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I. INTRODUCTION

Chaos. A simple word but by no means a simple concept. Traditionally, as in the Bible, chaos designates the initial states of the world before its organization by "God." It arises from *tohû wabohû*, a reality without any form, any order, or life (Bible, Genesis 1, verset 2). It is opposed to everything organized and structured.[1] In the early 1970s, Ruelle and Takens proposed an alternative theory for the transition to turbulence.[2] Thirty years before, Landau[3] and Hopf[4] described turbulence as the superimposition of many linear oscillators, each one producing a given frequency. Interacting with René Thom and physicists, Ruelle understood that nonlinear coupling through the fluid viscosity allowed one to get an infinite number of frequencies with a limited number of oscillators. Together with Takens, Ruelle transformed this into a theorem stating that four oscillators were sufficient to produce an aperiodic behavior which was not quasi-periodic, that is, a trajectory which was not structured in a torus T^n, with n being the number of different frequencies.

In 1971, Ruelle and Takens had no idea what could be such an attractor, so they named it *strange*. For these two mathematicians, turbulence is a "*very complicated, irregular, and chaotic*" fluid motion.[2] A few years later, Li and Yorke, in investigating quadratic maps, quoting May,[5] among others, discovered that very simple recurrence relation can produce very complicated and irregular sequences of numbers.[6] They designated them, in the title of their paper, as "*chaos*." At that time, the solutions were perceived neither as organized nor as structured. This is only a few years later that the underlying order—exhibited by Feigenbaum's discovery of the universal scaling law in period-doubling cascade[7]—was, in fact, understood. The word *chaos* was no longer appropriate but it was too late to change it because some scientists like Rössler started to designate systematically these aperiodic solutions, which were not quasi-periodic as being chaos.[8–11] As initiated by Lorenz, who entitled his founding paper *deterministic nonperiodic flow*, with Wegmann, Rössler described the concept of *deterministic chaos* by clearly distinguishing motion whose aperiodicity is the result of the underlying deterministic dynamics from aperiodicity induced by external perturbations (noise):[12] this distinction is not required when differential equations or discrete maps are considered,[13–17] but this is a fundamental question when experimental data are investigated. With the development of the properties exhibited by chaotic solutions, it became important for experimentalists to push the idea that aperiodicity and lack of long-term predictability are not contradictory to some *order within chaos*,[18] which Schuster popularized in his book entitled *Deterministic chaos*.[19]

This is only in 1984 that Grebogi *et al.* distinguished *strange*—associated with the fractal properties—from *chaotic*—associated with a sensitivity to initial conditions.[20] Thus, today, each time chaos is referred to in a scientific context, it is necessary to specify that chaos should be understood as deterministic, organized, structured, and long-term unpredictable and to illustrate what is meant with the example of meteorology, quoting Lorenz and his *butterfly effect*.[21] It should be specified that Lorenz never considered this type of behavior as disorganized and from his early contributions, he looked for the rules and the structures, which could help us to better describe nonperiodic behavior, distinguishing them from the quasi-periodic ones.[22,23] He used a tent map to describe how the so-called Lorenz attractor was visited.

Many names could be associated with *chaos theory*. We will not risk coming up with even a tentative list of them. There are also a few books devoted to the history of chaos. The book of Gleick is aimed at a wide audience and is mainly focused on the American contribution to the field.[24] Some recollections of the early times in chaos by Stephen Smale, Yoshisuke Ueda, Ralph Abraham, Edward Lorenz, Christian Mira, Floris Takens, James Yorke, and Otto E. Rössler were published in 2000.[25] How chaos emerged from the celestial mechanics and was applied to various fields is discussed in the book entitled *Chaos in Nature*.[26] The first paper of this Focus Issue, *Some elements for a history of the dynamical systems theory*, provides some excerpts of the recollections by some great contributors to chaos theory, which were never written before.[27] It appears that, in most of them, the important new tool in their scientific activities was the computer they used to investigate these solutions, which were seen as two-dimensional plots for the first time. Nevertheless, this is not necessarily required ingredients since many important contributions are still "pure mathematics" to provide some criteria for chaos[2,6,28,29] or a simple map producing a complex structure.[30]

To define a paradigm, there is a need to have (i) a set of principles and well-defined concepts, (ii) a set of benchmark systems, which can serve as examples for testing the techniques developed for characterizing these chaotic solutions, and (iii) some pictures to explain to a wide audience what is meant. Let us start with Devaney's definition of chaos:[13]

Let X be a metric space. A continuous map $f : X \to X$ is said to be chaotic *on X if*

(1) *f is transitive*;
(2) *the periodic points of f are dense in X*; and
(3) *f has sensitive dependence on initial conditions*.

