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Leptonic CP violation theory

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Abstract. I summarize the status of theoretical predictions for the yet to be measured leptonic CP phases, the Dirac phase δ and the two Majorana phases α and β . I discuss different approaches based on: (a) a flavor symmetry without and with corrections, (b) different types of sum rules and (c) flavor and CP symmetries. I show their predictive power with examples. In addition, I present scenarios in which low and high energy CP phases are connected so that predictions for the CP phases α , β and δ become correlated to the sign of the baryon asymmetry Y_B of the Universe that is generated via leptogenesis.

1. Introduction

Assuming three light neutrinos, lepton mixing can be parametrized with three mixing angles θ_{ij} , one Dirac phase δ and up to two Majorana phases α and β . It is encoded in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha/2} & 0 \\ 0 & 0 & e^{i(\beta/2+\delta)} \end{pmatrix}$$

with $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$ and $0 \leq \theta_{ij} \leq \pi/2$ as well as CP phases ranging between 0 and 2π . Neutrino oscillation experiments have made a considerable progress in the determination of the lepton mixing angles, see e.g. [1, 2], while there is no information yet on the leptonic CP phases. There are first indications from T2K and NO ν A as well as from global fits [1, 2] about a preferred value of δ , $\delta \simeq 3\pi/2$. Since it is yet unknown whether neutrinos are Majorana or Dirac particles, we have no information either about α and β .

2. Predictions of CP phases with a flavor symmetry

Flavor symmetries turned out to be a powerful tool in order to explain the values of the lepton mixing angles. Lepton masses are treated as free parameters in this approach and only distinguish the different generations. Two of the most prominent examples are the flavor symmetries A_4 and S_4 which can be the origin of tribimaximal (TB) mixing. In the following, I assume three generations of charged leptons and Majorana neutrinos that are grouped in three-dimensional representations $\mathbf{3}$ of a flavor symmetry G_f . The latter acts in a non-trivial way on the space of lepton generations. We focus on symmetries G_f which are non-abelian, finite and discrete. From experimental observations we know that such a symmetry cannot be unbroken at very low energies. The way how this symmetry is broken is crucial in order to constrain



the values of the lepton mixing parameters. The most predictive scenario is to require the existence of residual symmetries G_e and G_ν in the charged lepton and neutrino sectors [3, 4, 5], respectively. Since I assume that there are three generations of Majorana neutrinos the maximal size of G_ν is $Z_2 \times Z_2$ and in order to maximize predictive power I take $G_\nu = Z_2 \times Z_2$. In the same vein, in order to enhance the predictive power in the charged lepton sector I assume G_e to have a generator with three different eigenvalues. The form of the charged lepton mass matrix m_e and the Majorana mass matrix m_ν of the light neutrinos are restricted by the choice of G_e and G_ν . Consequently, also the contributions U_e and U_ν to the lepton mixing matrix $U_{PMNS} = U_e^\dagger U_\nu$, arising from the diagonalization of m_e and m_ν , respectively, are fixed and thus also the form of U_{PMNS} . Inspecting this situation in more detail, we find that the columns of the unitary matrix U_e can be rephased. Similarly, the columns of U_ν can be rephased, i.e. U_ν and $U_\nu K_\nu$ (K_ν unitary and diagonal) cannot be distinguished. While the former has no physical relevance, the latter freedom in the matrix K_ν tells us that this approach cannot constrain Majorana phases. In addition, the ordering of columns of U_e and U_ν and hence of rows and columns of U_{PMNS} is not fixed, since this approach does not comprise a theory of lepton masses. There are 36 different possible forms of U_{PMNS} that lead to (potentially) different numerical values for the lepton mixing angles θ_{ij} which have to be compared to the experimental data. On top of that, the CP phase δ is fixed to a numerical value, up to π . There has been an intensive search for groups G_f , that are subgroups of $SU(3)$ and/or $U(3)$, see [6, 7, 8, 9, 10, 11, 12]. Crucial results of these efforts are: (a) all mixing patterns that are compatible at the 3σ level or better with the experimentally measured lepton mixing angles have a trimaximal mixing column, i.e. the elements of the second column of the PMNS mixing matrix have all the same absolute value, and thus $\sin^2 \theta_{12} \gtrsim 1/3$ and (b) the CP phase δ is trivial, i.e. $\delta = 0$ or $\delta = \pi$. We note that these conclusions can change, if one neutrino is massless, neutrino masses are (partially) degenerate or neutrinos are Dirac particles, see [8, 13, 14, 15, 16]. In order to relax the two main results, $\sin^2 \theta_{12} \gtrsim 1/3$ and $\delta = 0$ or $\delta = \pi$, several approaches have been discussed in the literature. One idea is to reduce the residual symmetry in the neutrino sector [17], i.e. to use $G_\nu = Z_2$. In this way, one free parameter enters the PMNS mixing matrix and thus the CP phase δ becomes less constrained. One example is $G_f = \text{PSL}(2, 7)$, $G_e = Z_7$ and $G_\nu = Z_2$ [17] in which the correlation between $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ depends in such a way on δ that both mixing angles turn out to be close to their experimental best fit values, if δ is close $\pi/2$ or $3\pi/2$. In a second approach, inspired by models that realize the approach in a scenario in which G_f is spontaneously broken, one assumes that corrections arise to the lepton mixing matrix that is obtained from G_f and its residual symmetries [18, 19, 20, 21, 22, 23, 24]. In such a case a relation between the experimentally measured mixing angles θ_{ij} and the CP phase δ , i.e. a sum rule for $\cos \delta$, can be derived. Hence, based on the distribution of the experimentally measured values of θ_{ij} a probability distribution/likelihood for $\cos \delta$ can be obtained which can be centered around different values of $\cos \delta$, e.g. $\cos \delta \approx 0$ or $\cos \delta \approx -1$. Model-dependent effects from renormalization group (RG) running, higher-dimensional operators, additional states like sterile neutrinos, etc. are not considered.

