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A probabilistic approach to reliability analysis of district heating networks incorporating censoring: A report of implementation experiences

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Abstract: Reliability analysis has the potential to provide actionable insight into the failure probability of assets in district heating networks. Information about the failure rate and its trend may help operators and asset managers replace assets at the optimal time, which can increase the security of supply, save resources, and reduce operational and maintenance costs. In this paper, we employ a probabilistic proportional hazard modeling approach to reliability analysis, which has not been used for district heating pipes before, explore its potential and report our experiences. The model allows us to model the time-dependent survival probability of pipe assets as a function of asset-related and environmental predictors which have been shown to influence failure probability in previous studies. We find that the application of the model in this domain is challenged by several issues pertaining to data, one of which we attempt to remedy with a simple imputation strategy.

Keywords: Reliability analysis, District heating, Predictive maintenance, Data collection

1. Introduction

Breaks and leaks in district heating systems can cause losses of tens to hundreds of thousands of cubic meters of water yearly. Aside from water being expensive to replace, breaks in district heating pipes can decrease the supply to a level where consumers do not have access to the heat they require. While repairing breaks is expensive, pre-emptive replacement of pipes is inefficient in terms of both cost and other resources. Determining when is the optimal time to replace a pipe requires information about the expected time to failure, frequency of failure, time-dependent survival probability, or similar. “Reliability” or “survival” analysis sets out to determine exactly this optimal replacement time.

Survival analysis is a branch of statistics that is native to the medical industry but is also used (under other names) in other disciplines e.g. in the engineering domain it is referred to as reliability analysis. Purely physics-based models are rarely used to find the expected time to failure because they are very time-consuming to implement and can have unrealistic requirements for sensor input. Nevertheless, if the sensor data is available, physics-based degradation models have very high accuracy.

A common alternative to physics-based models is statistical models, which can be roughly divided into deterministic and probabilistic models. Deterministic models employ regression techniques such as least-squares regression to fit e.g., a Poisson-generalized linear regression model or a multivariate linear regression model [1] [2]. The probabilistic alternative to statistical modeling uses e.g. Monte Carlo simulation or Bayesian statistics [3] [4] [5], the former potentially including methods for Markov Chain Monte Carlo simulation of the conditional density of regression coefficients [3].

Rimkevicius et al. developed a methodology for assessing reliability of energy networks which was applied to a district heating network. The method consisted of both deterministic and probabilistic modelling elements and was able to identify the most failure-prone pipe sections and to estimate failure consequences at the consumer level [6]. Postnikov et al. used Markov random process theory to model the reliability of systems of district heating assets [7]. The simulated reliability study of combined assets showed that the reliability of district heating systems is mostly affected by individual assets with poor reliability. Valincius et al. used simple statistical analysis to identify failure-prone pipes in a district heating network and subsequently deterministically model only the most failure-prone pipes [8].

Event time information in survival data is rarely complete. E.g. a pipe can have been broken for several months before it is noticed, in this case the actual time the pipe broke is unknown, which can be referred to as incomplete event time information. This phenomenon biases survival models. To reduce the bias some probabilistic survival models account for incomplete event time information using censoring [3] [4] [5] [9] [10], which will be explained in more detail later in this paper.

A sub-category of probabilistic models referred to as Proportional Hazard Models (PHM) are unique in that they explicitly parameterize the effects of a set of covariates on survival time. These covariates typically explain the material type, nominal dimensions, etc. when employed for pipes [4] [5].

Recently there have also been some applications of machine learning for survival analysis of pipes, e.g., [2] in which a neural network was used for binary classification and remaining-life prediction. Recently, a combination of survival modeling and machine learning involving random survival forest techniques was applied to survival analysis of water pipes [11]. Random Forest considers the log-rank statistics between cumulative hazard functions of child nodes when performing splits, which means the model can be used on censored data.