This definition is, in fact, redundant, and Wiggins reduced it to conditions (1) and (3).[17] Note that the condition for a "continuous map" is the implicit condition for a deterministic dynamics. This very clear definition is not straightforward to apply to common chaotic systems. A more practical definition could be that a solution $\mathscr{S}$ to a dynamical system $f : \mathbb{R}^d \to \mathbb{R}^d$ is said to be *chaotic* if f is deterministic, $\mathscr{S}$ is bounded, and is sensitive to initial conditions.[31] Due to the apparent evidence for a bounded and aperiodic solution, the attention mainly focused on the sensitivity to initial conditions, which can be associated with a positive Lyapunov exponent.[32] A positive largest Lyapunov exponent turned to be a "*strong signature of chaos*,"[33] although there is a strong limitation to such a statement[34] and some strong requirements on the data.[35] The last condition—sensitivity to initial conditions—can be replaced with "$\mathscr{S}$ has a positive entropy,"[36] which presents the advantage of being easier to compute. In both cases, there is still the necessity to prove that the dynamics is deterministic, certainly the most difficult property to prove from experimental data. This led Leon Glass to exhibit that *prior to asserting that something is chaotic, there should be clear evidence that deterministic equations govern the dynamics*.[37] A possible tool for that is *global modeling*.[38]

For a paradigm, there is a need for benchmark systems. The Lorenz system[22] became the most quoted set of ordinary differential equations (23 417 quotations counted in February 2021). The simple continuous equation proposed by Rössler[10] is the second one (4200 quotations). Then, one may mention the driven Duffing equation[39] or the driven van der Pol equation:[40–42] it is worth noting that the first plot of a chaotic attractor in a two-dimensional projection was obtained in 1961 by Yoshisuke Ueda with the latter equation.[43] The Chua circuit[44,45] (1580 quotations) could also be mentioned, although its nonlinearity is a piecewise linear function: it produces a double scroll attractor as already observed by Rössler[46] or by Pikovski and Rabinovich.[47] All these systems are dissipative. Conservative systems are not so common. The first one to mention in a history of chaos is the restricted three-body problem as written by Jacobi[48] and in which Poincaré found homoclinic orbits.[49] Nevertheless, the first chaotic solution investigated through numerical simulations is most likely the four-dimensional conservative Hénon–Heiles system (2443 quotations).[50] For many reasons, it is often useful to consider iterated (or discrete) dissipative maps as the Logistic map (7892 quotations),[51] the Hénon map (3506 quotations),[52] and the Lozi map (397 quotations),[53] or the conservative Chirikov (standard) map (4724 quotations).[54]

To complete the Chaos paradigm, pictures are needed. The first one is certainly the butterfly effect: this is *stricto sensu* not a picture but it is such an easily understood example, which has been used in movies and novels. Then comes the Lorenz attractor but it is already a more complicated picture to understand since it is plotted in state space, which differs from the physical space: this difficulty is attenuated by its fascinating beauty. Another picture was discussed by Poincaré while investigating the complex homoclinic orbits: "*We will be struck by the complexity of this figure, which I do not even try to draw*."[49] It seems that Melnikov was the first to draw it in 1963 [Fig. 1(a)].[55] Another key picture is the bifurcation diagram of a quadratic map presenting a period-doubling cascade (in every book about chaos). The Poincaré section of a conservative system, as the Hénon–Heiles system [Fig. 1(b)], should also be mentioned with its chaotic sea and quasi-periodic islands.[50] Let us finally mention the amazingly beautiful mythic bird [Fig. 1(c)] produced by the not so well-known two-dimensional map discovered by Mira.[56]

We have here all the ingredients for a paradigm that emerged during the 1960s and 1970s with the popularization of computers. We took the opportunity of the 80th birthday of Rössler (born on May 20, 1940) to acknowledge him this Focus Issue for his contribution to the chaos theory. Our purpose was to promote chaos, its inherent properties, as well as its applications. *It consists of 18 papers,*[27,57–73] *which are briefly introduced in the subsequent part of this introduction.*