3. Predictions of CP phases with a flavor and a CP symmetry

In the recent past [25] the approach with a flavor symmetry has been extended by a CP symmetry which is also imposed on the underlying theory. This CP symmetry is represented by a CP transformation X that acts non-trivially on the flavor space [26], similarly to an element of the flavor group G_f . Its action on a set of n scalar (complex) fields ϕ_i , $i = 1, \dots, n$, is $\phi_i \rightarrow X_{ij} \phi_j^*$. The CP transformation X is a unitary matrix and, in addition, symmetric, if we require that applying the CP transformation twice shall be equivalent to the identity transformation: $X^* X = X X^* = 1$. In a theory with a flavor and CP symmetry certain consistency conditions have to be fulfilled [25, 27, 28]. The crucial change in the residual

symmetries lies in G_ν : instead of $G_\nu = Z_2 \times Z_2$ G_ν is taken to be the direct product of a Z_2 subgroup of G_f and the CP symmetry. In particular, the CP symmetry also constrains the form of m_ν , namely $X m_\nu X = m_\nu^*$. Consequently, also the choice of X constrains U_ν , the contribution to lepton mixing from the neutrino sector, which in turn puts constraints on the Majorana phases. The most prominent example is $\mu\tau$ reflection symmetry which exchanges a muon (tau) neutrino with a tau (muon) antineutrino [29, 30, 31]. Imposing this symmetry alone on m_ν in the charged lepton mass basis (i.e. $U_e = 1$), we obtain the following predictions: $\sin \theta_{23} = \cos \theta_{23}$, $\sin 2\theta_{12} \sin \theta_{13} \cos \delta = 0$ and the Majorana phases are trivial, i.e. $\sin \alpha = 0$ and $\sin \beta = 0$. Indeed, one can show that the mixing matrix U_ν arising from m_ν invariant under $G_\nu = Z_2 \times CP$ takes the generic form: $U_\nu = \Omega_\nu R(\theta) K_\nu$ where Ω_ν is a unitary matrix determined by the generator of the Z_2 symmetry and the CP transformation X , K_ν is a diagonal matrix with ± 1 and $\pm i$ and $R(\theta)$ a rotation in one plane through the undetermined angle θ . Like in the approach with a flavor symmetry only, this approach also does not comprise a theory of lepton masses, but only treats these as independent parameters. Thus, the PMNS mixing matrix depends, up to permutations of its rows and columns, on a single free real parameter θ . In particular, one column of the PMNS mixing matrix is fixed. All mixing angles and CP phases depend only on θ . Its preferred value is determined by the request to accommodate the measured lepton mixing angles well. In [32, 33] the series of groups $\Delta(3n^2)$ and $\Delta(6n^2)$, $n \geq 2$ and $3 \nmid n$, and several types of CP symmetries have been studied. All these groups have three-dimensional representations. Under the assumption that the residual symmetry G_e is given by a Z_3 subgroup of G_f all possible mixing patterns have been analyzed and it has been shown that only four different types of mixing patterns, compatible with the experimental data on lepton mixing angles, exist. These have different features and in the following I discuss one of them. In the analysis in [32] it is called case 3 b.1). The flavor group is $\Delta(6n^2)$. The first column of the PMNS mixing matrix is fixed and its particular form depends on the choice of the generator $Z(m)$, $m = 0, 1, \dots, n-1$, of the residual Z_2 symmetry, preserved in the neutrino sector. At the same time, the choice of the latter symmetry is constrained by the experimentally known value of the solar mixing angle, i.e. integer values with $m \approx n/2$ are favored. The free parameter θ is determined by the request to accommodate the reactor mixing angle well. For $m = n/2$, n even, strong constraints on the size of the CP phase δ can be derived

