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Charting standard model duality and its signatures

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We investigate high and low energy implications of a gauge dual description of the Standard Model. The high energy electric theory features gauge dynamics involving only fermionic matter fields, while the low energy magnetic description features a quasisupersymmetric spectrum testable at colliders. The flavor theory is constructed via operators generated at the Planck scale, possibly by gravitational global symmetry breaking interactions. We further show that duality opens novel avenues for theories of grand unification.

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I. INTRODUCTION

We show that the Standard Model of particle physics (SM), in its current form, can be interpreted as the magnetic dual of a more fundamental theory. The overall idea of gauge-gauge duality is inspired by Seiberg's work for supersymmetric theories [1,2], extended to nonsupersymmetric cases in [3–5]. It is based on the existence of two theories that describe the same physics at low energies, where usually one of them is strongly coupled and the other weakly coupled and perturbative. In our proposal, the ultraviolet (UV) electric description of the Standard Model, at high energy, features only gauge and fermion fields, while the infrared (IR) magnetic dual contains scalars. We use a non-Abelian gauge symmetry of the SM as pivot for the duality, hence intertwining the number of gauge colors with the number of matter generations [6]. The SM duality has striking and testable consequences:

- (i) The IR magnetic dual features emergent (partial) supersymmetry [7].
- (ii) Naturally light scalars are required by the duality.
- (iii) A light gauge-singlet scalar, transforming as a flavor bifundamental, can Higgs the electroweak sector and generate flavor structures in the theory.

(iv) A universal Yukawa coupling is generated in the magnetic theory.

(v) The scalarless electric theory spurs new possibilities for grand unified models at high energy.

SM dual scenarios are characterized by two or three scales of new physics: a mass scale Λ_S for the non-SM composite states in the magnetic theory, expected to appear around few TeVs; the actual duality scale Λ_D where the electric theory becomes strong and confines, appearing at intermediate energies between the TeV and Planck scales; if the gauge couplings in the electric theory unify, they would define a third scale Λ_{eGUT} below Planck.

In this letter, we present a concrete and experimentally testable scenario of a nonsupersymmetric dual QCD sector within the SM realizing all the features listed above. After briefly introducing the duality, we highlight the predictions testable at present and future colliders, as well as model building avenues in flavor physics and grand unification.

II. THE GAUGE-GAUGE DUALITY

We develop our idea around the duality first presented in [5], where the UV electric theory is based on a gauged $SU(N)$ with N_f Dirac fermions in the fundamental representation and one Weyl fermion in the adjoint. Dualities without the adjoint fermion have been discussed in [3,4], which lead to phenomenological predictions on electroweak observables [8]. Gauge-gauge duality is mainly based on matching the global anomalies, which depend only on the fermionic matter content. Hence, it is natural to ask whether Seiberg duality applies to nonsupersymmetric cousin theories where elementary scalars in the UV theory are absent, while composite scalars are dynamically

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TABLE I. Matter content for the UV electric (top block) and IR magnetic (bottom block) theories. Only the latter features scalar fields.

Electric theory (UV)					
Fields	SU(N)	SU(N_f) _L	SU(N_f) _R	U(1) _V	U(1) _{AF}
λ	Adj	1	1	0	1
Q	F	F	1	1	$-N/N_f$
\tilde{Q}	\bar{F}	1	\bar{F}	-1	$-N/N_f$
Magnetic theory (IR)					
Fields	SU(X)	SU(N_f) _L	SU(N_f) _R	U(1) _V	U(1) _{AF}
λ_m	Adj	1	1	0	1
q	F	\bar{F}	1	N/X	$-X/N_f$
\tilde{q}	\bar{F}	1	F	$-N/X$	$-X/N_f$
M	1	F	\bar{F}	0	$-1 + 2X/N_f$
ϕ	F	\bar{F}	1	N/X	$1 - X/N_f$
$\tilde{\phi}$	\bar{F}	1	F	$-N/X$	$1 - X/N_f$
Φ_H	1	F	\bar{F}	0	$2X/N_f$

generated in the IR. The magnetic and electric theories are expected to describe the same IR physics where the electric gauge coupling becomes strong. The minimal dual theories [5] are illustrated by the field content in Table I. The numbers of colors in the two theories are related via the flavor symmetry:

