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Anomalous dimensions of conformal baryons

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We determine the anomalous dimensions of baryon operators for the three-color theory as functions of the number of massless flavors within the conformal window to the maximum known order in perturbation theory. We show that the anomalous dimension of the baryon is controllably small, within the δ expansion, for a wide range of number of flavors. We also find that this is always smaller than the anomalous dimension of the fermion mass operator. These findings challenge the partial compositeness paradigm.

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I. INTRODUCTION

Determining the phase structure of gauge theories of fundamental interactions is crucial when selecting relevant extensions of the standard model [1]. Of particular significance are the critical exponents of composite conformal operators such as the fermion mass and the baryon anomalous dimensions in the conformal window. Large anomalous dimensions of these operators are often invoked when constructing composite extensions of the standard model, such as models of walking dynamics [2] and partial compositeness [3].

To gain a quantitative analytic understanding of these important quantities perturbation theory has been proven useful for the anomalous dimension of the fermion bilinear [4–6]. Here we determine the conformal baryon anomalous dimension for the SU(3) gauge theory when varying the number of massless flavors within the conformal window to the maximum known order in perturbation theory. These operators play an important role in models of partial compositeness.

II. BARYON ANOMALOUS DIMENSION

The perturbative expressions of the beta function and the fermion mass anomalous dimension for a generic gauge theory with only fermionic matter in the $\overline{\rm MS}$ scheme to four loops were derived in Refs. [7,8] and assume the general form

$$\frac{da}{d\ln\mu^2} = \beta(a) = -\beta_0 a^2 - \beta_1 a^3 - \beta_2 a^4 - \beta_3 a^5 + O(a^6), \tag{1}$$

$$-\frac{d\ln m}{d\ln\mu^2} = \frac{\gamma_m(a)}{2} = \gamma_0 a + \gamma_1 a^2 + \gamma_2 a^3 + \gamma_3 a^4 + O(a^5), \tag{2}$$

where $m = m(\mu)$ is the renormalized (running) fermion mass, μ is the renormalization scale in the $\overline{\rm MS}$ scheme and $a = \alpha/4\pi = g^2/16\pi^2$ where $g = g(\mu^2)$ is the renormalized coupling constant of the theory. The explicit four-loop values of the coefficients for generic fermion representations were generalized from the original references in Ref. [9] while the explicit formulas for the coefficients are shown in the appendix of Ref. [6].

We consider an SU(3) gauge theory with N_f fundamental massless Dirac flavors. Because of the triality condition, as for QCD, the baryon operator is constructed out of three quarks. In general for the renormalization of composite operators one needs to consider operator mixing. This results in a matrix of renormalization constants and a related matrix of anomalous dimensions. The renormalized baryon operators are given in terms of the bare ones $\mathcal{B}_j^b$ according to the standard relation

$$\mathcal{B}_i = Z_{ij}\mathcal{B}_j^b, \tag{3}$$

where the indices i, j range over all operators that mix. The matrix of anomalous dimensions is given by

$$\gamma_{ij}^{\mathcal{B}}(a) = \mu\frac{d}{d\mu}\ln Z_{ij}. \tag{4}$$

In Refs. [10,11] the anomalous dimensions of the proton-like baryon were derived considering the mixing of two three-quark operators. Using the same notation as in Ref. [11], the resulting three-loop eigen-anomalous dimensions are

$$\begin{aligned}\gamma_+^{\mathcal{B}}(a) &= 2a + [2n_f + 21]\frac{a^2}{9} - [260n_f^2 + [4320\zeta(3) \\ &\quad - 4740]n_f + 2592\zeta(3) + 22563]\frac{a^3}{162} + O(a^4), \\ \gamma_-^{\mathcal{B}}(a) &= 2a + [2n_f + 81]\frac{a^2}{9} - [260n_f^2 + [4320\zeta(3) \\ &\quad - 4572]n_f + 24399]\frac{a^3}{162} + O(a^4).\end{aligned} \tag{5}$$

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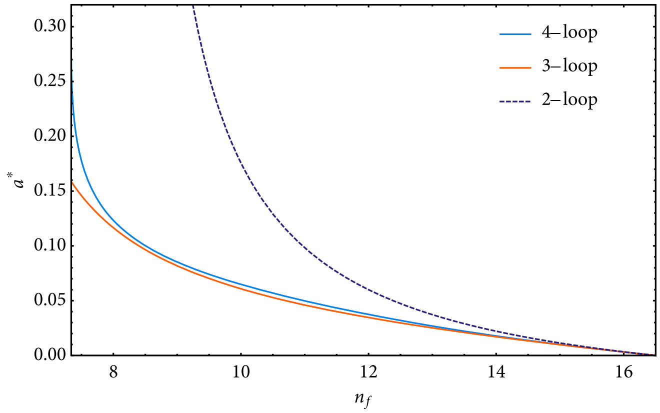

FIG. 1. Value of the fixed-point coupling a^* at four-, three- and two-loop order as a function of the number of flavors n_f. The four-loop fixed-point coupling reaches a finite value at the lower end of the four-loop conformal window at $n_f \approx 7.3$, while both the three- and two-loop fixed-point couplings diverge there.

