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Analysis of the Design Criteria for Ancient Greek and Roman Catapults

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Abstract: This paper will give a short overview of use of COMSOL Multiphysics for analyzing ancient Greek and Roman catapults with the main focus on the energy storing torsion springs. Catapults have been known and used in the Greek and Roman world from around 399 BC and a fully standardized design for powerful torsion catapults emerged around 270 BC, based on one basic factor, the diameter of the torsion springs. This value is then scaled to give all vital structural dimensions of the catapult. How optimal this design is has until now not been fully understood and earlier work has been dominated by trial-and-error methods. The use of COMSOL Multiphysics enables the construction of virtual catapult parts, non-linear analysis of structural parts where no analytical solution is known as well the analysis of the surviving designs, with respect to optimal performance. The result from COMSOL Multiphysics is compared to a traditional analysis and to data obtained from *Sigma X Section* software.

Keywords: Torsion constant, catapult design, spring.

1. Introduction

The reconstruction of ancient Greek and Roman catapults can be based on surviving design criteria from Greek and Roman authors. The types labeled “torsion-catapults” were developed around 350 BC and standardized around 270 BC, based on scaling of the size of all major components, after calculation of one prime parameter. It is necessary to combine information from several sources to be able to build a working catapult, as none of the sources have all the necessary information. A large stone throwing catapult is shown in fig. 1. There are differences in the surviving designs (mainly different measures) but until now has there been no detailed investigation in the significance of these differences for the performance of a given catapult type or size. It is not known if the standardized designs were optimal with regard to important parameters like energy storage versus

weight, projective speed (range) and maneuverability.

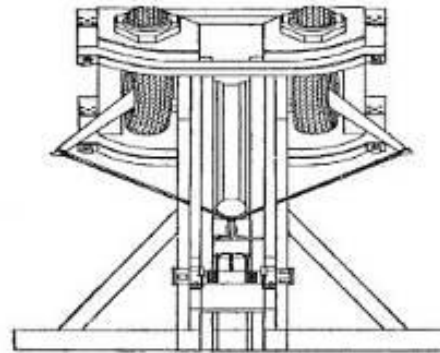


Figure 1: Drawing of a large stone throwing catapult. Note the rope based torsion springs. From [1,p.56] © Römerkastell Saalburg.

1.1 Terminology

The term “catapult” is used for projectile shooting engines, developed and used from approx. 400 BC until the fall of the Roman Empire, in the geographic areas under Greek-Roman dominance. Fig. 2 shows some of the basic parts of a torsion based catapult frame for an arrow shooter and indicates the terms used for the parts, seen from the front. For a full list see [2].

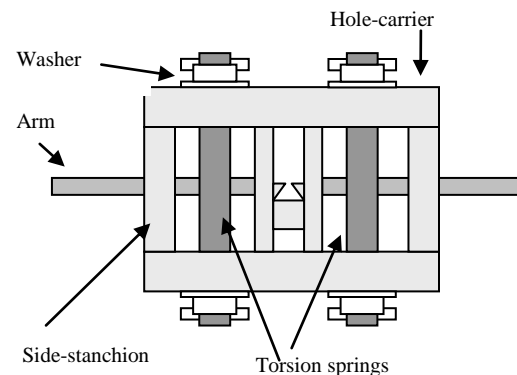


Figure 2: Main catapult parts and frame structure for a torsion catapult. Front view.

2 Ancient sources

The Greek sources give detailed (although not complete) design rules of 2 different catapult types, the arrow thrower (Greek: Euthytone) and the stone thrower (Greek: Palintone). The 4 most important sources for information about ancient catapult design are Biton, Philon, Heron and Vitruvius, together spanning the period from 240 BC to 25 AD. Bitons work is difficult to interpret, mainly due to the absence of diagrams (presumably lost). Philon mentions no catapults before fairly advanced torsion engines and his text is very important for understanding how to construct a working catapult. Herons *Belopoeica* provides almost no measurements, and the emphasis is on description of components. His *Cheiroballista* is fragmentary and very hard to decode. Vitruvius describes both the Greek and the Roman development in a historical perspective.

