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Metafrontier frameworks for estimating solar power efficiency in the United States using stochastic nonparametric envelopment of data (StoNED)

Haleh Delnava, Ali Khoosravi, Mamdouh El Haj Assad

1. Introduction

The growing industrial activity in both developed and developing nations has increased demands to fulfill the primary energy supply during the last decade. Conventional energy sources presently have the majority share in electricity generation. Unlike fossil fuels, renewable energy resources (i.e., geothermal, solar, hydropower, wind, and biomass) generate electricity with no threatening environmental issues [1,2]. In addition to electricity generation, these natural resources are dominant in producing heating, cooling, and freshwater.

Solar energy is one of the most abundant natural resources, with 1 h’s worth of electricity obtained from the sun almost equaling one year’s worth of human activities [3,4]. Solar technology, according to a report published by the International Energy Agency (IEA), can contribute to a 14% reduction in carbon dioxide emissions in the power industry by 2050, based on the BLUE Map scenario [5]. Solar energy’s main advantages are its widespread availability and accessibility, while its inconstancy makes forecasting difficult. There are various types of technologies [i.e., photovoltaic (PV), solar dish/Stirling, solar power tower (SPT)] to convert solar energy into electricity. The controversial issue is how to accomplish this energy more efficiently.

In the economic literature, performance evaluation and benchmarking have recently attracted many scholars’ attention. Parametric and nonparametric models are two bases of neoclassical production theory [6]. In the following, we go through them and elaborate on their strengths and weaknesses.

Data envelopment analysis (DEA) is a nonparametric technique that estimates production frontier empirically [7,8]. The efficiency scores are calculated for each decision-making unit (DMU) under linear programming by conducting a loop. The DEA efficient frontier graphically looks like piecewise linear segments based on undominated DMUs (multiple inputs/outputs), yielding a set of convex production possibilities. The

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Keywords: Efficiency Meta-technology US Solar power Stochastic nonparametric envelopment of data Convex and nonconvex frameworks

A B S T R A C T

Solar energy is one of the most promising energy sources as it significantly reduce greenhouse gas (GHG) emissions compared to fossil fuels. In this study, we employ the meta frontier framework to estimate US solar energy performance in 2019 using stochastic non-parametric envelopment of data (StoNED) under the convex and non-convex frameworks. This estimation allows us to monitor operating inefficiencies and technological gaps in each observation. In addition, we investigate the potential impact of the specification of a convex production technology in relation to the use of a nonconvex technology in the comparative analysis. This methodological reflection is mainly supported by the recent engineering literature that provides evidence of the non-convex hypothesis. The results indicate that a multifaceted approach must be taken to ensure the supply of energy. Given that sunny states have the potential to transmit energy to other states, the drawbacks, such as environmental concerns and high investment expenses, drive policymakers to look for other alternatives, such as adapting panels that are suitable for specific conditions.

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1 It aims to cut global energy-related CO₂ emissions in half by 2050 and looks at the most cost-effective ways to do it by deploying innovative low-carbon technology.
supporting argument regarding DEA does not demand any predefined assumptions about the geometrical shape of efficient frontier \([9]\). The fundamental drawback of this method is that it does not consider noise from data; therefore, distance from the Pareto frontier is due to inefficiency exclusively. Like DEA, free disposal hull (FDH) is a nonparametric metric for assessing performance based on a production frontier \([10]\). However, the distinction between these models is that DEA assumes convexity axiom, whereas FDH does not. Consequently, the FDH production possibility set is typically non-convex. \([11]\) developed a mixed-integer linear programming (MILP) formulation with a simple algorithm.

Alternatively, stochastic frontier analysis (SFA) proposed by Refs. \([12,13]\) is an econometric approach in productivity and efficiency analysis. For the survey of this parametric strand of production-efficiency literature, see Ref. \([14]\). Parametric methods assume some predefined assumption regarding the shape of the frontier that is considered a weakness \([15]\). The supporting argument regarding SFA is that the composite error term is a summation of one-sided disturbance (inefficiency) and two-sided disturbance term (noise).

In order to combine previous methodologies, a significant amount of research has been done on efficiency analysis. Recently, designing a model that incorporates both the nonparametric characteristics, i.e., axiomatic frontier estimation from DEA alongside the stochastic component from SFA has been a controversial issue among economists. \([16]\) were pioneers who proposed a two-step pseudo-likelihood estimator. They applied semi-parametric kernel regression to evaluate an average production frontier and maximize a pseudo-likelihood function for the remaining parameters, which are two favorable properties of these frameworks. The generalized form of \([16]\) developed by Ref. \([17]\) based on the local maximum likelihood principle \([18]\); or \([19]\) from the nonparametric aspect and localizing the parameters of the stochastic component, through one-step maximization procedure. Further contributions to this unified framework include the model by Ref. \([20]\) proposed to estimate the production frontier. This methodology provides a statistical foundation for estimating nonparametric frontier and avoiding any specific functional form. For an accurate comparison of some earlier approaches to these frameworks, see Vaninsky, \([2010, 21]\). However, most of these models require some further assumptions that make them computationally challenging to use. Stochastic nonparametric envelope of data (StoNED) is another framework that integrates SFA and DEA models \([22-24]\). This model’s notable characteristics that discriminate it compared with the previous model \([25]\) are that it excels in different Monte Carlo simulations, mainly when the data is subject to significant noise.