In recent work on relative fault vulnerability prediction, our results corroborated that the environment surrounding district heating pipes seems to affect their failure rate [12]. These environmental features can be integrated into the survival modeling of district heating pipes using the PHM approach, as has been done for water distribution systems [1].

In this paper, we implement the censor-adjusted Weibull Proportional Hazards Model (WPHM) in the district heating domain which, to the best of the authors' knowledge, has not been done before. We use the PHM approach to model environmental lying conditions' effects on district heating pipes. We explore the challenges that arise from working with a relatively new district heating network, i.e., a network in which only a small fraction of the pipes has reached the wear-out stage of their lifecycle. Furthermore, we consider the potential influence of suboptimal data collection, discuss the use of imputation, and suggest several criteria for good data concerning survival analysis.

The paper is structured as follows: First, we explain the methodology of applying the survival model, then we introduce the case study to which survival model is applied. Lastly, we present and discuss the results, emphasising finding reasonable explanations for the discrepancies that we observe between the values predicted for life expectancy of district heating pipes according to our model and industry belief.

2. Reliability model for maintenance of distribution systems

In this section, the theory behind the probabilistic survival analysis employed in this paper and its application is introduced. Kabir et al. [3] present a summary of survival analysis methods applied to water distribution systems. They find that the exponential model, Weibull model, Cox proportional hazard model (cox-PHM), and the Weibull proportional hazard model (WPHM) are particularly well-regarded and widely employed. The proportional hazard models share an interesting characteristic, in that they integrate a predictor term in their hazard function that depends on a set of covariates, meaning they can model the correlation between, e.g., pipes' lying conditions or asset information and their reliability directly [13].

The cox-PHM is described in [14]:

$$h(t|X) = h_0(t)e^{X\beta} \quad (1)$$

where $h(t|X)$ is the time- and covariate-dependent hazard function, h_0 is the baseline hazard function, t is time with reference to when the study period begins, and X and β are the covariates and the covariate coefficient vectors, respectively. From (1), it can be seen that the second factor is independent of time, i.e., the covariates' effect on the baseline hazard function does not change over time [4]. The survivor function is given by:

$$S(t|X) = S_0(t)e^{X\beta} \quad (2)$$

where S_0 denotes baseline survivor function.

In the WPHM, the survivor function is given by [15]:

$$S(t|X) = e^{-e^{\left(\frac{\log(t)-\mu-X\beta}{\sigma}\right)}} \quad (3)$$

where μ is a constant called intercept and σ is a scale parameter. The WPHM is derived from the log-linear relations [15]:

$$\log(t) = \mu + \beta X + \sigma\epsilon \quad (4)$$

where ϵ is an error term. The log-linear relationship shows that there is an interaction between the covariates and time in the WPHM which is not present in the cox-PHM. This is one reason the WPHM is gaining more attention [3]. For this study, we also use WPHM.

The parameters for the WPHM can be estimated using maximum likelihood estimation, the likelihood function of the WPHM is given by:

$$L(\beta, \mu, \sigma|data) = \prod_{i=1}^{n} f_i(\log(t_i))^{\delta_i} S_i(\log(t_i))^{1-\delta_i} \quad (5)$$

where δ_i is a censoring indicator and f_i is the density of the log time, given by

$$f_i(t_i) = \frac{1}{\sigma} e^{\left(\frac{\log(t_i)-\mu-X_i\beta}{\sigma}\right)-e^{\left(\frac{\log(t_i)-\mu-X_i\beta}{\sigma}\right)}} \quad (6)$$

where *i* is an index ordinal denoting a specific observation.