2.1 Early research history

The scientific investigation of the design and construction of Greek and Roman catapults started in the beginning of the 19. Century and can roughly be divided into a French and a German group. The French research was initiated by Emperor Napoleon III and is dominated by the works of the Generals Dufour and Reffye, who attempted to build reconstructions, based on the surviving ancient sources and also tried to develop a mathematical model for the shooting distance [3]. But Dufour and Reffye misinterpreted several crucial design information from the Greek writers and the reconstructions are therefore not correct, according to present knowledge [1]. The German group was dominated by Köchly and Rüstow [4], but their deep knowledge of Greek language was unfortunately not matched by their technical understanding, so they also did not succeed in developing correctly functioning reconstructions or usable mathematical models. The German tradition was later greatly influenced by artillery major Erwin Schramm, who in the period 1903-1920 experimented with the reconstruction of ancient catapults at the Saalburg museum near Frankfurt in Germany. The energy supplied to a given catapult is mainly stored in the torsion springs. Earlier attempts to calculate catapult

performance based on torsion theory did not generate usable results [5].

3. Catapult construction

The ancient sources state that all component sizes for a given catapult can be constructed from one base measurement, which is then scaled according to the needed size of the catapult. This standardization of the designs emerged around 270 BC, presumably in Alexandria. This base measurement was derived by two different methods, one for the Euthytone and another for the Palintone. The Euthytone (arrow thrower) was based on the length of the arrow and the Palintone (stone thrower) was based on the weight of the stone. The two formulas will briefly be discussed below. The factor in question is the diameter f of the torsion springs. Heron, Philon and Vitruvius give the formulas, but only Philon and Vitruvius have also supplied the ratios of the individual catapult parts, without those the calibration formulas would be worthless.

3.1 Calibration formulas

For an arrow shooter (Euthytone) will the diameter f of the torsion springs, according to both Philon and Vitruvius, have to be one ninth of the arrow length.

$$f = \frac{1}{9}L \quad (1)$$

An arrow shooter designed for 1 m arrows then required the diameter of the torsion springs to be 11.1 cm. For a stone shooters (Palintone) will the diameter f of the torsion springs (in dactyls, 1 dactyl=19.3 mm) have to be 1.1 times the cubic root of 100 times the weight W of the stone in attic mina (1 attic mina=436.6 grams).

$$f = 1.1 \sqrt[3]{100W} \quad (2)$$

A stone thrower for 50 minae stones (21.8 kg) required the diameter of the torsion springs to be 18.8 dactyls, equal to 36.3 cm. The largest stone throwers mentioned by Philon were shooting 3 talent stones (78.6 kg). The cubic root used in this formula is the first known appearance of a third degree equation in the history of

mathematics. Shooting distances in excess of 300 m are possible for both types.

3.2 Standardization

The standardization introduced in 270 BC linked all vital geometries to the calculated diameter of the torsion spring (eq. (1) and (2)), both for the arrow throwers and for the stone throwers. The ratios given by Philon and Vitruvius are not complete and it is also necessary to combine them for building a complete catapult. A catapult of any size can be constructed by applying the appropriate scaling factors. According to Philon were the rations discovered by experiment and experience, but the effect of a deviation of the individual multiplication factors has so far not been an object of research. Table 1 and 2 give for illustration some important ratios regarding the catapult frame and washers for euthytone and palintone. P indicates Philon data, V indicates Vitruvius data. For a full list, covering also winch and sliders, see [2, p.266].

Part	Height		Length/ diameter		Width	
	P	V	P	V	P	V
Hole carrier	1	1	6½	(6)	1 ½	1 ½
Washer	[3/4]	[3/4]	1 ¼	1 ¼		
Sides	3½	4			1½	1½

Table 1: Euthytone multiplication factors for f. (x) not provided, calculated from available information. [x] estimated value, difficult to calculate.

Part	Height		Length/ diameter		Width	
	P	V	P	V	P	V
Hole carrier	1	1	2 ¾	2 ¾	[2 ½]	2 ½
Washer	¾	¾	2	2	[2½]	1 5/12
Sides	5 ½	5 3/16			1 7/12	[1 7/12]

Table 2: Palintone multiplication factors for f. As table 1.