Sunbeams vary their accessibility and luminous intensity during the day. Apparently, on bright, cloudless days, the sunray reaches its maximum level when the sun is at its peak in the sky—at noon. Technically, firms across different regions or climatic zones encounter various production capabilities. These so-called technology sets differ due to differences in available natural wealth (e.g., ecological, meteorological, energy resources) and other socioeconomic characteristics in which production occurs. Because of these differences, efficiency researchers have estimated different production frontiers for each set of firms. Hence, policymakers encounter two scores: one refers to group efficiency that is measured under existing group frontier, and the other, called meta efficiency, corresponds to the best possible technology \([26-28]\); and \([29]\).

1.1. Knowledge gap and novelty

The goal of this research is categorized into two sides: computational and managerial aspects. From a computational viewpoint, the convex nonparametric least square (CNLS) formulation may face zero as a logarithm argument in the multiplicative case. Hence, finding a way to solve it is significant for the CNLS user community. In renewable energy applying the multiplicative form is effective as it helps to mitigate heteroscedasticity in higher levels of output by higher intercept and a smaller slope \([30]\). Considering a convex production frontier for the nonconvex electricity generation problem may likely generate doubtful results with less accuracy due to its nonlinear or mixed integer mathematical modeling nature \([31]\). Hence, under such circumstances, relaxing the convexity assumption is more convincing from an engineering viewpoint. In this study, to the best of our knowledge, we are the first to witness the impact of assuming or relaxing the convexity assumption in production frontier under StoNED frameworks. This framework allows us to distinguish primary inefficiency sources, whether operational activities or lack of technological advancement. Moreover, how we can give policymakers some alternatives to improve efficiency. Finally, what the managerial insight of this framework is.

2. Material and methods

2.1. Convex framework

2.1.1. Estimation of within-group efficiencies

Suppose there are \(n\) DMUs indexed by \(i (i = 1, \ldots, n)\) divided into \(K(> 1)\) which is indexed by \(k = (1, \ldots, K)\) groups, and \(\delta_k\) represents the number of observations in each group. Following a similar fashion as in SFA, the best practice production function represented as \(f(x)\), transforming inputs to desirable outputs where inputs denoted by the column vector \(x = (x_1, \ldots, x_p) \in \mathbb{R}^n\), the desirable output \(y^d \in \mathbb{R}^m\). The specific production possibility set for input-output combinations available to firms defined as: \(k^d = \{(x^d, y^d) : x^d \leq f(x)\}\), where \(f(x)\) are continuous, monotonic increasing, and concave. This definition is consistent with the StoNED framework as it acts as a bridge between DEA and SFA. Thus, the observed output \(y^d\) and \(f(x)\) of firm \(i\) may be mutually different, due to inefficiency and noise. Consistent with SFA framework, in each group \(k\), error term \(\epsilon_i = v_i = v_i^d - u_i^d\) is combination of inefficiency term \(u_i^d > 0\) and the stochastic noise term \(v_i\), expressed by:

\[
y_i = f(x_i) + v_i = f(x_i) + (v_i^d - u_i^d)
\]

The group inefficiency \((u_i^d)\) and noise \((v_i)\) random variables, follow half-normal, and normal probability distribution as \(v_i^d \sim N(\mu_i, \sigma_i^2)\), \(v_i \sim N(0, \sigma_i^2)\) \([12]\).

However, in the modeling production function, applying a multiplicative error model owing to the log-transformations applied to the data (Cobb-Douglas, translog) is more favorable among economists rather than an additive error \([24]\). The multiplicative form is effective solution to mitigate heteroscedasticity in higher levels of output by higher intercept and a smaller slope \([30]\). Thus, in this study a multiplicative form is built. Under multiplicative error assumption, the production function is represented as:

\[
y_i = f(x_i) \exp(x_i^d) = f(x_i) \exp(v_i^d - u_i^d)
\]

\[
\ln(y_i) = \ln(f(x_i)) + x_i^d
\]

As the composite disturbance term in (1) violates the Gauss–Markov properties that \(E(x^d) = E(u_i^d) = - \mu^d < 0\) where \(\mu^d\) is the expected technical inefficiency in \(k^d\) group. Therefore, the additive model is modified as \(E(x^d) = \ln[f(x_i) - \mu^d] + [\mu^d - u_i + v_i] = \ln[f(x_i)] + [\mu^d - u_i + v_i] = 0\). Following \([23,24]\); the StoNED estimator consists of multiple (4) steps. The CNLS estimator (step 1) to estimate conditional mean output in \(k^d\) group is calculated by the following nonlinear programming (NLP) problem as:

\(2\) refers to circumstances that the variance of the residuals is diverse over a range of measured values.