2.1. Censoring

Figure 1 illustrates the concept of censoring. Since most of the pipes in the case study have not failed yet, the observations in the data are predominately right-censored, meaning that the time, t_i, of a right-censored observation can be considered to be the pipe's minimum lifetime. Since the thermography measurements are not performed continuously but at regular intervals, when a fault is observed, the actual time of the fault is anywhere between the time of observation and the time of the previous observation of the pipe. This should be treated with interval-censoring. Pipes that have failed and been replaced without any record are omitted from the study and the dataset is therefore defined as left-truncated. Pipes that are currently installed and have failed before observations began should be left-censored since the fault has happened before *t*=0. In reality, faults have been repaired before the start of the study period, which means that pipes that should have been left-censored can only be represented as right-censored considering the available information.

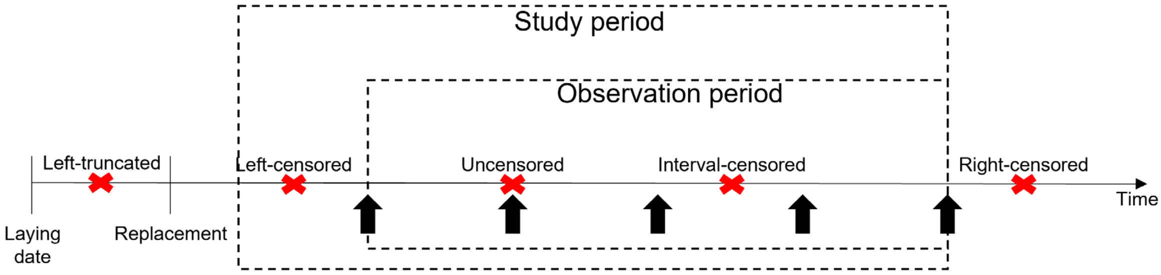

Figure 1: Illustration of types of censoring and truncation typically encountered in survival analysis. The black arrows represent observations, and the crosses represent faults.

The likelihood function, adapted to account for interval censoring, expressed as only right, and interval-censored observations can be described as [16]:

$$L(\beta,\mu,\sigma|data) = \prod_{i=1}^{n} S_i(\log(t_i))^{\delta_i} \prod_{i=1}^{n} (S_i(\log(t_{i,lb})) - S_i(\log(t_{i,ub})))^{1-\delta_i} \qquad (7)$$

where $\delta_i = 1$ denotes right censoring and $\delta_i = 0$ denotes interval censoring. $t_{i,lb}$ and $t_{i,ub}$ denotes the lower and upper bound for the survival time of pipe i respectively, assuming the pipe failure is immediately noticeable. The left product, therefore, pertains to right-censored observations only, and the right product pertains only to interval-censored observations.

2.2 Posterior sampling

To maximize insight into the parameters of the posterior distribution, maximum likelihood estimation (MLE) is first used to get an initial estimate of the parameters. Subsequently, samples are taken from the posterior distribution:

$$f(\sigma,\mu,\beta|Data) \qquad (8)$$

using the Metropolis-Hastings random walk algorithm [15], because of its efficiency and popularity [17]. This enables studying the uncertainty of the parameter estimates of the survivor model. We use a normal proposal density, define by a mean vector of zeros and the variance-covariance matrix of the parameters σ, μ, β. As suggested by [15] we set the scale parameter of the proposal density so that the acceptance rate of simulated draws is in the range 20-40 %.

In summary, an asset's survival probability is calculated using (3), the parameters of which are determined using the Metropolis-Hastings random walk algorithm, with the initial parameter estimate determined using the maximum likelihood estimation of (7). We use the Metropolis-Hastings random walk algorithm to simulate 10,000 draws from the posterior density (8), meaning the parameters of the model given in (3) given a specific dataset. Predictions are obtained by solving (3) for each simulated draw of the posterior density (8)(7) and characterizing the statistical distribution of the output survival probability.

3. Case study

We implemented the reliability model using data from a district heating network on the island of Funen, which represents Danish district heating pipe networks well. The network supplies more than 100,000 consumers and consists of more than 140,000 pipes. The pressure ranges from <25 bar in the transmission pipes to <6 bars in the distribution pipes. The dataset covers a span of 5 years.

