Graphs as Images vs Graphs as Diagrams: A Problem at the Intersection of Semiotics and Didactics

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Abstract. Apart from their role in the philosophy of science and philosophical logic Peirce’s concepts of diagrammatic reasoning and forms of iconicity are becoming increasingly important for didactic and cognitive semiotic studies of science education. One specific problem (“graphs-as-images”) in students’ graph and model comprehension will be analyzed in the context of chemistry and chemical engineering education. Rather than an educational philosophy the long-term goal of this type of research is to establish a semiotics of science within specific domains.

Diagrammatic Reasoning in the Context of Educational Semiotics

One of the great discoveries of Charles S. Peirce was the central role of diagrammatic reasoning in scientific reasoning as well as the role of representational forms as uncovered in his attempts to classify different types of signs. Around the time of his Lowell lectures in 1903 Peirce revised his early classification of signs as icons, indexes and symbols into a more complex conception, where these types are seen as aspects that can be combined in concrete representations. This is why Peirce begins to consider the classification of signs as a problem to be analyzed within a “speculative grammar” (Peirce 1903b), i.e. a part of semiotic treating the properties of signs in themselves independent of what they might represent. From this perspective the iconic is no longer primarily to be understood as a class of “iconic signs” (or “hypoicons” in Peirce’s new terminology), but as that aspect of a sign relation by which it can indicate a similarity with an object of the sign. In contrast the indexical aspect concerns the causal anchoring of a sign to its object in some context, and the symbolic aspect concerns the law-like or conventional regularity that might be expressed in the sign and its interpretation. One of Peirce’s own examples in the Harvard lectures (Peirce 1903c) is the hygrometer, i.e. a scientific instrument measuring humidity of the air. Peirce uses the hygrometer to illustrate the role of the index: the instrument is
constructed to have a “real relation” to its object and it will react to changes in local humidity. As a reactive mechanism it works independent of any observation and interpretation, but in order for it to actually convey any information the instrument must also embody “an involved icon” (Peirce 1903c). In a modern hygrometer this is the function of the pointer and the scale of the instrument interface. Reading the relative humidity (the ratio of water vapor in the air at a given temperature) on a hygrometer requires the scale to represent changes in humidity (measured indirectly through some other property such as electrical resistance) as corresponding changes in the displayed measurement, and this analogical relation is iconic even though points and distances on a scale do not “look like” relative humidity. It is important for our topic here to recognize that iconicity can be abstract, and the similarity implied should not be confused with a subjective judgement of “resemblance”. To complete the hygrometer example it can be added that any law-like or conventional regularity of humidity to other phenomena (e.g. temperature) expressed by the instrument or our interpretation of it would count as a symbolic aspect. A scientific instrument will generally involve all aspects (iconic, indexical and symbolic) of the relation of signs to their objects. In the following we will, however, not pursue the derivation of Peirce’s extended classification of signs, but focus on his proposed subcategorization of hypoicons into images, diagrams and metaphors (Peirce 1903b).

The subcategorization of iconic signs (“hypoicons”) is important in the context of higher education in mathematics, science and engineering, because of the multi-representational requirements of communication and research practices in science (cf. our instrument example). Specifically we will analyze a few examples associated with the representational form (“sign type”) of graphs and the models they represent. Graphs are basically diagrams, but importantly so are algebraic expressions, and this points to an operational conception of diagrammatic signs beyond their descriptive foundation in similarity relations (Stjernfelt 2010). Graphs and algebra are iconic forms associated with “necessary reasoning” (Peirce 1902). Peirce maintained the proposition of his father Benjamin Peirce (who was one of the founding fathers of linear algebra) that mathematics is “the science which draws necessary conclusions”, but with the modification
that mathematics deals with “hypothetical state of things” (Peirce 1902). For Peirce diagrammatic reasoning is inherently mathematical in this sense of necessary reasoning.