\(3\) Note that the composite error term in (3) is the modified version of composite error term in (1).
This algorithm evaluates CNLS regression with a multiplicative error term. The first constraint denotes the distance to the frontier as a nonlinear function of inputs and outputs. The second constraint ensures concavity among the hyperplanes in all pairs of observations [32]. The last one states that the estimated frontier is monotonic. The firm-specific coefficients βi diagnosed marginal effects of inputs. The objective function in (3) is nonlinear due to the natural logarithm functional. This is because the natural log is a concave function, and this appears inside of the squared function which is convex. Because both convex and concave functions appear in the objective function, it is generally non-linear and there does not seem to be a simple convex approximation.

To prevent logarithm argument not being zero, [33] recommended inserting one in argument is an impressive way during utilization of MINOS solver. CONOPT is another member of the NLP algorithm family that has considerable build-in logic and solution selecting technique [34, 35]. As the objective function of the CNLS problem (3) is convex, a local optimum is equivalent to a global optimum.

2.1.2. Computational aspects based on CONOPT algorithm

To avoid evaluating log function outside its domain of definition, assigning bounds on variables is an impressive alternative. In general, there are two purposes of using bounds in this type of models. First, bounds represent some restrictions in the model’s structure (e.g., βi ≥ 0) called model bounds. Second, bounds help the algorithm in avoiding system crash once a curve function is infinite in a particular domain called algorithmic bounds. Thus, model bounds have natural roots and do not cause any problems, specifying algorithmic bounds subject to the model’s functional format. In model (3), to deal with the computational issue, an intermediate variable (Ω) and an additional equation are introduced. This strategy is applied to the number of the Log function. Note that, the content of Ln should be greater than a small positive number, e.g., = 1 ϵ − 4. Thus, model (3) is reformulated as:

\[
\min \sum_{i=1}^{n} (\epsilon_i) \quad (3)
\]

\[
s.t. \ \ln y_i^* = Ln(a_i^0 + \beta_i x_i^0) + \epsilon_i \quad (4)
\]

\[
\alpha_i^0 + \beta_i x_i^0 \leq \alpha_i^* + \beta_i x_i^0 \quad (i = 1, \ldots, n)
\]

\[
\Omega(i) = \alpha_i + \beta_i x_i^0 \quad (i = 1, \ldots, n)
\]

By adding the estimated moments to the above equations, the (unconditional) estimators of \(\hat{a}_i\) and \(\hat{b}_i\) are obtained by the following equations:

\[
\hat{a}_i = \frac{\hat{M}_i \sigma_a^2}{\sqrt{\hat{\sigma}_i^2} \sqrt{\sigma_a^2}}
\]

\[
\hat{b}_i = \frac{\hat{M}_i \sigma_b^2}{\sqrt{\hat{\sigma}_i^2} \sqrt{\sigma_b^2}}
\]

The StoNED frontier (Step 3) is obtained by simply shifting the CNLS estimator upwards as:

\[
f^{-k}_{\text{StoNED}} = \hat{g}(x) + \hat{\mu}_k, \text{ where } \hat{\mu}_k = \hat{\sigma}_k \sqrt{\frac{\hat{\sigma}_i}{\hat{\sigma}_k}}.
\]

To calculate the efficiency score for each DMUs (Step 4), conditional expected value formula [36] is applied, which is equal to:

\[
E(u_i| \epsilon_i) = \mu_i + \varphi(-\mu_i / \sigma_i) / [1 - \Phi(-\mu_i / \sigma_i)]
\]

where \(\varphi\) and \(\varphi\) represent respectively the density and cumulative distribution function of the standard normal distribution N(0,1), \(\mu_i = -\epsilon_i^i x_i^0 / (\sigma_a^2 + \sigma_b^2)\), and \(\sigma_i = \sigma_a \sigma_b / (\sigma_a^2 + \sigma_b^2)\).