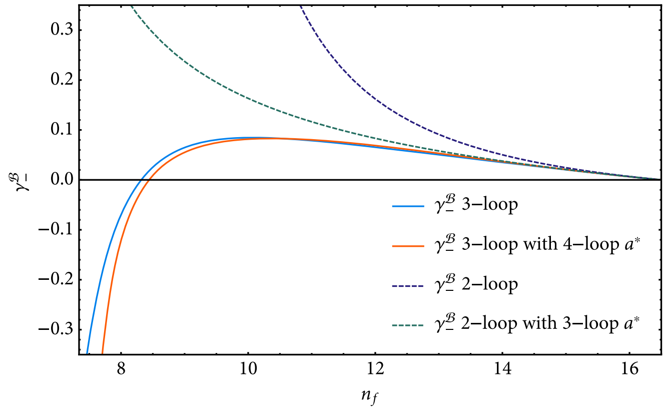

FIG. 3. To estimate the convergence of the perturbative series for $\gamma_-^{\mathcal{B}}$, we evaluate its two- and three-loop expression using the two-, three- and four-loop values of the fixed-point value a^*.

In the conformal window the anomalous dimensions at the infrared (IR) fixed point are physical quantities, which do not depend on the scheme. At fixed loop order, the scheme independence has been studied in Ref. [12].

Using the known expression for the β function (1), one can determine the value of the IR fixed-point coupling a^* at two, three and four loops, which we show in Fig. 1 [5,6]. From the figure it is clear that the two-loop result is only accurate very close to the upper limit of the conformal window, i.e. $n_f \geq 14$, while the agreement between the three- and four-loop results suggests that the fixed-point coupling so determined is reliable for a much wider range of number of flavors, i.e. $n_f \gtrsim 8$.

Using the value of the fixed-point coupling a^*, we can now determine the anomalous dimensions of baryons $\gamma_\pm^{\mathcal{B}}$. In Fig. 2 we compare the three-loop result for $\gamma_\pm^{\mathcal{B}}$ with the three- and four-loop results for the mass anomalous dimension γ_m at the IR fixed point. The range of validity of perturbation theory for γ_m can be estimated by comparing the three- and four-loop results, which suggests $n_f \geq 12$ [5,6]. In fact, the anomalous dimension of the fermion mass term γ_m has also been investigated on the lattice by several groups and for the purpose of this work we refer to the comprehensive review of these results presented in Ref. [13]. Although no universal consensus exists yet on whether or not the SU(3) theory with $n_f = 12$ has an IR fixed point [14], the lattice results measuring the anomalous mass dimension are compatible with the four-loop results. This is in agreement with the expectation that perturbation theory is valid until $n_f = 12$.

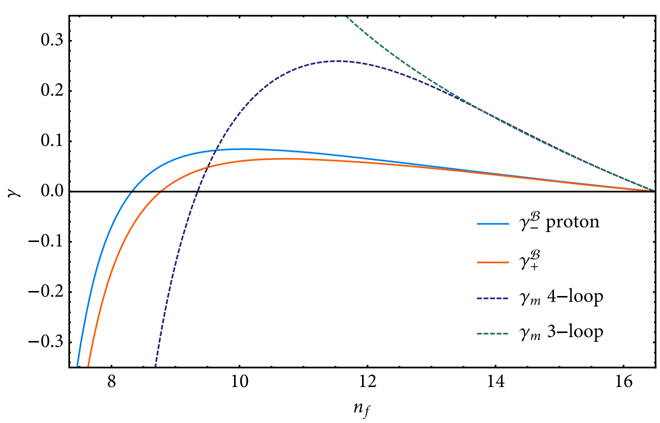

FIG. 2. Eigen-anomalous dimensions $\gamma_\pm^{\mathcal{B}}$ for three-quark composite operators compared to the anomalous dimension of the mass γ_m at three and four loops as a function of n_f. The anomalous dimension γ_- corresponds to the proton in QCD.

In this range, the baryon anomalous dimension is very small ≤ 0.07, about a factor of 4 smaller than the mass anomalous dimension. It is also apparent that the two eigen-anomalous dimensions of the conformal baryon are, perhaps not surprisingly, very close to each other. To investigate the stability of the perturbative result for $\gamma_-^{\mathcal{B}}$, we compare in Fig. 3 various perturbative estimates. We use the three- and two-loop expressions for $\gamma_-^{\mathcal{B}}$ and for each we insert the value of the IR fixed-point coupling at the same order in perturbation theory or one order higher. For $n_f \geq 12$, we observe a good agreement among three of these four estimates, corresponding to the three-loop $\gamma_-^{\mathcal{B}}$ computed at the three- and four-loop values of a^* and the two-loop $\gamma_-^{\mathcal{B}}$ computed at the three-loop value of a^*. The two-loop estimate of the baryon anomalous dimension shows significant deviations with respect to the other three estimates.This is consistent with the expectation that the two-loop computation is reliable only very close to the loss of asymptotic freedom. We report in Table I the perturbative values of the anomalous dimensions determined in this work as a function of the number of flavors.