In the later part of the 20th century diagrammatic reasoning was “rediscovered” by cognitive science, and reasoning with diagrams and external representations became a key issue in cognitive science starting from the AAAI symposium at Stanford in 1992 (Glasgow, Narayanan & Chandrasekaran 1995). However, Peirce’s contribution was largely disregarded at the time (Peirce 1903a; 1906). The motivation for this focus on the topic was primarily the importance of logic and knowledge representation for artificial intelligence research, and secondarily a growing interest within cognitive science in non-linguistic forms of thought and reasoning. Peirce was implicated because of his contribution to logic in a narrow sense, i.e. disregarding his conception of semiotics. The focus was on e.g. demonstrating the soundness of Venn diagrams or Existential Graphs as diagrammatic forms of logical reasoning, but the semiotic aspects of diagrammatic reasoning were later reintroduced (Pietarinen 2006, Stjernfelt 2007). The focus on mathematical logic points to a problem that paradoxically can be seen as inherited from Peirce himself: the tension between logical and discursive approaches to sign relations and signification. Peirce struggled all his life with this tension between semiotics as a logic of reasoning and semiotics as a construction of meaning within different genres and domains of discourse (Freadman 2004). The ambiguity that arises for the Existential Graphs between an operational iconic logic versus a diagrammatic reasoning with ontological implications (Stjernfelt 2011) can be seen as an instance of this general tension.

The focus here will be on diagrammatic reasoning in the sciences with respect to problems of graph and model comprehension in science education. Educational semiotics can be developed as a general philosophy of education (Semetsky 2010), but it is argued here that we should exploit the opportunity provided by cognitive semiotics (Zlatev 2012) to empirically investigate meaning construction and reasoning in educational settings within specific scientific domains.

A Phenomenographic Example from Chemical Engineering Education
A strange phenomena has been observed in science learning: students who adopt a “surface
“approach” to learning attempt to remember graphs independent of the models they represent. For example, when Cartesian graphs are detached from conceptual models and symbolic equations, students may attend to shape-features of graphs as if they were *images* rather than relational structures (diagrams). As we shall see in the examples below, this is a problem because different versions of the graph-as-image misconception all tend to short-circuit model comprehension.

Between 1995 and 1997 the author participated in a large scale investigation of problems of conceptual understanding in engineering education courses at the Technical University of Denmark. The focus was on problems of assessment associated with the use of computational exercises and examinations that did not evaluate students’ conceptual understanding. In the project students were exposed to concept tests and later interviewed to document their reasoning. Concept tests often involved free graph sketching questions rather than computational exercises (May 1997). In an advanced course on Transport Processes (mathematical models of the transport of mass, energy and momentum in fluid flows) we tested students understanding of concepts and models that should be familiar to students on entering the course. One question targeted Fourier’s law for heat conduction for a cylinder with a core heated to the temperature $T_1$, with radius $R_1$ and a flow of heat to the surface with radius $R_2$ and a lower temperature $T_2$. Students were asked to sketch the graph $T(r)$ of the temperature against the radius without doing any numerical computation. Phenomenographic studies often document a limited number of recurrent prototypical misconceptions within a given knowledge domain. A paradigmatic example was the analysis of conceptions of the Mole concept in chemistry (Marton, Lybeck, Strömdahl & Tulldal 1988). In the heat conduction question we found a similar simple distribution: even though 49 students sketched their own graphs, we only found three types of graphical answers (a, b and c below). In interviews we discovered that answers could be motivated in different ways. The correct answer is the hyperbolic curve $c$ in Fig. 1. This could reflect an adequate model understanding, but it could also arise from vague reasoning or rote learning, where students were not able to reconstruct how they came up with the correct answer.