$$|\sin \delta| \gtrsim 0.71 \quad (1)$$

and we find that the absolute values of the sines of the Majorana phases α and β are equal and are determined by the choice of the CP transformation $X(s)$, namely

$$|\sin \alpha| = |\sin \beta| = |\sin 6\phi_s| \quad \text{with} \quad \phi_s = \frac{\pi s}{n} \quad \text{and} \quad s = 0, 1, \dots, n-1. \quad (2)$$

For the choice $s = 0$ and $s = n/2$ both Majorana phases are trivial. Numerical examples, given in [34], can be found in table 1. One combination of the Majorana phases α and β is accessible, in principle, in the process of neutrinoless double beta ($0\nu\beta\beta$) decay. Using the examples shown in table 1, we find the following constraints on m_{ee} , the quantity measurable in $0\nu\beta\beta$ decay: for $X(s = 1)$ and $X(s = 2)$ m_{ee} has a non-trivial lower bound, whereas for $X(s = 4)$ a cancellation cannot be avoided for normal ordering (NO) of neutrino masses, since both Majorana phases are trivial. Apart from the constraints on CP phases also the lepton mixing angles are strongly restricted which further reduces the admitted values of m_{ee} . We show our results in figure 1. Another type of constraints on the Majorana phases can be derived from neutrino mass sum rules, see e.g. [35, 36, 37, 38, 39]. By now many more flavor symmetries have been combined with a CP symmetry, see e.g. [40, 41, 42, 43, 44]. In addition to the type of CP symmetries studied more general ones [45] and CP transformations with textures [46] have been considered as well as scenarios with two CP symmetries in the neutrino sector [47]. Considerable efforts

Table 1. Example of predictions of lepton mixing angles and CP phases from the approach with flavor and CP symmetries. The chosen group theory parameters are $n = 8$ for $\Delta(6n^2)$, $m = n/2 = 4$ for the generator $Z(m)$ of the Z_2 contained in G_ν and the different choices of s indicate the different choices of the CP transformation $X(s)$. The free parameter θ is adjusted in such a way that $\sin^2 \theta_{13}$ is accommodated well. The matrix K_ν is chosen as $K_\nu = 1$.

s	$\sin^2 \theta_{13}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin \delta$	$\sin \alpha = \sin \beta$
$s = 1$	0.0220	0.318	0.579	0.936	$-1/\sqrt{2}$
	0.0220	0.318	0.421	-0.936	$-1/\sqrt{2}$
$s = 2$	0.0216	0.319	0.645	-0.739	1
$s = 4$	0.0220	0.318	0.5	∓ 1	0