$$X = N_f - N. \quad (1)$$

The magnetic theory consists of a magnetic ‘‘gaugino’’ λ_m in the adjoint representation of SU(X), N_f Dirac ‘‘quarks’’ q and \tilde{q} , and a ‘‘mesino’’ field M . The latter is a gauge singlet, transforming as a bifundamental of the flavor symmetry and required by the matching of the global anomalies. Massless colored scalars ϕ and $\tilde{\phi}$ are also required by the decoupling of flavors in the electric theory [9]: giving a mass to one flavor of electric quarks is matched in the magnetic dual by a vacuum expectation value (VEV) of the scalars, which breaks $SU(X) \times SU(N_f) \rightarrow SU(X-1) \times SU(N_f-1)$ (i.e., $N_f \rightarrow N_f - 1$). Their quantum numbers match those of the magnetic quarks, where the U(1)_{AF} charge can be interpreted as the R-charge of the components of supersymmetric multiplets:

$$(\mathcal{G}_\mu, \lambda_m), \quad (q, \phi), \quad (\tilde{q}, \tilde{\phi}), \quad (M, \Phi_H). \quad (2)$$

A light scalar multiplet Φ_H is, therefore, necessary to accompany the mesinos. Both mesino M and mes-Higgs Φ_H fields are naturally interpreted as composite states of the electric theory [6]

$$M \sim \tilde{Q}\lambda Q, \quad \Phi_H \sim \tilde{Q}\lambda\lambda Q. \quad (3)$$

Besides kinetic terms and gauge interactions, the magnetic theory allows for the following interactions, invariant under all global symmetries

$$\begin{aligned} \mathcal{L}_m \supset & yq\tilde{q}\Phi_H + y'q\tilde{\phi}M + \tilde{y}'\tilde{q}\phi M \\ & + \xi_L\lambda_m q\phi^\dagger + \xi_R\lambda_m\tilde{q}\tilde{\phi}^\dagger + \text{H.c.} \end{aligned} \quad (4)$$

and scalar quartic couplings. Such couplings are compatible with supersymmetry if the first three emerge from a superpotential coupling ($y = y' = \tilde{y}'$) and the last two from gauge interactions ($\xi_L = \xi_R = g_m$). Note that the magnetic degrees of freedom are weakly interacting at low energies, hence they can naturally be identified as SM states. From the duality point of view, instead, they emerge as deeply composite states of the electric theory when the electric gauge coupling g_e becomes strong at the scale Λ_D . We therefore pursue the fascinating possibility that one of the asymptotic free gauge groups of the SM, i.e., either QCD’s SU(3) (and its extension to a fourth leptonic color SU(4) or the weak SU(2)_L, describes an IR magnetic dual [10].

III. THE QCD-DUAL SM

It is tantalizing to define the SM duality on QCD gauge symmetry, as the flavor symmetry can be understood as containing the electroweak interactions as follows:

$$SU(N_f)_{L/R} \supset SU(n_g) \otimes SU(2)_{L/R}, \quad (5)$$

where $n_g = 3$ is the number of generations (leading to $N_f = 2n_g = 6$) and SU(2)_R is partly hypercharge gauged. Hence, the mes-Higgs scalar field is composed by bidoublets of SU(2)_L \times SU(2)_R, one of which could be identified with the Higgs field. This would lead to a minimal SM duality, described in Table II. The electric and magnetic gauge groups feature the same number of colors. As hypercharge is defined as

$$Y = T_R^3 + \frac{1}{6}Q_V, \quad (6)$$

leptons are introduced as elementary states required by the cancellation of the U(1)_Y gauge anomalies, like in the SM. An alternative possibility, discussed in [6], leads to a more compact duality by considering leptons as fourth color [18], hence unifying quarks and leptons in SU(4) multiplets. Here the magnetic dual is a Pati-Salam model which, however, requires to be further broken down to the SM. Models based on supersymmetric Seiberg duality have been proposed for both QCD [11,12] and Pati-Salam [13]. Remarkably [6], requiring that the electric gauge group remains non-Abelian, for either $N = 3$ or $N = 4$, i.e., $N_f - N \geq 2$, sets $n_g = 3$ as the smallest number of SM generations compatible with the duality. This lends an

TABLE II. Matter content for the UV electric (top block) and IR magnetic (bottom block) dual theories of QCD.