II. CHAOTIC SYSTEMS

The first system in which chaos was detected is the restricted three-body problem.[49] Numerically, this system is not so easy to handle because it requires a symplectic integrator for reliable simulations.[74] As already mentioned, the Hénon–Heiles system[50] is preferred and is one of the most often quoted sets of ordinary differential equations. The amazing success of the Lorenz system is a result of its algebraic simplicity—only three differential equations with two quadratic terms—, it is dissipative (a simple Runge–Kutta scheme is sufficient) and it produces a rather simple attractor.[22] Moreover, this attractor has an aesthetic power and is even a very suggestive picture with its two wings for the butterfly effect! Nevertheless, from the dynamical point of view, this attractor has a rotation symmetry around the z axis, which can be the source of difficulty in interpreting the results.[75–77] This is the reason why the Rössler system[10] is

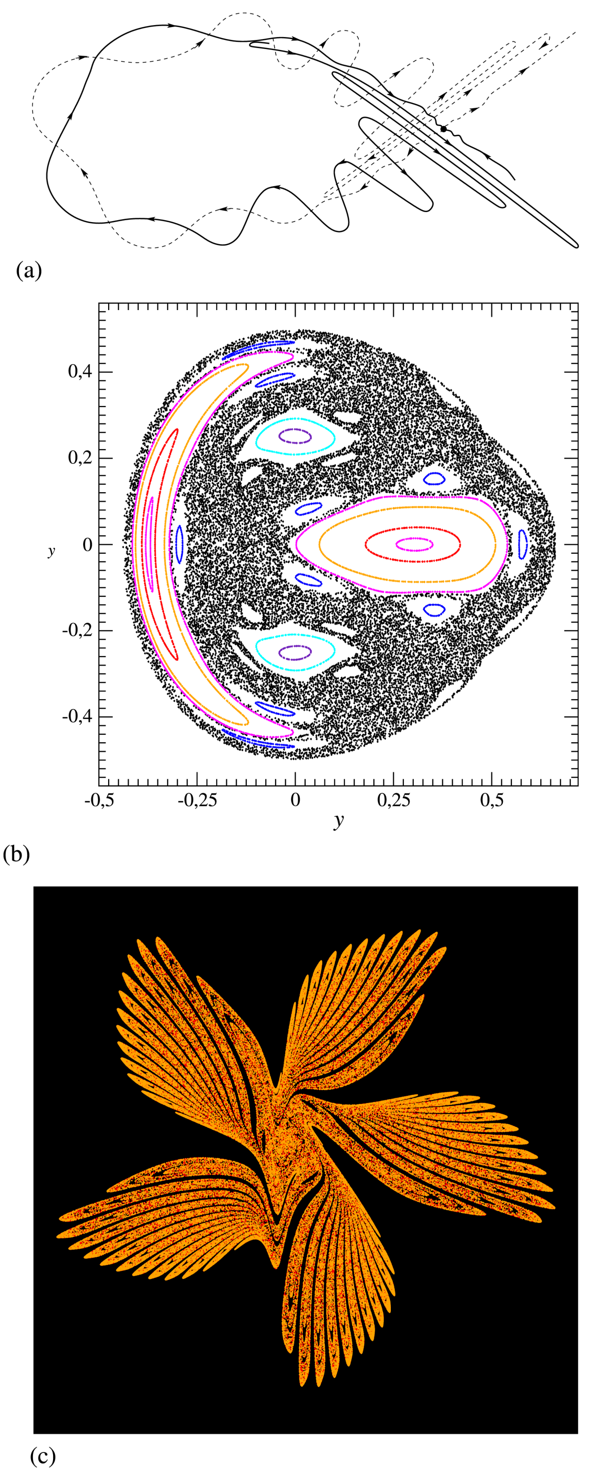

FIG. 1. Pictures to illustrate the *chaos paradigm*. (a) Homoclinic entanglement (redrawn from Melnikov, 1963). (b) Chaotic sea and quasi-periodic islands (Hénon–Heiles, 1964). (c) Mythic bird (Mira, 1973).

in fact a more recommendable system: it has no residual symmetry and the topology of its attractor—as characterized by a branched manifold—is the simplest one, with two branches with a single local torsion.[78–80] It could be replaced with an algebraically simpler system as the Sprott I, J, or N system[81] but the domain of the parameter space for which there is a chaotic attractor can be quite small, rendering too simple systems not structurally stable.[82] As a proof of the richness of the parameter space of the Rössler system and in spite of few studies,[78,83–85] new (hidden) attractors were found by Malasoma and Malasoma in this Focus Issue.[57] The structure of the Rössler attractors was explained by Malykh *et al.* in terms of homoclinic bifurcations.[58] Finding a homoclinic bifurcation is considered as a proof of chaos.[86] The role of the second singular point is also exhibited in this contribution. The fact that a centered version of the Rössler system—which is more elegant than the original one—is used is irrelevant since its parameter values can be expressed in terms of the parameter values of the original Rössler system.[87] Thus, just like the Lorenz system, the Rössler system is sufficiently rich in dynamic complexity to still trigger new studies.