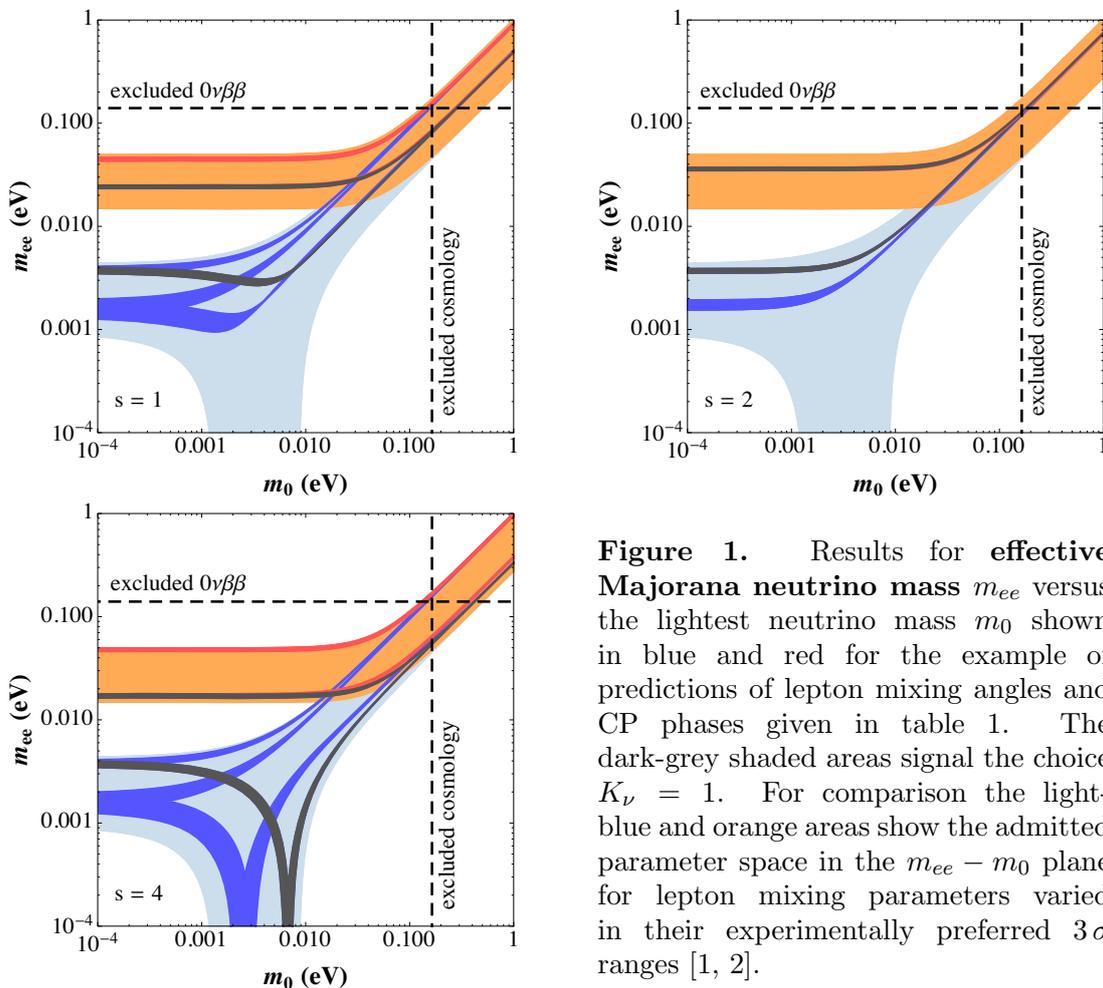


Figure 1. Results for **effective Majorana neutrino mass** m_{ee} versus the lightest neutrino mass m_0 shown in blue and red for the example of predictions of lepton mixing angles and CP phases given in table 1. The dark-grey shaded areas signal the choice $K_\nu = 1$. For comparison the light-blue and orange areas show the admitted parameter space in the $m_{ee} - m_0$ plane for lepton mixing parameters varied in their experimentally preferred 3σ ranges [1, 2].

have also been made to construct models in different contexts that implement this approach, see e.g. [48].