In a previous study [12] we created a dataset based on the geographic information system (GIS) of the district heating system, its historical maintenance record, and relevant GIS data representing the external environmental lying conditions of the pipes. Detailed information about the environmental datasets is given in [12]. The raw datasets were retrieved from [18] [19] [20]. The environmental conditions give insight into the chemical and mechanical stresses the pipes may be exposed to.

Using this dataset, in [12] we predicted number of faults and ranked the pipes according to their relative vulnerability. Using our ranking method on this dataset showed that 30 % of the network was responsible for 60 % of the historical faults in the test data. In this paper, we use a subset of that dataset consisting of the 8 most important, non-redundant, features. These are listed in TABLE I.

TABLE I: Descriptions of selected features.

Feature	Description
Number of joints	Number of joints along a pipe section
High-risk area	Whether a pipe is located in an area with a high failure risk
Nominal dimension	The nominal internal dimension of a pipe
5 Meter minor road proximity	The proportion of a pipe that is closer than 5 meters to a minor road
Level	Ordinal encoding of the hierarchical classification of a pipe, e.g., transmission level or distribution level.
Mean redox depth	The average depth to anaerobic soil conditions along a pipe section
1 Meter track proximity	The proportion of a pipe that is closer than 5 meters to a track
DSG type soil coverage	The proportion of a pipe that is located in meltwater-sand and -gravel

4. Results and discussion

This section presents results relevant to the evaluation of the survival model, comparing the model with results from our previous work [12]. The median survival probability as a function of time for the entire population of pipes is investigated and compared with life expectations by the Danish district heating association. Lastly, we explore the distribution of age and faults in the data, and test an imputation method to identify the reason for the discrepancy between predictions of expected lifetime according to our model and industry belief.

Figure 2 shows the survival probability for the most and the least at-risk pipe according to our previous study [12]. The plots are produced by calculating the survival probability, using (3), at various times for each simulated draw of parameters given by the posterior density (8).

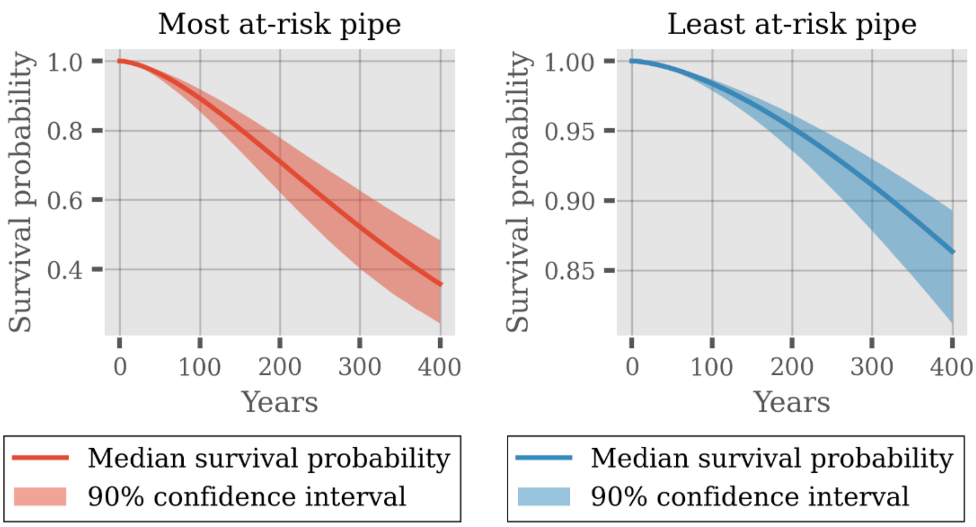

Figure 2: Survival probability, calculated using the WPHM with simulated draws of its parameters, for the most at-risk pipe (red) and the least at-risk pipe (blue) according to our previous study. The lines denote median survival probability, and the areas denote 90% confidence intervals.