Another important system among the benchmark systems is the four-dimensional hyperchaotic Rössler system (1489 quotations).[88] It is a direct extension of the three-dimensional Rössler system but producing dynamics characterized by two positive Lyapunov exponents. Its four-dimensional nature induces an insurmountable difficulty to characterize it because topological techniques based on the knot theory as branched manifolds can no longer be used. Alternative techniques must be developed, for instance, by using homology groups.[89] In the present Focus Issue, the parameter space is investigated by Stankevich *et al.*,[59] and a transition to hyperchaos is found through toroidal chaos: notice that toroidal chaos is here discovered in this hyperchaotic Rössler system.

An additional system, perhaps not (yet?) so well-known (116 quotations) was proposed by Thomas.[90] It is very simple, with three ordinary differential equations coupled in a cyclic way. The great interest of this system is, when the single parameter value b is varied, to progressively switch from a dissipative to a conservative dynamics ($b = 0$), the so-called *labyrinth walk*. The nature of the conservative dynamics is an open question since it is not bounded. This dynamics is indeed puzzling because it is (apparently?) similar to a random walk.[60,91] In this Focus Issue, it is shown that constructing a network of such systems can produce complex dynamics as chimera states.[60] Many interesting features associated with this simple system are still to be discovered. Since the 1980s, there is a particular type of chaos, which was sporadically investigated: toroidal chaos. Here, we retain as a definition for toroidal chaos, a behavior, which is contained between two concentric tori. The first picture of toroidal chaos was in fact plotted by Ueda[43] (see Fig. 1 from the first paper of this Focus Issue) by using a driven van der Pol equation.[80] A route to toroidal chaos was proposed by Curry and Yorke with the help of a simple two-dimensional map.[92] Shaw proposed a modified driven van der Pol equation for beautiful bi-folded toroidal chaos.[41] In his book written in the early 1980s (but only published in 2020), Rössler proposed a few other examples.[93] Toroidal chaos exists in three-dimensional state space.[94–97] In such a space, characterizing chaotic attractors, when produced by sufficiently dissipative systems, can be performed by using branched manifold (template)[79,98] but was never achieved for toroidal chaos. This breakthrough is now completed in this Focus Issue by "opening" the torus using an allowed slit in one of its branches: a three-branch manifold is thus proposed for a unimodal—with a single folding—toroidal chaos.[61]

III. CHEMICAL CHAOS

Chemical reaction systems may be simulated using mass-action kinetics. Since even relatively simple chemical reactions are nonlinear, then, on reflection, it may come as no surprise that chaos may exist in such systems when they are open to the surroundings. In fact, a pioneering paper by Rössler in 1976 directly demonstrates that simple (bio)chemical reaction systems may show chaotic dynamics.[8] Shortly thereafter, chaos was found experimentally in two (bio)chemical reactions systems,[99,100] and later on in multiple other chemical reactions.[101,102] An article in the Focus Issue discusses chaos and other complex dynamics in the peroxidase–oxidase (PO) reaction,[62] which is the first experimental (bio)chemical reaction system to display chaotic dynamics. The PO reaction involves two substrates, which are both continuously supplied to the reaction mixture containing a single enzyme (peroxidase). In the paper, a new realistic model is presented, which is capable of simulating most of the experimental behaviors such as multiple different bifurcation scenarios and quasiperiodicity, chaos, and hyperchaos.

An important description of chemical reaction networks involves thermodynamics. Non-equilibrium chemical systems are characterized by entropy production. A paper in the Focus Issue investigates entropy production in chemical reaction networks with chaotic behavior but produced by the underlying stochastic process in the large volume limit.[63] It is concluded that the methods from stochastic thermodynamics may be used also on chemical reaction networks that follow mass-action reaction laws and display behaviors running from stationary states to limit cycle oscillations and chaos. The reaction networks are decomposed into individual cycles and the entropy production of each cycle is calculated. Interestingly, some cycles may show negative contributions to entropy production. This is consistent with the second law of thermodynamics since these negative contributions are balanced by positive contributions from other cycles such that the overall entropy production is positive.