4. High energy CP phases in the lepton sector

A prominent explanation for the smallness of (Majorana) neutrino masses is the assumption that they are induced by heavy right-handed (RH) neutrinos N_i through the type 1 seesaw mechanism, i.e. $m_\nu = -m_D M_R^{-1} m_D^T$ with m_D being the Dirac neutrino mass matrix, $m_D = Y_D \langle H \rangle$, Y_D : Dirac Yukawa couplings and H Higgs field, and M_R being the Majorana mass matrix of RH neutrinos. In the following, I consider a scenario in which three such states are present in the theory. For Dirac Yukawa couplings Y_D of order one the correct order of the light neutrino masses is achieved for RH neutrino masses M_i in the range $10^{12} \text{ GeV} \lesssim M_i \lesssim 10^{14} \text{ GeV}$. Considering the type 1 seesaw formula we note: albeit M_R can contain phases, these are in general unrelated to the low energy CP phases δ and α, β . The phases in M_R are relevant for leptogenesis [49], an elegant mechanism for explaining the baryon asymmetry Y_B of the Universe. In the simplest scenario of unflavored leptogenesis Y_B is given by $Y_B \sim 10^{-3} \epsilon \eta$ with ϵ being the CP asymmetry due to the decay of RH neutrinos and η the efficiency factor. Experimental observations [50] determine Y_B precisely, $Y_B = (8.65 \pm 0.09) \times 10^{-11}$. For $10^{-3} \lesssim \eta \lesssim 1$ the generated CP asymmetry ϵ has to have values between 10^{-7} and 10^{-4} . Given that we have shown that low energy CP phases can be predicted with a flavor and CP symmetry it is tempting to search for a scenario in which also the high energy CP phases, playing a crucial role for leptogenesis, are predicted by some symmetry and its breaking. Indeed, there is an appealing framework in which this happens: the Majorana mass matrix M_R of RH neutrinos is invariant under the residual symmetry G_ν , while the Dirac Yukawa couplings Y_D , connecting heavy and light fields, are invariant under the entire flavor and CP symmetry. In this way, the light neutrino mass matrix m_ν is also invariant under G_ν . In addition, light neutrino masses m_i are inversely proportional to RH neutrino masses M_i and the contribution U_ν from light neutrinos to lepton mixing is given by the matrix U_R , diagonalizing M_R . The residual symmetry in the charged lepton sector is still given by G_e . For convenience, I choose a basis in which U_e is trivial. Thus, the PMNS mixing matrix is given by U_ν . Computing the CP asymmetry ϵ in such a scenario we find $\epsilon = 0$, simply because the relevant matrix $\hat{Y}_D = Y_D U_R$ is proportional to a unitary matrix and thus $\epsilon \propto \text{Im}((\hat{Y}_D^\dagger \hat{Y}_D)_{ij}^2)$ with $i \neq j$ vanishes. This result has already been known in scenarios in which lepton mixing is explained with a flavor symmetry alone [51, 52, 53, 54]. In explicit models we expect the form of the Dirac Yukawa couplings Y_D to be subject to corrections, which are in general invariant under symmetries other than G_f , CP and G_ν . In particular, I consider the case in which these corrections δY_D are invariant under G_e . They are proportional to a small symmetry breaking parameter $\kappa \sim 10^{-(3 \div 2)}$. Evaluating ϵ with the corrected form of the Dirac Yukawa couplings we find that ϵ becomes non-vanishing and proportional to κ^2 . Thus, the question why ϵ is small can be answered. Furthermore, the sign of ϵ , which is crucial for the (known) sign of Y_B , can be fixed, because the CP phases entering ϵ are – like the low energy CP phases – determined by the flavor and CP symmetry and their residuals in the neutrino and charged lepton sectors. As example I present the results for the choice of symmetries used in table 1 in figure 2. The dominant contribution to Y_B depends on one of the two Majorana phases, i.e. it is proportional to $\sin \alpha$. We observe for $\sin \alpha < 0$ ($s = 1$) and NO $Y_B > 0$ only for larger values of the lightest neutrino mass $m_0 = m_1$. In contrast, for $\sin \alpha > 0$ ($s = 2$) and again NO we find $Y_B > 0$ only for small values of m_0 . For $s = 4$, instead, both Majorana phases are trivial, while the Dirac phase δ is maximal, see table 1. Then, Y_B depends on δ as well as the parameters in δY_D . This makes a prediction of the sign of Y_B impossible.

For a study of flavored leptogenesis in a framework with a flavor and CP symmetry see [55, 56].