3.3 Catapult materials

The catapult frames were made of hardwood, mostly oak, and plated with iron plates. The washer for holding the springs were cast in bronze. The bolts holding the springs were of iron. The type and quality of spring material was

very important. Heron considered sinew to be the best material for springs but also mentions that women's hair was a suitable substitute in emergencies. Vitruvius also favored sinew to hair, but indicates that women's hair the best substitute in emergencies. Another source mentions that also silk and hemp could be used. Tests conducted at Reading University (UK) in the 1970'ies indicated that modern nylon rope behaved better than hemp rope, perhaps as good as sinew [6]. Schramm was unable to produce springs of sinew rope and used ropes of horse hair instead [1]. Table 3 below lists some mechanical properties for typical materials for comparison.

Material	Young's mod. (GPa)	Resilience (kJ/m ³)	Resilience (kJ/kg)
Spring steel	210	6700	860
Sinew	1.2	4100	3200
Carbon	410	5000	2000
Kevlar	130	27000	18000

Table 3: Mechanical (bulk) properties for some selected materials [8, p.68].

It can be seen that the maximum elastic strain energy (resilience) of sinew per volume is close to modern spring steel and 4 times as high when compared per weight. The properties of sinew are extraordinary and it is not surprising that it was recommended for catapult springs.

4. The torsion spring

The energy storage in the catapult takes place in the torsion springs and they have traditionally in literature been treated as solid cylinders, often of twisted ropes. But according to the ancient sources are the spring cords pre-stretched during insertion, so much that the diameter is reduced by 1/3. If pre-stretching was used would it not have been possible to twist the springs into solid cylinders.

4.1 Energy storage

The energy storage due to strain in a homogeneous linear-elastic cylinder of length L, when fixed at one end and with a symmetric torque T [Nm] applied in both ends resulting in the twisting angle θ , is given as

$$U = \int_0^\theta T d\theta = \frac{1}{2} T_1 \theta \quad (3)$$

where T_1 is the equivalent single torque [9,p.697].

In 1820 the French engineer A. Duleau derived analytically that the torsion constant is identical to the polar moment of inertia for a beam. This however holds only for a circular cross section. The angle θ is a function of the torque T_1 , length of the material L , the shear constant (modulus of rigidity) G and the polar moment of inertia J .

$$\theta = \frac{T_1 L}{JG} \quad (4)$$

G can be calculated from Young's modulus E and Poisson's ration ν for the specific spring material

$$G = \frac{E}{2(1+\nu)} \quad (5)$$

J is also called the torsion constant, specific for the cross sectional shape but is only valid for a circular cross section. The polar moment of inertia for a circular rod of diameter f can be shown to be

$$J = \frac{\pi f^4}{32} \quad (6)$$

Rearranging eq. (3) and eq. (4) gives

$$U = \frac{1}{2} \frac{T_1^2 L}{JG} \quad (7)$$

This shows that the energy depends linear on the length L , for a given applied torque. Philon defines the optimum length of the torsion spring L as 9 times the spring diameter f , thus the total energy stored in the catapult (4 half-springs) as a function of rotation angle equals

$$U_{Tot} = \pi \frac{f^3 \theta^2 G}{72} \quad (8)$$

4.2 Non-circular cross section

For non-circular cross sections there is no exact analytical function for the torsion constant. J has in eq. (4) to be substituted by K , being the generalized torsion constant. K will always for non-circular cross sections be lower than J and may only be a very small fraction of J [7, p.347]. The torsion constant K depends only on the cross sectional shape and not on any material

parameter, as the material parameters are contained within G .

Earlier modeling by Dufour [2], Cotterell [8] etc. have used the polar moment of inertia J for a full circle and have not considered the influence of opening in the torsions spring. But the openings are mandatory for fixing ropes are limbs. Numerous different cross-sectional shapes have been investigated by approximations, numerical methods, curve fitting to experimental data etc. [7,table 20]. But a solution for the shape of a circle with a slit like opening, which is the case for a real catapult spring, is not known to have been investigated earlier.

