metasurfaces, a task that is extremely important for further technological progress in their practical applications. The corresponding experimental investigations are in progress, and will be reported elsewhere.

References


Figures captions

Fig.1. Schematic SDHM configuration, where $f_1$ and $f_2$ – shifted optical frequencies of two probe beams, $f_0$ – unshifted optical frequency, $f_i$ – heterodyne frequency, $\delta$ – the distance between the probe beams at the object plane, $I$ – signal from a photodetector.

Fig.2. Basic example of SDHM response calculation: (a) Amplitude profile of the SDHM probe beams with the half width $w = \lambda$ and separation $\delta = 0.25\lambda$ (solid), $\delta = 4\lambda$ (dashed lines); (b) SDHM response on 60°-phase step object. Phase response: $\delta = 0.25\lambda$ (solid), $\delta = 4\lambda$ (dash-dotted line); Amplitude response: $\delta = 0.25\lambda$ (dashed), $\delta = 4\lambda$ (dotted line); (c) SDHM response on 60°-phase 5\lambda-wide ridge object. Phase response: $\delta = 0.25\lambda$ (solid), $\delta = 4\lambda$ (dash-dotted line); Amplitude response: $\delta = 0.25\lambda$ (dashed), $\delta = 4\lambda$ (dotted line).

Fig.3. SDHM phase responses for the phase-gradient metasurface modell with various parameters: (a) Supercell phase profile, $\delta \phi$ – phase deviation in one (third) unit cells; (b) Responses for $\delta = \lambda$, $\delta \phi = 0$ and various supercell widths: $\lambda = 4\lambda$ (dash-dotted), $8\lambda$ (dotted), $14\lambda$ (solid), and $30\lambda$ (dashed line); (c) Responses for $\lambda = 4\lambda$, $\delta \phi = 0$ and various beam separations: $\delta = 0.25\lambda$ (solid), $0.5\lambda$ (dotted), $\lambda$ (dashed), and $1.5\lambda$ (dash-dotted line); (d) SDHM phase responses for $\lambda = 30\lambda$, $\delta \phi = 0$ and various beam separations: $\delta = 0.25\lambda$ (1), $0.5\lambda$ (2), $\lambda$ (3), $2\lambda$ (4), $3\lambda$ (5) and $4\lambda$ (6).

Fig.4. SDHM phase responses for the phase-gradient metasurface with $\lambda = 3\lambda$, undisturbed/disturbed profile and $\delta = 0.25\lambda$ (solid), $0.5\lambda$ (dashed), $\lambda$ (dotted), and $1.5\lambda$ (dash-dotted line); (a) Responses for the undisturbed profile ($\delta \phi = 0$, $\delta R = 0$); (b) Responses for the phase-disturbed profile $m = 3$; ($\delta \phi = \pi/20$); (c) Responses for amplitude-disturbed profile, $m=3$, ($\delta R/R = 15\%$)

Fig.5. Supercell width dependences of the ripple amplitude in the SDHM phase responses for the phase disturbed profile, $m=1$, ($\delta \phi = \pi/60$) and various beam separations: $\delta = 0.5\lambda$ (solid), and $\delta = \lambda$ (dash-dotted line).
Figure(s)
Figure(s)

Fig. 4a

Fig. 4b

Fig. 4c
Figure(s)

![Graph showing phase response versus \( \Delta/\lambda \) with three curves representing different scenarios.](image)

Fig. 5