Fourier’s law for heat conduction can be understood as a law schema (Kuhn 1970) for heat flow that has to be specified for each geometry under consideration. Heat flow will be
proportional to the product of an area and a heat flux through it, i.e. the rate of heat transfer per unit surface area. The heat flux will generally have the form – k \( \frac{dT}{dx} \) where the constant \( k \) is the thermal conductivity of the specific material and \( \frac{dT}{dx} \) is the temperature gradient. Heat is conducted in the direction of decreasing temperature and the flux is therefore negative in the positive direction of the \( x \)-axis. The linear answer (a) to the question will be appealing to some students, because they remember (correctly) that the heat flow through a metal wire can be considered as proportional to the temperature gradient over some distance from a heat source (Fig. 2). This is however the case for heat flow in one dimension (along the length \( L \)), i.e. not considering the area of the wire. This is how students are often introduced to heat conduction in high school physics.

Fig. 1. Concept question testing the adaptation of Fourier’s law for (steady-state) heat conduction to the special case of a heated cylinder (cross section left) and graph sketching answers of students in chemical engineering (right).

Fig. 2. The simple case of heat conduction in a thin metal wire where the linear answer is correct is typically the way students have been introduced to Fourier’s law for heat conduction.
As stated above Fourier’s law should be understood as a law-schema, i.e. a law that has to be specified for any given problem situation in order to be applied (Kuhn 1970), i.e. it has to be specified to fit the geometry of each case. In the case of a cylinder with radius \( r \) the area is \( 2 \pi r L \) and this will affect the temperature curve from \( T_1 \). In the cylinder the temperature gradient will depend on \( 1/r \) (Fig. 3) and the graph will be a hyperbolic curve (c in Fig. 1). A model-based understanding of the law schema will imply that the graph for \( T \) will always have to depend on the geometry. Without doing any computations students should be able to reconstruct the qualitative graphs for prototypical cases like cuboid (“boxes”), cylindrical and spherical objects. This is in itself an example of iconic variation regulated by symbolic constraints and as such a form of diagrammatic reasoning, but our focus here is on the nature of the second misconception (b in Fig. 1). This answer is different by not having a foundation in the model.

![Law-schema](image)

In interviews we discovered that students answering b did not attempt to argue with reference to the conceptual model or the equations of heat conduction, but rather with a direct reference to what particular graphs “looked like” in textbooks. A short interview fragment can illustrate this. The student had originally drawn the correct graph (c), but had changed it to (b).

(Interviewer): Can you remember the kind of reasoning that was behind your correction of one answer to the other?
The “immediate idea” that the student forms has no foundation in model comprehension but attempts to recall a mental image of graphs of temperature difference as recalled from textbooks, or lectures. The student furthermore generate his own heuristic rule: “if it is something with temperature differences, then there are bended curved”. This is inadequate in several ways. First of all it is an overgeneralization since the graph will depend on the geometry, as we have seen, but it is also inadequate by being underspecified (vague), since curves can “bend” in many ways. To explore this type of graph comprehension problem in more detail, we will go back briefly to the first reporting of this phenomena in cognitive science and educational research.

From Phenomenography to Semiotics
Under the influence of cognitive science Claude Janvier and John Clement pioneered studies of graph comprehension problems in science and mathematics education in the 1980-ies, and they identified a “graph-as-picture” misconception in secondary school students (Janvier 1981; Clement 1985; Leinhardt, Zaslavsky & Stein 1990). An example is shown in Fig. 4: students were asked to sketch the graph of speed as a function of time for a problem situation where a bicycle runs down a hill. Some students would draw a graph depicting the problem situation visually (Fig. 4) rather than the actual graph of speed over time.

Clement described this as a mathematical literacy problem, where students apparently created a figurative correspondence between visual characteristics of the problem situation (e.g. the slope of the hill) and the shape of the graph. Clement reports two versions of the graph-as-
picture misconception. In the bicycle example there is a global correspondence error associated with the shape of the graph. In other cases a local visual feature of the sketch of a problem situation is seen as corresponding to specific features of a graph. This is called a feature correspondence error. In one study students were presented with graphs plotting car speed against distance along different racing tracks, and in this case some students established a false correspondence between peaks of the graphs and bends of the tracks when asked to infer how many bends they could “see in” (infer from) the graphs (Janvier 1981).