Thus, the conditional expected value of inefficiency is equal to:

\[
E(u_i| \epsilon_i) = \frac{\hat{a}_i \hat{b}_i}{\sqrt{\hat{\sigma}_i^2} \sqrt{\hat{\sigma}_i^2}} \varphi \left( -\frac{\epsilon_i^i x_i^0 / (\hat{\sigma}_a^2 + \hat{\sigma}_b^2)}{1 - \Phi \left( -\frac{\epsilon_i^i x_i^0 / (\hat{\sigma}_a^2 + \hat{\sigma}_b^2)}{\sqrt{\hat{\sigma}_a^2 + \hat{\sigma}_b^2}} \right)} \right)
\]

Following Farrell’s definition (1957) and (2), the group efficiency score for ith DMU is obtained as:

\[
TE_i = \exp \left(-E(u_i| \epsilon_i) \right)
\]

2.1.3. Estimation of meta-efficiencies

The concept of meta-technology was first introduced by Ref. [26]. They assumed meta technology envelops the group-K frontier. Indeed, the concept of meta-technology is based on the simple assumption that all units can access the best technology. Mainly, meta frontier facilitates units comparison belonging to different groups. Meta-technology (TM) is convex hull of group frontiers (1) as below:

\[
(1) \quad TM = \text{Convex Hull } \{T^1 \cup T^2 \cup \ldots \cup T^k\}
\]

\[
(2) \quad \text{If } (x,y) \in T^k \text{ for any } k \text{ then } (x,y) \in T^M ;
\]

\[
(3) \quad \text{If } (x,y) \in T^M \text{ then } (x,y) \in T^k \text{ for some } k ;
\]

\[
(4) \quad TE_i \geq TM_i
\]

As the group technologies are a subset of meta-technology (2), the technical efficiency regarding group technology is equal or greater than the efficiency measure regarding meta-technology (4).

The CNLS estimator based on meta-technology may not overlap all the group frontiers as in some cases corresponding hyperplanes based on meta-technology placed below the \((x_i, y_i)\) that means \(\epsilon_i^0 > 0\). To eliminate such an inconsistency, following to Ref. [37]:

\[
\min \sum_{i=1}^{n} \sum_{k=1}^{K_i} \left( \epsilon_i^k \right) \quad (11)
\]

s.t. \(Ln(y_i^*) = Ln(\Omega(i)) + \epsilon_i^0 \).
\[ a_i^k + \beta_i^k x_i^k \leq a_i^k + \beta_i^k x_i^k \forall k, \forall h, \forall q \]
\[ \Omega(i) = a_i + \beta_i x_i \]
\[ \beta_i \geq 0 \text{ and } \Omega(i) \geq \epsilon_i \]
\[ \epsilon_i \leq 0. \]

where \( y_i^k = \frac{y_i^k}{x_i^k} \) as an expansion-adjusted output level [38] and estimated shape function denoted as \( f^M \) which is equal to \( \ln(y_i^k) = \ln(f^M(x_i^k)) + \epsilon_i \) such that \( \epsilon_i < 0 \). This leads to \( \ln(y_i^k)/\ln(f^M(x_i^k)) = (y_i^k/\overline{TE}_i)/\ln(f^M(x_i^k)) = \overline{TE}_i / \overline{TE}_i \). Model (12) determines technological gap ratio (TGR) of each firm i.e., \( \text{TGR}(x_i^k, y_i^k) = \ln(y_i^k)/\ln(f^M(x_i^k)) \) and \( \overline{TE}_i \) is:

\[ \overline{TE}_i = \text{TGR}(x_i^k, y_i^k) \]  

(12)

Equation (13) implies that the measure of efficiency relative to meta-technology decomposes into two components group efficiency (identifies group technology in terms of regional, social, and economic) and technological gap ratio (indicating group technology position from meta frontier). Commonly \( \text{TGR}(x_i^k, y_i^k) \leq 1 \) whenever both group frontier and meta frontier overlap \( \text{TGR}(x_i^k, y_i^k) = 1 \).

Fig. 1 illustrates these concepts.

2.2. Non-convex framework

2.2.1. Estimation of within-group efficiencies

In production theory, technology construction was based on two usual axioms: convexity and monotonicity (free disposability). DEA, CNLS, and StoNED estimators have these two properties. However, some other techniques rely exclusively on monotonicity without maintaining convexity assumption. In statistics literature, the monotonicity condition is called isotonic regression [39–41]. Non-convex version of the StoNED model developed by Ref. [42] partially differs CNLS as \( f(X) \) are assumed to be continuous, monotonic increasing. The best practice production function represented as \( f(x) \), transforming inputs \( X = \{x_i \in \mathbb{R}^n_+ \} \) to desirable outputs \( \bar{y} \) are isotonic respect to partial order if \( \forall i, h \in X, x_i \preceq x_h \) result in \( f(x_i) \leq f(x_h) \). Note that, a partial order on \( X \) is a relation that is reflexive (\( x_i \preceq x_i \)), anti-symmetric (\( x_i \preceq x_h \) then \( x_h \preceq x_i \)); except if \( x_i = x_h \) and transitive (\( x_i, x_j, x_k \) such that \( x_i \preceq x_j, x_j \preceq x_k \) then \( x_i \preceq x_k \)).