5. The COMSOL Multiphysics model

COMSOL Multiphysics has been used to estimate the value the torsion constant K due to the introduction of the slit and thus removal of mass in a torsion spring. The results have been compared with results obtained by "Sigma X Section", specialized software to calculate cross-sectional properties (www.sigma-x.net). A cylinder geometry has been defined, based on the known dimensions of the Ampurias catapult ($f=7.9$ mm), with a slit of variable width inserted and a center bar. See figure 3.



Figure 3: Geometry used in the COMSOL simulations. The diameter f and slit width w are scalable and the center bar is automatically adjusted to fill the slit.

A small force was applied symmetrically on each side of the center bar end surfaces as normal to the surfaces, thus forcing the cylinder to rotate a small angle θ . The displacement of the cylinder surface normal to the bar is used to calculate K , based on eq. (4). All cylinder end surfaces have been fixed by constraints in the model. The stationary solver in COMSOL 4.0a was used with standard settings and automatic

meshing. Changing mesh size showed no influence to be considered here. Different materials were tested (steel, nylon etc.) determining the shear modulus G given by E and ν via eq. (5), but also showed no influence to be considered here.

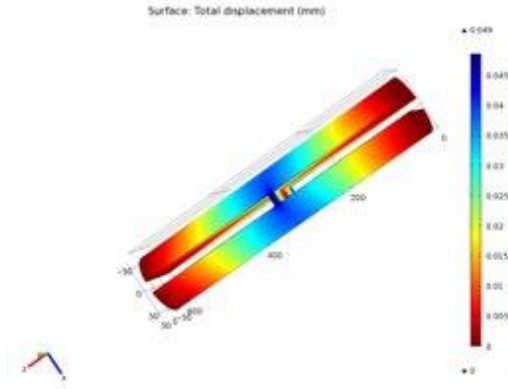


Figure 4: Simulated total displacement of a catapult torsion spring during applied torque.

For a cylinder without slit, the theoretical torsion constant K equals J . For $f=7.9$ cm the value of J is 382 cm^4 , acc. to eq. (6). COMSOL, with the present simple linear elastic model and stationary solver, returned 402 cm^4 . The deviation of around 5 % from the theoretical values is considered acceptable for this investigation. But how does the introduction of the slit for the catapult limbs and bars influence the torsion constant K ? The optimal width w of the slit, acc. to Philon, is equal to $1/5 f$. The variation of w from 0.5 to 2.0 cm has been investigated by COMSOL, showing a clear decline of K as a function of slit width w . See figure 5.

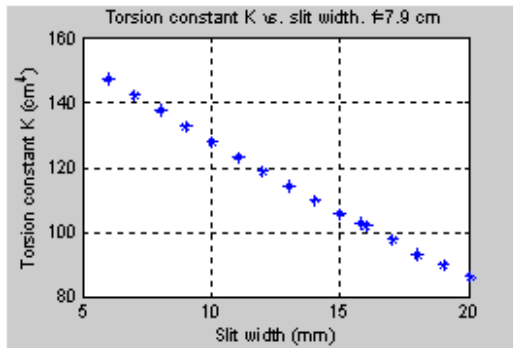


Figure 5: Torsion constant K as function of slit width w , determined via COMSOL.

It is also clear that K is much lower than for the solid cylinder, approx. 102 cm^4 for $w = 1/5 f$, actually approx. a factor of 4 lower! This simple investigation shows that the energy storage in solid torsion springs build acc. to the ancient sources is much lower than earlier anticipated.

As catapult fragments with spring diameters up to 30 cm are known it is relevant to investigate the ratio p between J (circle) and K (circle with slit) as a function of f (spring diameter), for $w = 1/5 f$. The bar width w was held at the same width as the slit in the model. See figure 6.

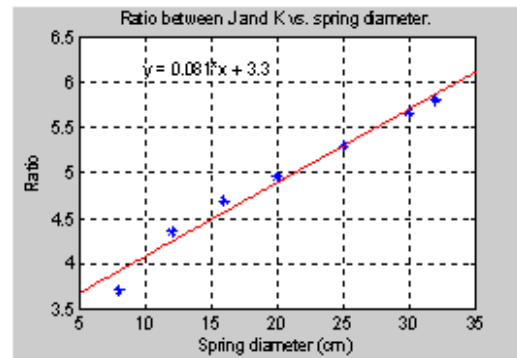


Figure 6: Ratio p between J and K for varying spring diameters.