Electric theory (UV)					
Fields	SU(3)	SU(6) _L	SU(6) _R	U(1) _V	U(1) _{AF}
λ	Adj	1	1	0	1
Q	F	F	1	1	-1/2
\tilde{Q}	\bar{F}	1	\bar{F}	-1	-1/2
L	1	F	1	-3	0
\tilde{L}	1	1	\bar{F}	3	0
Magnetic theory (IR)					
Fields	SU(3)	SU(6) _L	SU(6) _R	U(1) _V	U(1) _{AF}
λ_m	Adj	1	1	0	1
q	F	\bar{F}	1	1	-1/2
\tilde{q}	\bar{F}	1	F	-1	-1/2
$l \equiv L$	1	F	1	-3	0
$\tilde{l} \equiv \tilde{L}$	1	1	\bar{F}	3	0
M	1	F	\bar{F}	0	0
ϕ	F	\bar{F}	1	1	1/2
$\tilde{\phi}$	\bar{F}	1	F	-1	1/2
Φ_H	1	F	\bar{F}	0	1

elegant dynamical understanding of the need for at least three generations in Nature.

A. The dual Higgs sector

The SM magnetic dual in Table II lists all SM fields plus some new states. In particular, the mes-Higgs field (and similarly the mesino) contains nine complex bidoublets [11]

$$\Phi_H = \{H_{ij}\}, \quad i, j = 1, 2, 3, \quad (7)$$

with

$$H_{ij} = (H_{ij}^u, H_{ij}^d), \quad (8)$$

where $H^{u,d}$ are distinguished by their hypercharge. In total, the magnetic SM contains 18 ‘‘Higgs’’ doublets, one of

which should be identified with the SM one. Henceforth, the following states must acquire a mass at around the TeV scale or above: the gaugino λ_m , the squarks ϕ and $\tilde{\phi}$, all mesinos M , and all but one of the doublets in Φ_H . The TeV scale is generically required by collider bounds, with the precise values depending on the details of the spectrum. This scale Λ_S is the analog of an emerging supersymmetry breaking scale. The theory is characterized by the following regimes below the Planck scale:

- (i) At $E < \Lambda_S$, the SM is valid.
- (ii) At $\Lambda_S < E < \Lambda_D$, the full spectrum of the magnetic theory appears.
- (iii) At $E > \Lambda_D$, the electric theory describes the weakly coupled elementary degrees of freedom.
- (iv) Optionally, at $M_{\text{Pl}} > E > \Lambda_{\text{eGUT}} > \Lambda_D$, the electric theory can merge into a dual grand unified theory.

To illustrate this, we show in Fig. 1 a schematic evolution of the gauge couplings from the electroweak scale up to Planck. The figure was obtained via one-loop renormalization evolution of the gauge couplings, with $\Lambda_S = 10$ TeV, and $\Lambda_D \approx 10^{15}$ GeV chosen to obtain gauge unification in the electric theory at $\Lambda_{\text{eGUT}} \approx 10^{19}$ GeV. Such scale values are only indicative, at this stage. At the scale Λ_D , the electric and magnetic SU(3) gauge couplings are matched as follows:

$$\frac{1}{\alpha_e} \sim \alpha_m = \frac{g_m^2}{4\pi}. \quad (9)$$

A precise determination of the scales involved requires higher order running of the couplings, including the Yukawa interactions in Eq. (4) in the magnetic regime, which we leave for future studies. Also, the presence of unification in the electric theory and the value of Λ_{eGUT} crucially depend on the matching of gauge couplings between the dual theories, which is beyond perturbative control. Note that, above Λ_D , only three gauge couplings are present in the electric theory, which simplify enormously the construction of an electric grand unified theory (eGUT).