The INLS estimator (step 1) to estimate conditional mean output in \( k \)th group is calculated by the following nonlinear programming (NLP) problem:

\[ \min \sum_{k=1}^{K} \sum_{i=1}^{n} (e_i^k)^2 \]  

(13)

s.t. \( \ln(y_i^k) = \ln(a_i^k) + e_i^k \)
\[ p_o a_i^k \leq p_o a_i^k \forall i, h \]

The preference matrix \( P = [p_i]_{i,n} \) transforms poset ordering among elements of \( X \) into binary values. Precisely, such a transformation is equal to: if \( \forall i, h \in X, x_i \preceq x_h \) then \( p_{ih} = 1 \), otherwise \( p_{ih} = 0 \). GAMS, as cutting edge mathematical modeling, imposing conditionals involves using the ($) statement syntax. In step 2, similar framework is applied to evaluate \( \tilde{a}_g \) and \( \tilde{e}_g \) under the assumed probability distributions (Eqs. (7) and (8)). Given \( \tilde{a}_g \) the non-convex StoNED frontier (step 3) is obtained by simply shifting the INLS step-function upward as:

\[ \tilde{f} \nabla \text{INLS} = \tilde{a}_g + \tilde{\mu} \]


Table 1

<table>
<thead>
<tr>
<th>StoNED</th>
<th>Steps</th>
<th>Convex</th>
<th>Non-Convex</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Estimating conditional mean (Eq. (4)) and (14)</td>
<td>CNLS</td>
<td>INLS</td>
<td></td>
</tr>
<tr>
<td>2. Estimating the variance parameters (Eqs. (7) - (8))</td>
<td>$\tilde{a}_g = \frac{M_0}{\sqrt{\frac{2}{3}}}$</td>
<td>$\tilde{a}_g = \frac{M_0 - \sigma^2}{\sqrt{\frac{2}{3}}}$</td>
<td></td>
</tr>
<tr>
<td>3. Shifting the estimator</td>
<td>$\tilde{f}(x) = \tilde{a}_g \theta \nabla \text{INLS} \quad \tilde{f}(x)$</td>
<td>$\tilde{f}(x) = \tilde{a}_g \theta \nabla \text{INLS} \quad \tilde{f}(x)$</td>
<td></td>
</tr>
<tr>
<td>4. Estimating conditional expected value of inefficiency (Eq. (10))</td>
<td>$\tilde{e}_g(x_i^k) = \mu_i + \sigma \sqrt{\tilde{a}_g^2 + \sigma^2}$, $\sigma = \sigma \tilde{a}_g \sqrt{\tilde{a}_g^2 + \sigma^2}$</td>
<td>$\tilde{e}_g(x_i^k) = \mu_i + \sigma \sqrt{\tilde{a}_g^2 + \sigma^2}$, $\sigma = \sigma \tilde{a}_g \sqrt{\tilde{a}_g^2 + \sigma^2}$</td>
<td></td>
</tr>
</tbody>
</table>
(\(v_i\)) using MM and the residuals of the INLS estimations, and then adjust the step function estimated by INLS by the expected value of in-efficiency. Group step function denoted as: \(F^g(x^g_i) = \min(\alpha^g_i x_i^g|\alpha^g_i x_i^g < 1 \text{ if } x_i^g < x_i^g_{\text{w}} \text{, otherwise } x_i^g = 0)\). Meta frontier is built as \(F^m(x^m_i) = \max(\min(\alpha^g_i x_i^g|\alpha^g_i x_i^g | k = 1, \ldots, K)\) which cover all group frontier and provides a homogeneous boundary for all heterogeneous groups. Meta efficiency is formulated as: \(TE_m^g = \exp(-E(u_i^g|x_i^g + \lambda))\) where \(M\) as a sufficiently large constant denotes the maximum in-efficiency of corresponding group, \(M = \max E(u_i^g|x_i^g, k = 1, \ldots, K)\). Note that, \(M\) cannot be fixed for all DMUs as each group has distinct properties. The TGR can be obtained by the division of \(TE_m^g\) and \(TE_m^g\), i.e., \(TE_m^g/TE_m^g\). Consequently all the \(TE_m^g\), \(TGR\) and \(TE_m^g\) in \([0, 1]\) are consistent with the usual definition of efficiency scores. Fig. 2 illustrates these concepts.

2.2.3. Comparison of convex and non-convex

Both convex and non-convex frameworks are techniques to form a production frontier under predefined: assumptions (monotonicity, with/without concavity). These estimators allow analyzing the performance of production units concerning technical efficiency in multiple inputs and outputs; even prices are unavailable or unreliable. Moreover, these estimators are structured base on unlike mathematical modeling. In an original setting, CNLS and INLS are formulated as quadratic programming (QP) and mixed-integer linear programming (MILP), respectively. From a computational viewpoint, INLS is easier to solve. It uses a preference matrix to represent dominance relationship by elements of binary values to help in reducing computational burden as an initial step. From a graphical aspect, the second constraint in CNLS model \(\alpha^g_i + \beta^g_i x_i^g \leq \alpha^g_i + \beta^g_i x_i^g\), called Aheilov inequality [32] gives convexity postulate to production technology. In contrast, the preference matrix in the second constraint of INLS \((\rho_0, \alpha^g_i \leq \rho_0, \alpha^g_i)\) provides a “step-wise” structure to the technology.