Both pipes seemingly have incredible long lifetimes, with expected life being approximately 300 years for the most at-risk pipe and well past 400 years for the least at-risk pipe, here expected life is define as the time where the survival probability falls below 50 %. This is not aligned with experts' knowledge, which is that the pipes likely will not survive hundreds of years. The Danish district heating association, "Dansk Fjernvarme", claims that the expected lifetime of Danish district heating networks is 50-100 years [21]. The Danish district heating association claim 50-100 years is a long expected lifetime and that it is in part credited to the relatively low temperatures that Danish district heating networks are operated at, 70-80 degrees celsius.

Figure 3 visualizes all pipes' predicted median survival probability as a function of service life. According to the WPHM, the expected median survival probability at 400 years is close to 90%. This emphasizes the order of magnitude for the difference between the expected life according to the model the expectation of industry experts.

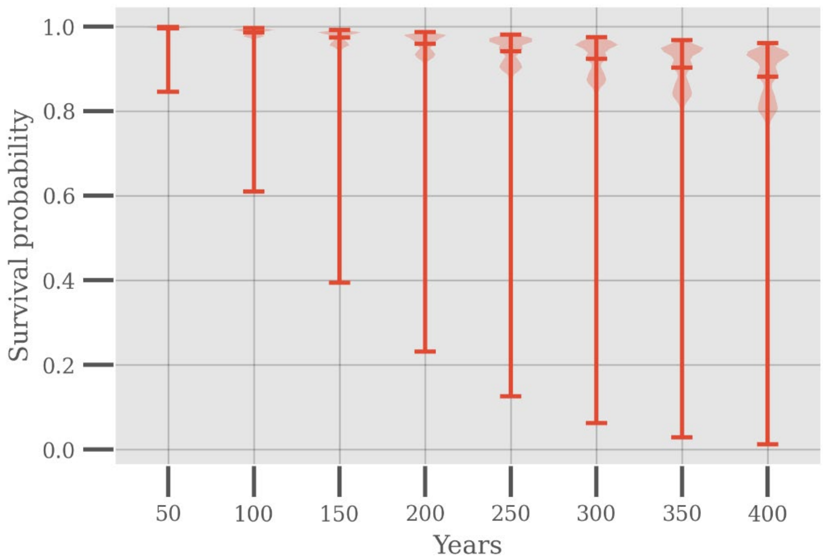

Figure 3: Violin plot of median survival probability of the entire population of pipes at different service lives. The horizontal lines represent max (top), median (middle), and min (bottom) median survival probability.

Three reasons, in particular, can explain why the observed trend deviates so much from what is expected. Firstly, the maintenance record, i.e., thermographic imaging in this case, only spans 5 years, while the oldest pipes are more than 40 years old, see Figure 4. The data is in that sense, predominately left-censored, which results in a discrepancy between the observed number of failures and the actual number of failures, with actual failures including failures that happened before the study period. This biases the model towards a longer survival time, as pipes that broke and were repaired or replaced before the study period began are not represented with accurate failure times. Since it is unknown which pipes broke and were repaired before the study period began, the pipes that should be left-censored are represented as right-censored observations, and pipes that have been replaced before are truncated.