It is seen that the ratio p between J and K is around 4.5 for all diameters investigated. A first order fit for the COMSOL results gives

$$p = 0.081f + 3.3 \quad (9)$$

The values returned by *Sigma X Section* are all 5.3 for all spring sizes, thus being in the same range. *Sigma X Section* only calculates on the cross section and does not take any deformation at load surfaces etc. into account. All results indicate that the energy stored by torsion in a solid spring with a slit is at least between 4 and 5 times lower than estimated. It is expected that for most practical applications will a constant value of 4.5 for p be adequate.

The present modeling does not take into account that torsion springs are composed of numerous parallel rope sections with air between, so a further reduction in effective K is to be expected, depending on the achievable packing density of the rope. Tests on a full size reproduction of a Greek catapult frame have

shown that only around 72 % of the available space can be filled with rope material, if rope with a diameter of $1/5 f$ is used as specified by Philon.

6. Experimental results

Tests have been carried out with a torsion spring inserted into a reconstructed full size catapult frame, constructed according to Philon. The rope as made of PE. The spring length L was 0.53 m, $f=7.9$ cm, rope pre-stress σ was around 40 MPa, the slit opening w was $1/5 f$. The applied torque T_1 was 310 Nm and resulted in an angle of rotation θ of 0.68 rad. With the parameters given/measured above can the stored energy in the experiment via eq. (3) be calculated to 104 joule. Cotterell calculates the energy in a torsion spring, constructed of ropes, as [8,p.191].

$$U = \frac{\sigma \pi f^4 \theta^2}{16 L} \quad (10)$$

where σ is the initial strain of the ropes in the spring. With the data given above will eq. (10) give approx. 240 joule stored in the spring, which is a factor 2.3 larger than the energy determined by experiment. Compensating for the slit area and the spring fill factor of approx. 72% in eq. (10) yields a stored energy of 110 joule, close to the experimental value.

7. Conclusions

The finite element analysis focused on catapult torsion springs and different calculations schemes for modeling have been investigated. Simulation with COMSOL Multiphysics has shown that the generalized torsion constant K must be used instead of the polar momentum of inertia J , when generalized torsion theory is used. This is due to the small slit, which is generated by winding rope around the fixation bars according to the ancient sources. No analytical solution exist for the resulting geometry and the COMSOL simulations have shown that K is a factor 4-5 lower than J . Comparison with experimental data has also shown that the torsion theory must be used with extreme care, as the torsion springs cannot be treated as solids, but should be treated as individual ropes. The results obtained demonstrate that the classical modeling methods,

using solid cylinders, result in an estimated energy storage level several times larger than results obtained by experiment with real rope based torsion springs with slits.

Ongoing and future work with COMSOL Multiphysics include non-linear modeling of rope-based torsion springs (especially sinew), geometry analyses of arbitrary sized catapult frames and other catapult parts as well as comparison with experimental data obtained from a reconstructed full size catapult.

“Accurate computation. The gateway to knowledge of all things and dark mysteries”.
Rhind Mathematical Papyrus. Egypt, circa 2500 BC.

8. References

1. E. Schramm, *Die antiken Geschütze der Saalburg*. Saalburg Museum (1918, reprint 1980)
2. E.W. Marsden, *Greek and Roman Artillery*, Oxford University Press (1969)
3. G.H. Dufour, *Mémoire sur l'artillerie des anciens et sur celle du moyenâge*. Paris–Geneve (1840)
4. Köchly and Rüstow, *Griechische Kriegsschriftsteller*, vol. I, Leipzig (1852).
5. D. Baatz, *Bauten und Katapulte des Römischen Heeres*. Franz Steiner Verlag Stuttgart (1994).
6. G. Landels, *Engineering in the ancient world*. University of California Press (1978).
7. W.C. Young, *Roark's Formulas for Stress and Strain*, 6th ed. McGraw-Hill (1989)
8. B. Cotterell and J. Kamminga, *Mechanics of pre-industrial technology*, Cambridge University Press (2000).
9. F. Beer et al., *Mechanics of Materials*, 3rd ed. McGraw-Hill (2002)

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