In production economics, improving efficiency is one of the key factors to boost productivity [43,44]; Su et al., 2022). As StoNED estimation merges both axiomatic characteristics of parametric and non-parametric models into a unified framework, the resulting technical efficiency scores have higher discrimination power than traditional models. As argued by Ref. [45]; convex cones containing the nonconvex cone (i.e., \(TE_m^g = TE_m^g\)) in deterministic case, same behavior is consistent in stochastic one that results, \(TE_m^g \leq \sigma_{\text{CNLS}}\).

3. Empirical study of U.S. Solar power industry

In this section, an empirical survey is conducted to estimate the efficiency of US solar power plants for 45 US states in 2019. In 2003, researchers at the Department of Energy (DOE) established five major climatic areas throughout the U.S. (Baechler et al., 2015). In this estimation, we follow the rule of thumb [46,47] that says the number of units should be at least twice the number of inputs and outputs. Hence, considering each climatic zone as a group contradicts this rule. We categorized climatic regions into three groups, namely (1) hot-humid, hot-dry, (2) mixed humid, (3) cold/very cold/marine (Fig. 3). According to some criteria (temperatures, precipitation, and heating degree days), these climatic regions classified, denoted in Table 2.

3.1. Experimental data

The data collecting source is Emission and Generation Resource Integrated Database [48] which is issued by the EPA. Three inputs and one desirable output are used. In detail, the three inputs are the land area [49], nameplate capacity of solar power sectors, and annual heat input. The annual heat input denotes the amount of heat energy that panels require to produce 1 MWh of electricity. The considered output is the annual net generation\(^5\) in megawatt-hours (MWh). Table 3 gives the summary of the statistics.

General Algebraic Modeling System (GAMS) software, and the CONOPT solver (step 1) and excel spreadsheet (step 2-4) are applied to compute the efficiency scores. Fig. 4 denotes the overall procedure of the implemented algorithm.

4. Results and discussion

Overall, solar energy consumption in the United States raised from around 0.06 trillion British thermal units (Btu) in 1984 to around 1.246 trillion (or roughly 1.2 quadrillions Btu) in 2020. This amount of power output accounted for about 95% to fulfill energy demand and the remaining 5% for heating purposes. Utility-scale PV power plants, distributed/small-scale PV systems, and thermal-electric power plants allocate 66%, 31% and 2% of electricity generation, respectively.

There are two sub-figures (a) and (b) corresponding to each of the figures from 5 to 7, denoting efficiency estimation with or without the convexity hypothesis. The elimination of the convexity assumption yields a lower bound for both groups and meta frontiers, demonstrated by comparing (a) with (b). Consequently, this aspect yields lower group and meta efficiency scores against maintaining the convexity

Fig. 3. The continental United States contains five of the eight US climate zones identified by Building America. Only Alaska is dedicated to the sub-arctic climate zone, which is also not depicted on the map (Source: Volume 7.3, Guide to Determining Climate Regions by County - August 2015).

5 The annual net generation is equivalent to total electricity generation subtracted by internal energy consumption.
Table 2
IECC climate zone.

<table>
<thead>
<tr>
<th>Category</th>
<th>Climatic Zone</th>
<th>Temperature</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hot-Dry</td>
<td>&gt; 45°F (7°C) throughout the year</td>
<td>Arizona (AZ), California (CA), Texas (TX)</td>
<td></td>
</tr>
<tr>
<td>1 Hot-Humid</td>
<td>&gt; 67°F (19.5°C) for 3,000 h during the warmest six consecutive months.</td>
<td>Florida (FL), Georgia (GA), Hawaii (HI)</td>
<td></td>
</tr>
<tr>
<td>2 Mixed-Humid</td>
<td>&gt; 73°F (23°C) for 1,500 h during the warmest six successive months.</td>
<td>Alabama (AL), North Carolina (NC), Delaware (DE), Maryland (MD), Iowa (IA), Kentucky (KY), Oklahoma (OK), Mississippi (MS), South Carolina (SC), Tennessee (TN), Arkansas (AR), Virginia (VA), Kansas (KS)</td>
<td></td>
</tr>
<tr>
<td>2 Mixed-Humid</td>
<td>&lt; (65 °F) during heating degree days.</td>
<td>&lt; (45°F)/7°C during the winter months.</td>
<td></td>
</tr>
<tr>
<td>3 Cold</td>
<td>5400 &lt; heating degree days &lt; 9000 (65°F)</td>
<td>Colorado (CO), Idaho (ID), Illinois (IL), Maine (ME), Nevada (NV), Nebraska (NE), Massachusetts (MA), Montana (MT), Pennsylvania (PA), Rhode Island (RI), Connecticut (CT), Utah (UT), Ohio (OH), New Mexico (NM), Vermont (VT)</td>
<td></td>
</tr>
<tr>
<td>3 Very cold</td>
<td>9,000 &lt; heating degree days &lt; 12,600 (65°F basis).</td>
<td>Oregon (OR), Washington (WA), Minnesota (MN), Michigan (MI), Wisconsin (WI), Wyoming (WY)</td>
<td></td>
</tr>
<tr>
<td>3 Marine</td>
<td>(27°F)/3°C &lt; coldest mean temperature &lt; (65°F)/10°C</td>
<td>(72°F)/22°C, warmest mean temperature month</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&gt; (50°F)/10°C average temperatures for at least four months.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3
Statistics for 44 U.S. State-level solar power units operating in 2019.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land area nameplate capacity</td>
<td>MW</td>
<td>61,865</td>
<td>47,984</td>
<td>261,222</td>
<td>1,545</td>
</tr>
<tr>
<td>Total annual input heat generation</td>
<td>MMBtu</td>
<td>14,100,080</td>
<td>38,877,284</td>
<td>250,841,988</td>
<td>13,410</td>
</tr>
<tr>
<td>Annual net generation</td>
<td>MWh</td>
<td>1,578,908</td>
<td>4,338,426</td>
<td>27,971,766</td>
<td>1,506</td>
</tr>
</tbody>
</table>