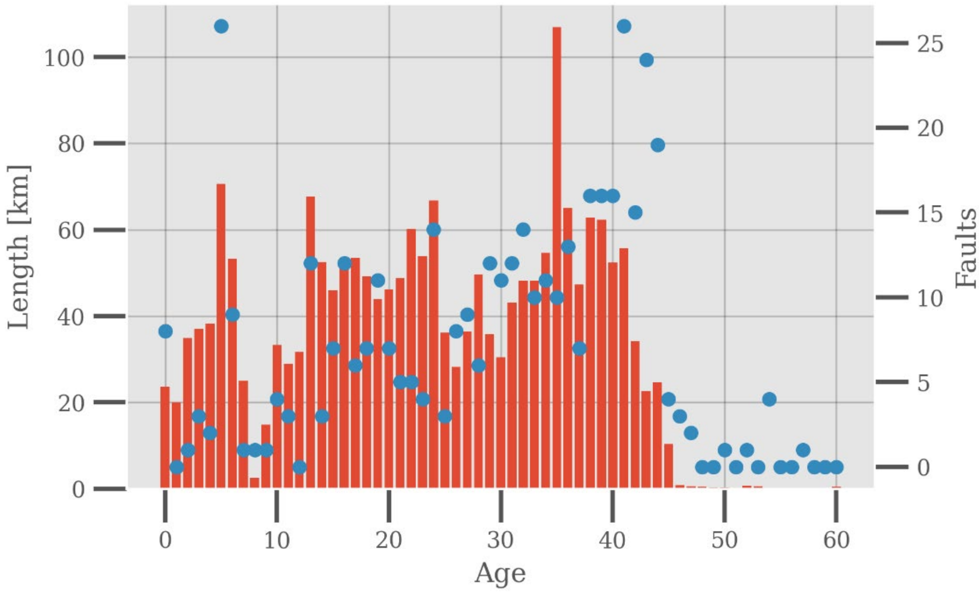

Figure 4: Histogram of the age distribution of the district heating pipes. The blue dots denote the number of historic faults for pipes in that age bin.

To study the magnitude of the bias introduced due to left-censoring, we employ a basic imputation strategy and reevaluate the parameters of the WPHM. The imputation strategy makes use of the observed time-dependent failure rate, visualized in Figure 5. The figure shows the failure rate as a function of age. Since the fault observations span multiple years, note that the individual pipes at a specific age change from year to year. The histogram, therefore, shows the average length of pipes at each bin.

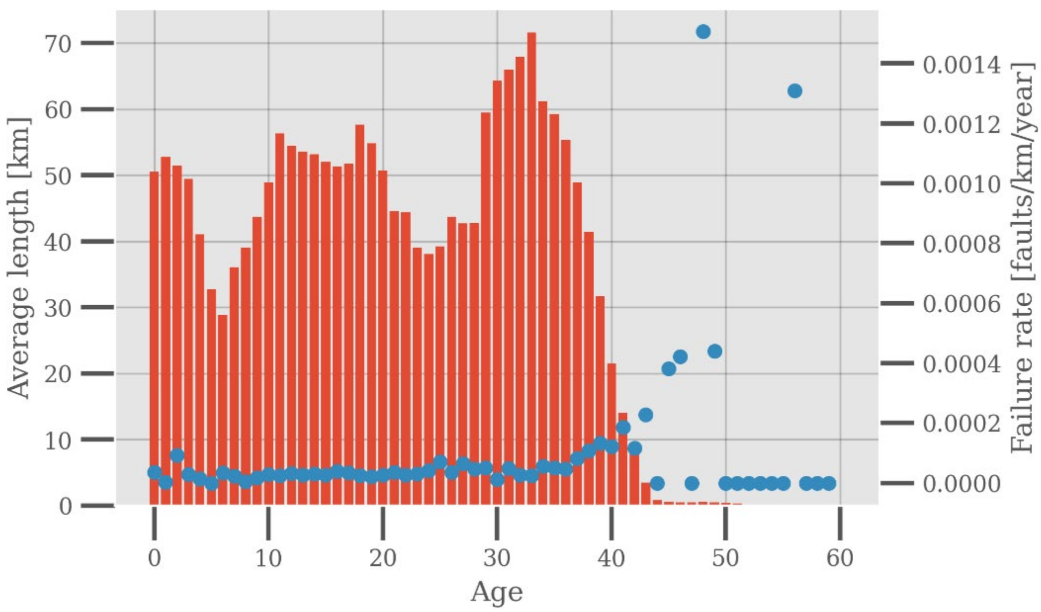

Figure 5: Histogram of the distribution of average length for each observation bin of 1 year. The blue dots denote the failure rate for each age bin.