IV. OBSERVABILITY AND CONTROLLABILITY

One of the main contributions to nonlinear dynamics is the Takens theorem.[103] Roughly, it states that the state portrait is reconstructed with a diffeomorphical equivalence from a scalar time series $\{s\}$ if the reconstructed space is spanned by a least $2d_{\rm H}+1$ (delay or derivative) coordinates, where $d_{\rm H}$ is ideally the Hausdorff dimension (well approximated by a box-counting dimension of the attractor). One of the conditions of the applicability of the Takens theorem is that the measurement function $h:\mathbb{R}^d(\boldsymbol{x}) \to \mathbb{R}^{d_{\rm R}}(\boldsymbol{X})$ is generic, a property that is not possible to check in many practical situations. Moreover, this theorem does not prevent to having a diffeomorphical equivalence between the attractor embedded within the original state space and the attractor embedded within the space reconstructed with less than $2d_{\rm H}+1$ coordinates. Indeed, there are many cases for which the optimal diffeomorphical equivalence is obtained with a d-dimensional reconstructed space. This is obtained when the Jacobian matrix of the coordinate transformation $\Phi:\mathbb{R}^d(\boldsymbol{x}) \to \mathbb{R}^{d_{\rm R}}(\boldsymbol{x})$ is never singular.[104] This Jacobian matrix is equivalent to the observability matrix[105] used in control theory when $d_{\rm R}=d$.[104] Thus, when the measured variable provides full observability of the original state space, there is a diffeomorphism between the original state space $\mathbb{R}^d(\boldsymbol{x})$ and the reconstructed one $\mathbb{R}^{d_{\rm R}}(\boldsymbol{X})$. This is now quite matured topics, even for networks,[106–108] at least when the governing equations are known. It remains a challenge to assess the observability when these equations are not known. This is addressed in this Focus Issue by using delay differential analysis, that is, by using a roughly approximated model for one-step-ahead prediction: the error appears to be a good marker for the observability provided by the measurements.[64] It complements the two existing techniques.[109,110]

Controllability is the duality of observability, and consequently, controllability can be assessed by using an approach, which is the duality of those used for controllability.[105,111] In this Focus Issue, a four-step feedback procedure is used to control logistic map: it is thus shown that chaos can be used to control chaos toward periodic orbit.[65] An application to a model for traffic control is thus developed. A growing interest is devoted to the control of networks,[112–114] although most of the contributions are devoted to their controllability.[115,116] An optimal strategy would be to perturb a single node or a small set of them which can affect all the other nodes, ideally controlling the entire network to the desired behavior but this remains a great challenge. In this Focus Issue, a network of Rössler systems with a time-delay coupling scheme is controlled by using the pinning control. This is a feedback control strategy for synchronizing complex dynamical networks by acting on a virtual leader (the pinner), which is added to the network for driving the network along a desired trajectory.[66] Such a pinner directly exerts a control action only onto a small fraction of the nodes. This is based on a pinning error vector whose ith component is given by the difference between the output of the pinner and the output of the ith node. The condition for the network synchronization is established. In this Focus Issue, the synchronization of two delayed memristive neurons with external unknown disturbance via an observer-based active control is realized.[67] To reach synchronization, a disturbance-observer is designed to approximate the external unknown disturbance in the response system.

Sometimes, as in secure communication problems, it is desirable to develop some systems which are very difficult to synchronize.[117–119] In this Focus Issue, a bandwidth-enhanced secure communication system with time-delay signature concealment is proposed and analyzed by numerical simulation.[68] The system is based on two mutually coupled electro-optic phase feedback loops driven by a common chaotic source, thus increasing the complexity of the chaotic carrier. By insertion of an electro-optic feedback loop, the synchronization performance is improved to be a good candidate for applications in secure communication systems.

V. NETWORKS

Our world is extensively made of networks whose nodes can be neurons (brain), subjects (social networks, ecology), devices (power grids), and elements (chemical reactions and biological systems). In networks, one of the challenging problems is to understand the route to synchronization.[120,121] One of the possible puzzling observed behaviors is a chimera.[122–125] For investigating all these features, networks of Rössler systems are very often used. In this Focus Issue, a ring network made of 200 of them is investigated: the fractal structure of the basins of attraction for chimera states is determined.[69] The route to synchronization in networks is definitely not trivial.