5. Comments on CP violation in the quark sector

Unlike in the lepton sector, CP violation has been measured in the quark sector. The CP phase δ_q is indeed large. Here I also briefly summarize the status of the explanation of δ_q with flavor (and CP) symmetries. The main problem that has to be overcome in order to satisfactorily describe

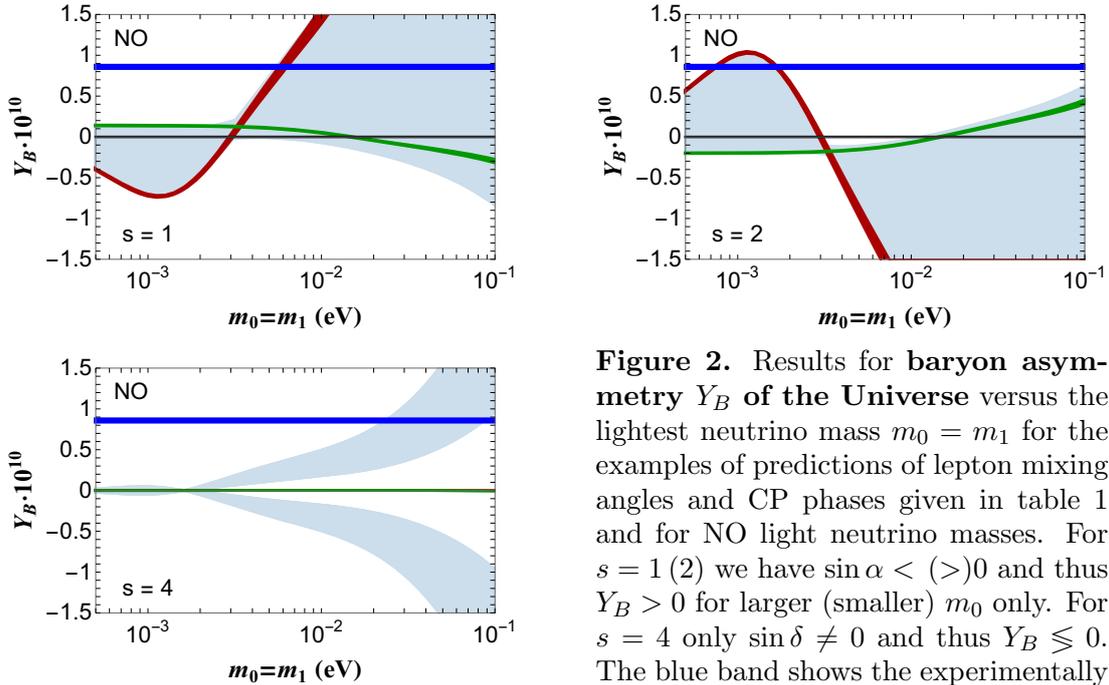


Figure 2. Results for **baryon asymmetry Y_B of the Universe** versus the lightest neutrino mass $m_0 = m_1$ for the examples of predictions of lepton mixing angles and CP phases given in table 1 and for NO light neutrino masses. For $s = 1$ (2) we have $\sin \alpha < (>)0$ and thus $Y_B > 0$ for larger (smaller) m_0 only. For $s = 4$ only $\sin \delta \neq 0$ and thus $Y_B \leq 0$. The blue band shows the experimentally preferred 3σ range for Y_B .

δ_q with flavor (and CP) symmetries is the adequate description of all three quark mixing angles, since only in the case of three non-trivial mixing angles the CP phase has physical meaning. First steps towards the understanding of the Cabibbo angle in terms of a flavor symmetry and its residuals have been made [3]. They show that the absolute value of the element V_{us} (and V_{cd}) of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix can be derived from dihedral symmetries D_7 and D_{14} that are broken to different residual Z_2 symmetries in the up and down quark sectors. As numerical value $|V_{us}| = \sin \pi/14 \approx 0.2225$ is found [3]. However, the fact that the other two quark mixing angles are (at least) one order of magnitude smaller than the Cabibbo angle makes it difficult to also describe them well. Intensive searches (with computer aid) have been performed [57, 58, 59, 60, 61]. Albeit these efforts it still seems a likely explanation that the two smaller quark mixing angles arise as corrections in this type of approach. It is, thus, not clear whether it is possible to understand the observed CP violation in the quark sector in terms of flavor (and CP) symmetries and their breaking. So, CP violation in the quark sector still remains a mystery.

6. Conclusions

Flavor and CP symmetries represent the most powerful tool for predicting the yet to be measured leptonic CP phases. Flavor symmetries alone can predict the Dirac phase δ . If the theory also has a CP symmetry, additionally both Majorana phases can be predicted. Furthermore, in a scenario with RH neutrinos high energy CP phases are constrained as well and thus the sign of the baryon asymmetry Y_B of the Universe can be fixed from symmetries.

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