![Graph diagram](image)

Fig. 4. A graph-as-picture misconception in school physics redrawn after (Clement 1985).

Graph comprehension will always involve these two levels: a reading of the graph seeing and recognizing it as a particular type of representation, and seeing the conceptual model and its possible iconic variation in the graph. These levels have been described in aesthetic philosophy of pictures as “seeing as” and “seeing in” (Wollheim 1980; May 1999).

The focus on the erroneous “dyadic” correspondence between sketches of problem situations (as semiotic objects) and graphs (as external representations) will, however, tend to hide the triadic nature of the sign relations involved. Correspondence errors are linked to the interpretation of graphs, and we need to focus on the conceptual structures acting as interpretants. In our example there is no correspondence error in associating the heat flow graphs with the problem situations. University students do not make the mistake of seeing the graphs as directly depicting heat flow. The linear answer (a) will usually be a motivated model-based mistake in
disregarding the geometry. It is motivated in the sense that it is based in a correct reference to Fourier’s law (for the linear case), although they forget to modify the law schema according to the geometry, whereas the (b) answer seems to result from failed attempts to remember graph features independent of the model or law schema.

Graphical objects interpreted as graphs will represent relational structures, but what happens in answer (b) is that textbook graphs are treated by the perceptual cognitive system as images to remember. This is the significant aspect that is overlooked, if we disregard the role of triadic sign relations, i.e. the “graph-as-picture” misconception is not generally based in direct correspondence errors, but in the cognitive consequences of a desymbolization of diagrams to images. It is, in other words, a typological error of a semiotic nature: when a graph is seen as an image, students cannot examine the graph and its possible variations in diagrammatic reasoning. Thus, the seeing in is lost (seeing the model in the graph). Instead the graph-image is exposed to cognitive operations valid for images such as mental rotation and mirror image formation. These operations make sense for images, but not for diagrams here since they should be interpreted as graphs representing a model. We can of course perform image-based layout operations on diagrams as graphical objects, but only insofar as they do not distort the intended interpretation, cf. the infamous examples of misleading 3D perspective rendering of bar graphs (Kosslyn 1993).

Diagrammatic operations on relational structures in the form of parameter variation is involved in the thought experiments essential to model-based comprehension. For example, students will change selected terms of an equation to see “what happens if…”. Such mental experimentation is a core aspect of Peirce’s notion of diagrammatic reasoning, and it is necessary for the individual construction of mental models leading eventually to the conceptual understanding “behind” the equations and graphs. Diagrammatic reasoning is necessary for the accomplishment of intended learning outcomes in higher education. In the attempt to remember graphs based on their graphical features students risk violating significant differences between representational forms within “iconic signs”. C.S. Peirce understood algebraic equations as well as their graphical expressions as diagrammatic representations resting not only on icons of relations but relations aided and regulated by habits of law or convention (i.e. symbols) within
consistent representational systems (Peirce 1903a, CP 4.418). The desymbolization process proposed here (Fig. 5) takes place between different types of iconic signs, i.e. the images, diagrams and metaphors indicated in the speculative grammar (Peirce 1903b, EP 2:272-288). In exemplifying iconic signs we have to take into account that Peirce’s second classification of signs operate on aspects that are combined in concrete signs. For example, we can consider X-ray images to be good examples of the image type of iconic signs, but this is only an aspect since X-ray images are also indexical traces of the objects (e.g. organs, tissues) exposed in the imaging process. Furthermore, modern X-ray images are digitally manipulated to enhance desired features based on relevant medical and biophysical knowledge, and as such they embody conventional as well as natural regularities and will have to be considered as the diagrammatic result of a symbolization as well. X-ray images are not simply “images”. They rely on indexical, iconic image-like and iconic diagram-like (or even symbolic) features for their intended interpretation.