assumption. In Fig. 5 (b), all states are efficient with respect to TEk and also efficient with respect to TGR, while in Fig. 5 (a), we observe different behavior. In Texas (TX), the difference between these two estimations for group efficiency and TGR is 14% and 23%, respectively. In the second climatic zone, some states report low performance that their corresponding mean in Fig. 6 (a). Such a low performance is highlighted in Arkansas (AR), which reports 0.49 in both group and meta efficiency scores. However, estimation is no longer too complicated as we relax the convexity assumption, and observe almost all of the states have higher performance than their average. By scrutinizing the results in Fig. 7, we witness the impact of different axioms on our estimation. Under traditional axiomatic assumptions, i.e., convexity and strong disposability, the average group efficiency (0.52) is much lower than TGRs (0.98), indicating that enhancing operating efficiency with existing technology outweighs the technological breakthroughs. Under minimal axiomatic assumptions, i.e., strong disposability that is consistent for any monotone boundary suggests that technological advancement is crucial in the US compared with operational efficiency. According to Ref. [31], convexity could play a key role in bridging the gap between economic studies using an aggregate approach (black box) and engineering surveys based on disaggregate modeling. Considering only strong disposability generates robust estimators for any not decreasing boundary. Consequently, there is no reason to impose convexity.

The well-developed industrial infrastructure has contributed to these states’ ability to generate energy from solar panels. The performance of the topmost productive states can be a practical example for other states to apply their information to plan their efforts despite extreme climatic conditions. Factors such as weather conditions, temperature, operation, monitoring and maintenance should be considered to ensure the optimum efficiency of the solar panels. However, unlike the conventional energy more specifically coal power, the labor force as an input is being overshadowed. Cleaning solar panels is one of the beneficial actions to take full advantage of the sunlight to turn into the power supply. However, cleaning solar panels consumes large amount of water supply per year (about 1 billion gallons). To this end, [50] developed an automatic cleaning approach for solar panels and mirrors of solar thermal plants that is water-free. This technological innovation uses electrostatic repulsion to ensure that panels are not blocked from dust or snow to absorb the sunlight properly. Operation and maintenance (O&M) services are another impressive factors to guarantee solar panels’ technical and economic performance over their predefined lifespan. The primary purpose of these operations services is remote monitoring and controlling solar power plants’ conditions through monitoring software. Last but not least, each module has a different spectral frequency depending on the type of the module. Thus, reducing the spectral reflection of the light can enhance energy efficiency.

The most fascinating analysis feature, which compares the average technical efficacy of the group frontier (TEG) and the meta frontier (TEM) under two frameworks, is shown in Table 4. When it comes to group technical efficiency, states in the hot dry/hot and humid climatic zone are more effective. The technical group efficiency is lower in two other climatic zones (κ = 2, 3), which even under the convexity assumption
makes the result extremely low. States create about 90% of the maximal level of output under the first group of technologies for both convex and non-convex frameworks, which is consistent with the data on $\text{TE}^k$’s superiority. Some states, including IN, NJ, NY, and MO, fall into either mixed humid or cold climatic zones, according to the IECC’s classification of climatic zones. In the present research, we took into account the second zone’s stated states. However, as we proceed to the cold zone, we notice certain variations in the group efficiency scores. As a result, the third group’s average operational performance increased under the convexity and nonconvexity frameworks, respectively, from 0.52 to 0.55 and 0.85 to 0.91. $\text{TE}^m$ supports the findings obtained from both group frontiers and TGR. They represent an average of 53% (convex) and 66% (nonconvex) of their production level, according to $\text{TE}^m$. Therefore, the results for meta efficiency under the convexity assumption were primarily attributable to operational inefficiency, whereas, under the nonconvexity assumption, deficiencies in technology were a major factor in the estimation.