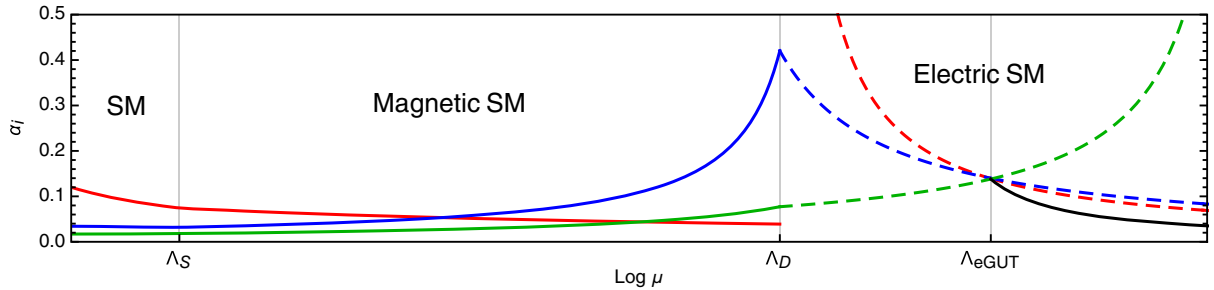


FIG. 1. Schematic evolution of the SM dual gauge couplings from the electroweak to the Planck scales. The three colors refer to SU(3) (red), SU(2)_L (blue), and U(1)_Y (green). Above Λ_{eGUT} , the three couplings could be replaced by a unified one, here shown in solid black. The figure is obtained via one-loop renormalization evolution of the gauge couplings, with $\Lambda_S = 10$ TeV, and $\Lambda_D \approx 10^{15}$ GeV chosen to obtain gauge unification in the electric theory at $\Lambda_{\text{eGUT}} \approx 10^{19}$ GeV. Such scale values are only indicative, at this stage.

B. Origin of flavor structures

A crucial aspect of the SM duality, as well as of any model of new physics, is the generation of flavor structures in the quark and lepton masses. In the magnetic dual, a single Yukawa y is generated, cf., Eq. (4), while 18 Higgs doublets emerge as composite states within the mes-Higgs field Φ_H . In terms of flavor components, the magnetic Yukawa term can be expanded as

$$yq\tilde{q}\Phi_H = y\sum_{i,j}(q_L^i u_R^j H_{ij}^u + q_L^i d_R^j H_{ij}^d), \quad (10)$$

where i, j are the generation indices. We stress that this democratic multi-Higgs structure [19] is a prediction of the SM duality on QCD. Hierarchical quark masses and mixing can, therefore, be generated by giving hierarchical VEVs to the Higgses, so that the effective Yukawa couplings read

$$Y_{ij}^u = y\frac{\langle H_{ij}^u \rangle}{v} \quad \text{and} \quad Y_{ij}^d = y\frac{\langle H_{ij}^d \rangle}{v}, \quad (11)$$

where

$$v^2 = \sum_{ij}(\langle H_{ij}^u \rangle^2 + \langle H_{ij}^d \rangle^2) = \frac{1}{2}(246 \text{ GeV})^2 \quad (12)$$

is the SM Higgs VEV.

Similarly to Ref. [19], the patterns in the VEVs is generated via suitable mass terms among the 18 Higgs doublets. As an example, the doublet that couples to the top can be identified as the SM Higgs, $H_0 \equiv H_{33}^u$, and it must acquire the largest VEV $\langle H_0 \rangle \approx v$. A smaller VEV can be communicated to the next-to-largest values, e.g., the doublet that couples to the bottom H_{33}^d . In practice, this is achieved via a linear mixing of H_{33}^u with H_{33}^d :

$$-(\mu_b^2 H_{33}^u H_{33}^d + \text{H.c.}) - M_{d,33}^2 |H_{33}^d|^2, \quad (13)$$

which induces, therefore, a VEV of order

$$\langle H_{33}^d \rangle = \frac{\mu_b^2}{M_{d,33}^2} v \sim \frac{m_b}{m_t} v. \quad (14)$$

Following this scheme, hierarchical quark masses and mixing can be generated via patterns in the mass terms for the 18 Higgs doublets in the magnetic theory, hence flavor structures are naturally recast in the scalar mass sector. The origin of these patterns, therefore, rests within the origin of the composite scalar masses within the UV theory. We stress that such masses are not generated by the duality, as the mes-Higgs field is required to be massless by the emerging supersymmetry, which links them to the massless mesinos. Hence, the masses are generated by operators in the electric theory that violate both flavor

symmetries and the emergent supersymmetry, hence we expect them to be radiatively stable.