A population of pipes that are older than the observation period can be assumed to have developed faults at about the same rate, so the number of faults in those pipes in a given year can be calculated as the product of the failure rate of pipes at the age the pipes were at that time and the total length of the same cohort of pipes. This can be expressed as:

$$n_f = \lambda_{yb} L_{yb} \tag{9}$$

where n_f is the number of faults, λ_{yb} is the failure of pipes aged yb, and L_{yb} is the total length of pipes aged yb. This assumption is used for all years in which the failure statistics for a population of same-age pipes was not recorded. Employing this imputation strategy results in 1075 additional faults scattered over the lifetime of the pipes. The feature vectors of all imputed fault observations are sampled randomly from the distribution of the observed pipes that have failed. This is an attempt to avoid wrongfully impacting the covariate coefficients of the WPHM.

Figure 6 shows the median survival probability based on the imputation of left-censored observations. Compared to Figure 3, the survival probability declines faster, which means that expected life is lowered. This suggests that the imputation of left-censored observations can reduce the bias toward longer expected lives of survival models. Nevertheless, the expected life is still incredibly high, so the bias from left-censoring cannot alone explain the deviation between the model outcome and industry belief.

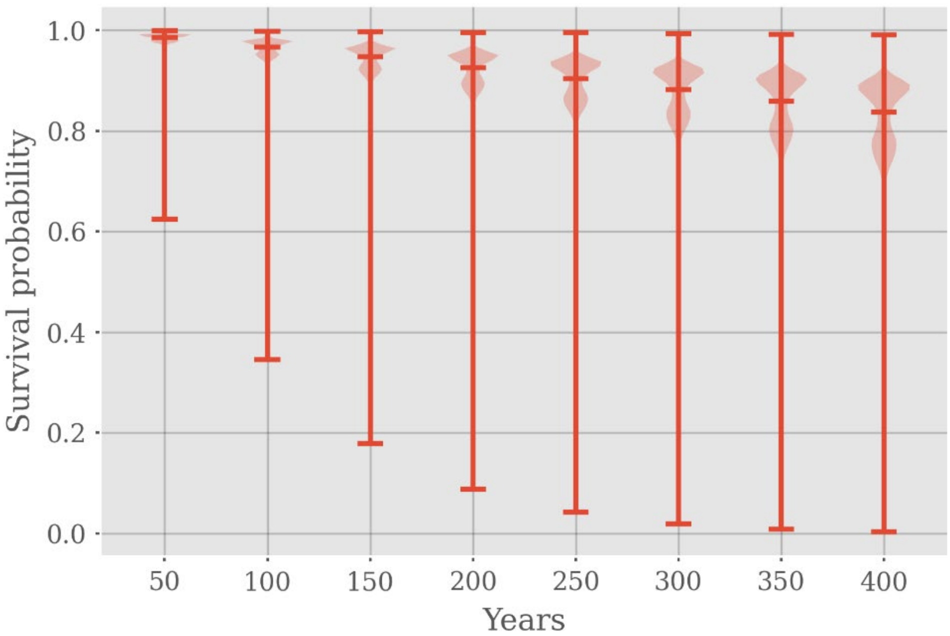

Figure 6: Violin plot of the median survival probability of the entire population of pipes at different service lives based on parameters determined based on the imputation of left-censored observations. The horizontal lines represent max (top), median (middle), and min (bottom) median survival probability.

Another potential reason for the discrepancy is that the vast majority of pipes have likely not reached the wear-out stage of their lifecycle. The lifecycle is generally believed to be described by the bathtub curve, as presented in [22], which has three distinct stages. The first stage is described as infant mortality, which is caused by manufacturing or installation faults. The second stage is characterized by a constant failure rate, and the last stage, the wear-out stage, describes an increasing failure rate as the materials reach the end of their lifetime.