We use a parametric statistical test (Table 4) to formally assess whether these two frameworks (convex and nonconvex) are distinct from each other. As it is more accurate when the variances of the two samples are not equal, Welch’s $t$-test is an adaption of the two sample $T$-test (Welch, 1947). The alternative hypothesis contends that the average densities are different, whereas the null hypothesis assumes that they are same. The null hypothesis should be rejected in favor of an alternative hypothesis if the p-value is less than the threshold. We denote outcomes using asterisks to show the significance level ($\alpha$) associated with it for the sake of simplicity. Additionally, rather than reporting an actual value, the p-value is reported as “$<0.001$” if it is very low.

4.1. Policy implications

- While it might be more efficient to concentrate solar panels in sunny states, there are several factors to consider. The cost of transmission infrastructure, potential power losses during long-distance
transmission, and environmental impacts of the transmission lines are among the factors that might offset the benefits of concentrating solar panels in sunny states. Moreover, diversification of energy sources is crucial for the security of the power supply. Consequently, a mix of strategies, including investing in solar power in different climatic conditions, should be pursued to ensure a stable and secure energy supply.

- Tolerating technology gaps for the sake of security of power supply: Tolerating technology gaps might not be the optimal solution, as it could lead to inefficiencies in the long run. Instead, a focus on improving the performance of solar power in various climatic conditions will contribute to a more secure and sustainable power supply. Therefore, it is essential to invest in research and development to identify and implement the most effective technologies and strategies for each climatic zone.

- Developing panels suited for different types of climate conditions: Our findings suggest that there is potential for improvement in the performance of solar power in different climatic conditions. Investing in the development of solar panels that are specifically designed for different climates could significantly enhance the overall efficiency of solar power generation. Such advancements would not only address the technological gaps but also contribute to a more robust and secure power supply across the country.

In conclusion, our analysis highlights the need for a multi-faceted approach to improve solar power performance and address the technology gaps among different states. Strategies should include concentrating solar panels in sunny states and investing in transmission capacity, while also focusing on research and development to create solar panels suited for different climatic conditions. The policy implications outlined above will provide decision-makers with valuable insights and a basis for future investments in the solar energy sector.

Fig. 8 characterizes top ten states based on convexity assumption. California remains in the first place among states generating electricity from solar power, generating more than 31% of the U.S. total of 11,042 thousand megawatt-hours, according to the U.S. Energy Information Administration (EIA) followed by North Carolina, Delaware, New Jersey, Florida and Arizona are the next three states on the list.

Figs. 9 and 8 show that California and Florida have the same ranking score, while Hawaii and Arizona are ranked fourth and fifth, respectively. Due to the high expense of importing energy (coal and petroleum) to this island compared to mainland places, it is not surprise that Hawaii has become one of the top users of solar energy. Applying such a renewable energy source provides their inhabitants with a cost-effective clean energy alternative and generates great returns on investment.
Texas has historically produced electricity using coal-fired power stations. Due to abundant sunlight and declining solar technology costs, businesses have installed enormous solar frames during the past ten years, moving them up to the top two positions on the list.

5. Conclusion

This paper measured the solar power performance of 44 U.S. states in 2019, using StoNED methodology. We classified the 44 US states into three climatic zones, namely (hot dry/hot humid areas), (mixed humid areas), and (cold/very cold areas and marine). While the fact that convexity axiom has a long-known effect on benchmarking, few empirical applications have demonstrated this effect. This study examines this issue through parametric statistical testing. In addition, we derive some management insight-related conclusions. In order to increase the productivity of US solar energy systems are typically four scenarios. A state may keep putting up new solar panels if it receives high rankings above its mean score for both the TE* and TGR. This state is referred to as a leader. Authorities must take some fundamental steps if both TE* and TGR scores fall below their mean values. In this case, the emphasis should be on research and development to enhance operational performance. Consider a state whose TE* is higher than its mean score and TGR is lower than its mean score. In that case, it advises the state to put more of an operational focus on using cutting-edge technologies than on performing any other activities. The findings indicate that most states struggle with breath of operations and maintenance strategies for solar systems. In this case, the best course of action is to invest in the production of solar panels that have been adapted for diverse climates in order to enhance the overall effectiveness of solar power.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References


CRediT authorship contribution statement

Haleh Delnava: Conceptualization, Data gathering, Investigation, Methods, Software, Writing – original draft, Validation, Visualization. Ali Khoosravi: Conceptualization, Methods, Visualization, Writing – Reviewing and Editing. Mamdouh El Haj Assad: Conceptualization, Visualization, Reviewing and Editing.