In the electric theory, these masses can be written in terms of four-fermion interactions that violate the flavor symmetries. A natural source of such terms is gravity itself, which is expected to break all global symmetries of the theory via nonperturbative quantum effects [20]. Hence, at the Planck scale, we can expect the presence of operators in the form

$$\mathcal{L}_{\text{Planck}} \supset \frac{c^{abcd}}{M_{\text{Pl}}^2} (Q^a \tilde{Q}^b)(Q^c \tilde{Q}^d)^\dagger, \quad (15)$$

which are the lowest order ones in the electric quark fields. Note that the coefficients c must be nonuniversal, and possibly small, $c \lesssim 1$ [20]. The patterns in flavor space, however, cannot be predicted and the observed SM ones emerge as one of the many possible models generated by gravity. The main advantage of this approach is that the only potentially dangerous flavor violating effects are carried by the magnetic Higgses, in a calculable and controllable way, while other effects are safely suppressed by the Planck scale. At the duality scale Λ_D , they generate mass terms for the scalars of the order of

$$\mu^2 \sim \xi \frac{\Lambda_D^4}{M_{\text{Pl}}^2}, \quad (16)$$

where ξ is a numerical coefficient depending on the c couplings and on form factors between the four-fermion interactions and the composite scalars. Requiring $\mu \sim \text{TeV}$ gives an estimate of the smallest natural duality scale

$$\xi^{-1/4} \Lambda_D \sim \sqrt{\mu M_{\text{Pl}}} \sim 10^{11} \text{ GeV}. \quad (17)$$

Note similar operators related to gravity can generate masses for the mesinos and the gluino, hence endowing the duality with some predictive power at low energies.

Lepton masses could be generated via the same mechanism if they are included in the duality via the fourth color [6,12]. In the QCD dual model, they require the presence of another four-fermion interaction in the electric theory

$$\mathcal{L}_e \supset \frac{h_l}{M^2} (L\tilde{L})(Q\tilde{Q})^\dagger, \quad (18)$$

which is invariant under the flavor symmetries. At the duality scale, this will generate a direct coupling to the mes-Higgs field in the form

$$\mathcal{L}_m \supset h_l \mathcal{F}_\Phi \frac{\Lambda_D^2}{M^2} \tilde{l} \Phi_H^\dagger, \quad (19)$$

where \mathcal{F} is a form factor. Once expanded in components, we obtain

$$\tilde{l}\tilde{\Phi}_H^\dagger = \sum_{i,j} (l_L^i \nu_R^j (H_{ij}^d)^\dagger + l_L^i e_R^j (H_{ij}^u)^\dagger). \quad (20)$$

Interestingly, the largest VEV couples naturally to the τ lepton, explaining its large mass, while the hierarchy with the top mass is explained by the further suppression given by the scales. While the operator in Eq. (18) respects the flavor symmetries in the electric theory, additional flavor structures could be generated via flavor indices in the coupling h_i . Finally, large masses for the right-handed neutrinos can be included, close to the scale Λ_D , to implement a seesaw mechanism of type I.

C. Collider signatures

At current and future colliders, the SM duality can reveal itself via the discovery of new states in the magnetic theory, at the scale Λ_S . Due to the emerging supersymmetry in the QCD sector, the magnetic gluinos and squarks provide the same signatures as in the minimal supersymmetric SM, hence their masses are constrained to be above roughly 2 TeV at the LHC [21–23]. A smoking signature is the presence of a large number of Higgsino-like states, emerging from the mesino field. If provided a Majorana mass, the lightest mesino could act as dark matter candidate [24]. Another smoking gun consists of the 17 heavy doublets, responsible for the generation of flavor, however their masses in the multi-TeV range can only be discovered at future higher energy colliders and/or via precision flavor tests. The details of the low energy phenomenology depend

on the specific model, and we leave them for future investigation.