![Diagram](image)

Fig. 5. Desymbolization of graphs to graph-images within types of iconicity. Other processes of symbolization and desymbolization between iconic signs are possible but not our focus here.

Treating graphs as images degrades an intended inquiry about similarity of relations (at the level of graphs and their underlying models) into an inquiry about similarity of qualities (e.g. graphical features of images)\(^1\). When Cartesian graphs are handled as images, the operations allowed will no longer be constrained symbolically and limited to what is meaningful in the model (e.g. Fourier’s law-schema). If the mere shape of the graphs is what is recalled, then the

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\(^1\) In the extended sign classification (with 10 classes of signs) we might describe this transformation as going from genuine graphs as rhematic iconic legisigns to desymbolized graph-images handled as rhematic iconic qualisigns, cf. Houser’s classification of models based on types of iconicity (Houser 1991).
spatial orientation of graph-images and their symbolic interpretation relative to a coordinate system tend to be ignored.

Although mental imagery of 3D objects was an issue in early perceptual psychology since the work of Hermann von Helmholtz, the experimental study of mental rotation was first introduced by (Shepard & Metzler 1971), and mental rotation of spatial molecular structures is now recognized as an important representational competence in domains such as organic chemistry (cf. the subdomain of “stereochemistry”). For example, spatial manipulation of 3D molecular models is necessary for grasping many spatial problems in chemistry from simple isomers (molecules with similar composition but different structure) to complex protein structures; but it is significant from a semiotic point of view that some of these diagrammatic tasks in chemistry can be performed by alternative symbolic and heuristic means (Stieff 2007). Less attention has been paid to the inadvertent effects of 2D mental rotation of Cartesian graphs. Recent educational studies in mathematics stress the constructive role of image schemas, metaphorical projection and visualization for teaching and learning Cartesian graphs (Font, Bolite, & Acevedo 2010), but we should also recognize that “mental imagery” can disturb mathematical reasoning (Aspinwall, Shaw & Presmeg 1997)

It is well known that a graph or diagram in general is never a “direct image of a certain reality”, but rather a “figural expression of an already elaborated conceptual structure, like any other symbolic system” (Fischbein 1997, p. 157). However, Pierce’s concept of diagrammatic reasoning lifts up and important didactic issue, namely, that to understand the model expressed by graphs and equations, students need to perform thought experiments and work through the possible iconic variation of the graphical forms as expressions of relations in the model (May 1999). The confusion of graphs-as-images can also appear in cases where graph representations really do “look alike” each other as graphical objects, but where their intended interpretations differ because the represented dimensions and units are different. An brief example from physical chemistry is given below to show the generality of the phenomenon.
An Example from Physical Chemistry

In chemical reaction kinetics students will learn about the rate of chemical reactions and their classification as zero, first, or second order reactions. Reaction orders refer to the exponential character of the rate by which chemical reactions depend on substance concentrations. For a chemical reaction schematically written as \( aA \rightarrow bB + cC \), and where \( v \) is the rate at which the substance \( A \) is consumed in the process, the rate is given by \( v = k [A]^n \) where \( k \) is a rate constant for the specific process and \( n \) is an exponent which in simple cases is 0, 1, or 2. For a 0-order reaction the reaction rate only depends on the rate constant \( k \) and not on the substance concentration (\( [A]^0 = 1 \)). In this case the graph of \( [A] \) plotted against time \( t \) will be a straight line expressing the relation: \( [A] = -kt + [A]_0 \), where \( [A]_0 \) is the initial concentration of \( A \).

Problems can arise for students because they are often trained in the computation of reaction rates before they have a sufficient background in thermodynamics to understand the causal mechanisms involved in chemical reactions.

Fig 6. Prototypical plots for zero order and first order chemical reactions.