Legged robots are normally composed of rigid bodies linked by simple kinematic connections; they have higher mobility than wheeled robots. For such robots, it is necessary to design walking patterns by using low-dimensional models.[126–128] In this Focus Issue, gaits are produced by using a small network of Rössler systems coupled by delayed schemes.[70] A real-time robot is navigating an arena using a brain-machine interface in a paranomic view experiment, which was anticipated by Rössler in the early 1980s.[129]

The human cerebral cortex is an amazingly complex network that can be described by using brain parcellation, that is, by the determination of a partition into areas or networks. This is a required step for understanding brain organization[130] since one of the main characteristics of the brain is functional differentiation, meaning that each area can be associated with a specific function. This differentiation can emerge from various constraints during the developmental process. In this Focus Issue, an evolutionary neural network based on an extended reservoir computer is developed.[71] It is thus shown that neurons differentiation is the result of some specific input stimulations. In these simulations, the network topology changed from a random network to a feedforward network including feedback connections as observed in animal and human cortical network structures.

VI. APPLICATIONS TO REAL DATA

When real data are concerned, some additional points deserve careful attention. The numerical techniques used to investigate the measurements should be able to tackle both chaotic and stochastic dynamics since none of these two situations can be excluded beforehand. The analysis should be sufficiently robust against noise contamination always present in the real world. Finally, as already mentioned, chaotic dynamics require determinism and high sensitivity to initial conditions. Consequently, to claim chaos, the techniques should have the ability to capture the deterministic part of the studied system, at least when its dynamics is sufficiently decoupled to the surrounding environment. If reliable techniques have been introduced to characterize the geometry of the behavior,[131–134] its sensitivity to initial conditions,[135,136] and its topological properties,[137–139] the detection of determinism is an absolute necessity when dealing with observational data since the equations that govern the dynamics are generally unknown. Until now, only two tests can permit the detection of determinism, that is, the direct test proposed by Kaplan and Glass[140] and the global modeling technique.[141] The latter has been proved to be quite robust against spurious detection.[142] It aims to obtain delayed discrete equations[143] or ordinary differential equations[144–146] directly from measured time series. The global modeling technique is presently the only approach that offers the potential to the governing equations in an algebraic formulation, contrary to the machine learning approach.[147]

Recent developments have shown that global modeling can be used to retrieve the system original equations, to detect weak and strong couplings, and to identify dynamical behaviors. Applying the approach to biological and environmental data was even more challenging since such systems are generally high-dimensional and may exhibit numerous complex couplings. Despite these limitations, during the last 15 years, the approach was applied to numerous realms including ecology, epidemiology, eco-epidemiology, medicine, real-time pandemic, cereal crops cycles, cycles of snow surface area, hydrogeology, microclimatology, which reveals the promising potential of the approach and the liveliness and the dynamism of chaos in science today. In this Focus Issue, a recent version of the global modeling technique extended to multivariate time series[142] is used to study the dynamical coupling between earthworms and soil water content based on a small set of *in situ* data gathered under real tropical conditions. The analysis returns a six-dimensional model coupling two subsystems: they are coupled through a strong and nontrivial coupling from soil water content dynamics to earthworms dynamics and a weak coupling in the reverse direction.[72] The present approach is in very good agreement with the deductive biology advocated by Rössler in the 1970s:[148] it is a top-down approach that fosters the global behavior in contrast to the more traditional procedure starting from elementary processes to build the global behavior. The analysis also reveals that, when isolated, the two subsystems are chaotic. Surprisingly, earthworm dynamics converge to a weakly dissipative chaotic attractor of toroidal structure with a Poincaré section, which looks like the two-dimensional map published by Mira and his colleagues.[56]

A second study of data from the real world is devoted to epileptic seizures based on invasive measurements [intracranial electroencephalograms (iEEGs)] using depth electrodes. Before a surgery to remove the seizure onset zone, a visual inspection of iEEG allows us to determine which brain area is involved in the transition from the background state (assumed to be pseudo-stationary) to the ictal state. It is thus shown that the seizure onset may result from network interaction emerging at different (nearby or distant) locations.[149] In this Focus Issue, the approach is to project the brain dynamics into a low-dimensional space in which ictal and interictal episodes are well discriminated and, consequently, allowing us to detect the seizure onset.[73] This is based on the construction of a weighted network from each iEEG leading to the localization of the seizure onset.

Hopefully, this Focus Issue would provide an overview of the present liveliness of chaos theory, the relevance of its historical contributions, its applications, and its remaining challenges.

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