IV. CONCLUSIONS

In summary, in this paper we investigated the low and high energy implications of a possible dual description of the Standard Model. This provides a new intermediate scale associated with the duality and new directions for theories of grand unification. The emergent supersymmetry at low energies leads to new states, potentially around the TeV scale, which can be produced and studied at current and future colliders. In fact, the nonsupersymmetric gauge duality can also be tested via lattice simulation. Finally, the duality investigated here offers a novel origin of the flavor sector of the SM stemming directly from operators generated at the Planck scale.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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- [1] N. Seiberg, Exact results on the space of vacua of four-dimensional SUSY gauge theories, *Phys. Rev. D* **49**, 6857 (1994).
 - [2] N. Seiberg, Electric—magnetic duality in supersymmetric non-Abelian gauge theories, *Nucl. Phys.* **B435**, 129 (1995).
 - [3] F. Sannino, Higher representations duals, *Nucl. Phys.* **B830**, 179 (2010).
 - [4] F. Sannino, QCD dual, *Phys. Rev. D* **80**, 065011 (2009).
 - [5] M. Mojaza, M. Nardecchia, C. Pica, and F. Sannino, Dual of QCD with one adjoint fermion, *Phys. Rev. D* **83**, 065022 (2011).
 - [6] F. Sannino, The standard model is natural as magnetic gauge theory, *Mod. Phys. Lett. A* **26**, 1763 (2011).
 - [7] O. Antipin, M. Mojaza, C. Pica, and F. Sannino, Magnetic fixed points and emergent supersymmetry, *J. High Energy Phys.* **06** (2013) 037.
 - [8] F. Sannino, Magnetic S-parameter, *Phys. Rev. Lett.* **105**, 232002 (2010).
 - [9] J. Preskill and S. Weinberg, Decoupling constraints on massless composite particles, *Phys. Rev. D* **24**, 1059 (1981).
 - [10] Models based on the supersymmetric Seiberg duality have been proposed for QCD’s SU(3) [11,12], Pati-Salam’s SU(4) [13], and the weak SU(2) [14,15] (for AdS/CFT analogs of the latter, see e.g. [16,17]).
 - [11] N. Maekawa, Duality of a supersymmetric standard model, *Prog. Theor. Phys.* **95**, 943 (1996).
 - [12] N. Maekawa and J. Sato, Duality of a supersymmetric standard model without R-parity, *Prog. Theor. Phys.* **96**, 979 (1996).
 - [13] N. Maekawa and T. Takahashi, Duality of a supersymmetric model with the Pati-Salam group, *Prog. Theor. Phys.* **95**, 1167 (1996).
 - [14] C. Csaki, Y. Shirman, and J. Terning, A Seiberg dual for the MSSM: Partially composite W and Z, *Phys. Rev. D* **84**, 095011 (2011).
 - [15] N. Craig, D. Stolarski, and J. Thaler, A fat Higgs with a magnetic personality, *J. High Energy Phys.* **11** (2011) 145.
 - [16] Y. Cui, T. Gherghetta, and J. D. Wells, Emergent electroweak symmetry breaking with composite W, Z bosons, *J. High Energy Phys.* **11** (2009) 080.
 - [17] S. Abel and T. Gherghetta, A slice of AdS₅ as the large N limit of Seiberg duality, *J. High Energy Phys.* **12** (2010) 091.

- [18] J. C. Pati and A. Salam, Lepton number as the fourth color, *Phys. Rev. D* **10**, 275 (1974); **11**, 703(E) (1975).
- [19] C. T. Hill, P. A. N. Machado, A. E. Thomsen, and J. Turner, Scalar democracy, *Phys. Rev. D* **100**, 015015 (2019).
- [20] R. Kallosh, A. D. Linde, D. A. Linde, and L. Susskind, Gravity and global symmetries, *Phys. Rev. D* **52**, 912 (1995).
- [21] G. Aad *et al.* (ATLAS Collaboration), The quest to discover supersymmetry at the ATLAS experiment, [arXiv:2403.02455](https://arxiv.org/abs/2403.02455).
- [22] ATLAS Supersymmetry searches, <https://twiki.cern.ch/twiki/bin/view/AtlasPublic/SupersymmetryPublicResults>.
- [23] CMS SUS Physics Results, <https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSUS>.
- [24] A. Belyaev, G. Cacciapaglia, D. Locke, and A. Pukhov, Minimal consistent dark matter models for systematic experimental characterisation: Fermion dark matter, *J. High Energy Phys.* **10** (2022) 014.