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2 Here the small letters \( a, b \) and \( c \) are the “stoichiometric” coefficients indicating the number of molecules of a substance taking part in a chemical reaction. Large letters \( A, B \) and \( C \) indicate the substances (reactants and products) involved. Chemical notation allows abstraction from a specific chemical process just like a mathematical equation can abstract from specific numerical values.
This can lead to attempts to remember features of graphs in reaction kinetics detached from the underlying models. If students reason with graphs as being simply “linear” or “curved”, as we saw in the previous example, this will lead to misinterpretations because the symbolic constraints on the graph interpretation in the form of variables and units can be overlooked. Fig. 6 shows prototypical graphs of zero and first order chemical reactions plotting substance concentrations \([A]\) against time, reaction rates against time\(^3\), and the natural logarithm of the concentration, \(\ln[A]\), against time (for the first order reaction). The purpose of the logarithmic plot is that it will show a linear relation if a reaction is a first order reaction, but not if the reaction is a second order reaction. The logarithmic plot is a cognitive artifact in the sense that it imposes a purely symbolic transformation on the data that students might obtain in experiments to determine reaction rates of a chemical process: the transformation does not produce anything “new”, but it makes certain properties of the data visually salient. If the plots, however, are treated in memory as graph-images rather than symbolically regulated graph-diagrams, the logarithmic plot for \(\ln[A]\) will exhibit a misleading visual similarity with the plot of \([A]\) for a zero order reaction, and they can accordingly sometimes be confused.

**Conclusion**

Educational research in science should not focus entirely on pedagogical issues or on the disciplinary knowledge itself, but should move into the domain of the discursive forms of knowledge as expressed in the *multiple representational forms* utilized in science and their different affordances for learning and understanding. Students often have literacy problems in working with graphs in science, and graph and model comprehension have to be supported and trained explicitly in the form of external representations corresponding to the different (valid) cognitive operations on graphs (Vogen, Girwidz, & Engel 2007). Thought experiments and working through variations of graphs and models (e.g. parameter variation) within a domain is necessary for students’ conceptual understanding. It is also necessary to support and train the

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\(^3\) This will be a negative differential expression for the substance being consumed, here A, i.e. the concentration of A will gradually be reduced by the chemical reaction in which it is used.
multiple perspectives implied by scientific theories and models as well as the complex transformations (conceptual and mathematical) imposed on scientific representations in order to modify their affordances for reasoning (Giere 2006; Ainsworth 2006). In modern science the complexity of these representational forms, transformations and perspective is considerable, but nevertheless we do not pay much attention in university teaching to the “language of science”, and to the literacy problems associated with these mathematical, philosophical and semiotic aspects of learning scientific domains.

Looking back at the examples mentioned here, it is clear that we intend high-school students to be competent in using Cartesian graphs and associated simple models in e.g. physics and chemistry, but in teaching science at a university level we tend to expose students to a multitude of representational forms, mathematical transformations and different perspectives on theories and models without explicitly addressing this as a literacy issue. Digital and web-based technologies seem to provide a foundation for external support of diagrammatic reasoning, but we need a cognitive semiotics and didactics of science for the careful analysis of how external representations can actually support conceptual understanding.

Some difficulties in learning science seem to be associated with literacy issues rather than real difficulties of the scientific content. The graph-as-image issue exemplifies this. In schools some pupils will fail to separate a graph representation of relations from a depiction of the problem situation (cf. the bicycle example in Fig. 4), and even if science students at the university should be competent in graph reading, they can still be confused about visual similarities between graphs that “look alike” although they represent different things (cf. the reaction order graphs in Fig. 6). Even students at an advanced level can fall back into learning strategies where they rely on mental images of graphs rather than model comprehension (cf. the heat conduction example in Fig. 1). The didactic conclusion is that we underestimate the importance of representational competences and the need to train students – not just in “learning the content” of their subject domains – but also in the multiple representational forms, transformations and perspectives appearing at this intersection between didactics, mathematics and semiotics.
References