Firstly, infant mortality is not observed, see Figure 5. This is because these faults are detected using insulation resistance monitoring with copper wires integrated into the insulation and this datatype was notavailable for this study. The technology was adopted fairly recently, so it primarily pertains to the newest pipes. This is, of course, onereason why the number of observed faults is lower than the actual number of faults in this study and is likely to bias the model towards longer expected lives.

The failure rate is close to constant until the pipes reach an age of approximately 40 years. At this point, the amount of data used to calculate the failure rate is fairly low (low number of pipes and correspondingly low number of faults), meaning that the failure rates are very uncertain. This, coupled with the information that lifetime is expected to be 50 years at the very least, renders it challenging to confirm whether a small portion of the pipes has reached their wear-out stage. The time following the expected lifetime of the pipes is an important object of study, as it represents the wear-out stage's increase in failure rate, and this object of study is unavailable in the dataset, as very few if any, pipes have reached the wear-out stage of their lifecycle.

Survival data for pipe networks are widely analysed in the water distribution domain. Old maintenance records are very helpful contributors in this context [10], as they can cover decades of maintenance data [3], [10], [9]. However, several studies emphasize that an extensive maintenance database is not necessary for accurate survival modeling [4], [2], [23], e.g. [4] states that "*short maintenance records (5-10 years) give as good results as long maintenance records*". Additionally, multiple studies suggest that survival analysis can be useful even under left-censoring or left-truncation [4], [23]. However, left-censoring and left-truncation can still create bias in survival modeling by underestimating the apparent time to failure, which we show is not the only bias in the current model.

The general characteristic of the water distribution domain's survival data is that at least parts of the pipe network are older than their expected life [3], [10], [9]. A concrete example of this is the network in [2], which is more than 50 years old while the expected lifetime of the network's pipes is approximately 30 years. This means that the evolution of the pipes' failure rates is well expressed in the maintenance and asset data. Performing survival analysis on relatively young pipe networks, therefore, runs the risk of not having the pipes' wear-out phase represented in the data.

While this can explain a model bias towards much longer estimates of the service life of district heating pipes, it is also possible that the district heating network is much more reliable than initially thought. It is possible that the Danish district heating association have underestimated the expected life of the district heating grid. However, the order of magnitude difference between the expectations and the model output makes it very unlikely that this is the only explanation.

5. Conclusion

This paper explored probabilistic survival modeling for pipes used in district heating and found that the model predicted a much longer service life for the pipes than the industry expects. We identified several reasons for this discrepancy, with truncation and censoring bias being two of them. This bias can be reduced by imputating censored observations -- however, even with imputation, the model was still biased towards much longer lifetimes than expected. A short review of survival analysis in the water distribution domain suggested that the primary reason for the large discrepancy is that the case network is relatively young, so the increase in failure rate during the wear-out stage of the pipes' lifecycle is not well represented in the data. Good data collection for survival analysis, based on the work presented in this paper, entails tracking all failures from when the pipe network is commissioned, though imputation can help if this is not done. Lastly, survival analysis is more applicable for relatively old infrastructures for which the data describes all parts of the pipes' lifecyle.

Future research might perform survival analysis using more complete data, with less need to censor observations, which represent an older network. This would provide a case study, where the findings of this paper could be corroborated. Additionally, the value of censor-adjusted survival modeling could be studied and demonstrated

Declarations

Availability of data and material

The parts of the dataset relating to the district heating system are confidential. The parts of the dataset relating to environmental condition before any processing are publicly available at [18] [19] [20].

Competing interests

The authors declare that they have no competing interests.

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Authors’ contributions

LKM prepared the data used in the study, implemented, and interpreted the survival model, and prepared the first draft of the manuscript. HRS and CTV provided supervision and reviewed the manuscript. All authors read and approved the final manuscript.

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