The physical dimension of the conformal baryon is given by

TABLE I. Anomalous dimension of the mass γ_m and of three-quark operators $\gamma_{\pm}^{\mathcal{B}}$ as a function of the number of fundamental Dirac fermions n_f to three-loop order.

n_f	10	11	12	13	14	15	16
γ_m	0.1559	0.2497	0.2533	0.2098	0.1474	0.0836	0.0259
$\gamma_-^{\mathcal{B}}$	0.0816	0.0802	0.0688	0.0531	0.0365	0.0207	0.0064
$\gamma_+^{\mathcal{B}}$	0.0542	0.0641	0.0597	0.0484	0.0344	0.0200	0.0064

$$\mathcal{D}[\mathcal{B}_\pm] = \frac{9}{2} - \gamma_\pm^{\mathcal{B}}, \tag{6}$$

and therefore it will remain very close to the engineering dimension 9/2 for $n_f \geq 12$.

III. APPROACHING THE LOWER BOUNDARY OF THE CONFORMAL WINDOW

It is also possible to expand the anomalous dimensions at the fixed point in terms of the physical parameter $\delta = n_f^{\mathrm{AF}} - n_f$ where $n_f^{\mathrm{AF}} = 16.5$ is the value for which asymptotic freedom is lost [4,15,16]. It is worth summarizing the properties of the δ expansion [4,16]: this is the expansion of a physical quantity, here the anomalous dimensions at the IR fixed point, in terms of the physical parameter δ around the point where asymptotic freedom is lost; the nth coefficient of the series can be computed from the perturbative expansion at $n+1$-loop order, but it is exact to all higher orders. Furthermore, another virtue of this expansion is that it offers a more immediate evidence of scheme independence [4,16] and, as tested in Ref. [16], it converges rapidly in the entire conformal window, to the exact results in supersymmetry. We observe a similar convergence pattern for the coefficients of the series in δ also for the conformal baryon anomalous dimensions. The approximated numerical expressions, with rounded coefficients, are

$$\gamma_-^{\mathcal{B}} = 0.012461\delta + 0.000845\delta^2 + 0.000042\delta^3 + \mathcal{O}(\delta^4),$$
$$\gamma_+^{\mathcal{B}} = 0.012461\delta + 0.000586\delta^2 + 0.000029\delta^3 + \mathcal{O}(\delta^4). \tag{7}$$

We plot in Fig. 4 the conformal baryon anomalous dimensions in the δ expansion and compare them with the results presented above.

From the analysis above clearly the anomalous dimensions are largest for lower numbers of flavors. It is therefore relevant to know the location of the lower end of the conformal window. This is being investigated by several lattice groups [17–26]. However, at present there is no general consensus on the location of this lower boundary: even if most studies indicate a number of flavors around eight, there are claims that it could be as large as 12 flavors [19].

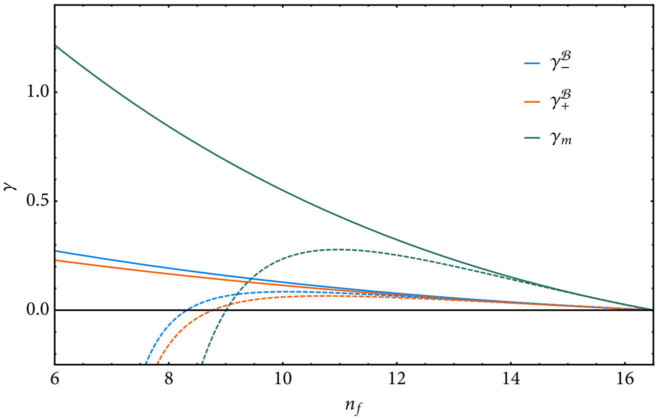

FIG. 4. Anomalous dimensions of the quark mass and of the three-quark operators $\gamma_\pm^{\mathcal{B}}$ as a function of the number of fundamental Dirac fermions n_f in the physical $\delta = n_f^{\mathrm{AF}} - n_f$ expansion (solid lines). We compare it with the perturbative results (dashed lines) at three loops.

If the conformal window extends down to six flavors we have argued that the δ expansion converges down to this value with the baryon anomalous dimensions never exceeding the value of about 0.3.

Another possible estimate of the lower boundary of the conformal window can be made by observing when the anomalous dimension of the mass operator approaches unity. Within the δ expansion, at order δ^3, this occurs around seven flavors for which $\gamma_m \simeq 1.02$ [16]. We report in Table II the values of the anomalous dimensions considered in this work computed in the δ expansion.

IV. PHENOMENOLOGICAL IMPACT

Models of partial compositeness typically require the presence of baryonic operators with large anomalous dimensions such that

TABLE II. Anomalous dimension of the mass γ_m and of three-quark operators $\gamma_\pm^{\mathcal{B}}$ as a function of the number of fundamental Dirac fermions n_f in the δ expansion.

n_f	γ_m	$\gamma_-^{\mathcal{B}}$	$\gamma_+^{\mathcal{B}}$
6	1.2160	0.2725	0.2295
7	1.0190	0.2305	0.1965
8	0.8435	0.1927	0.1663
9	0.6874	0.1586	0.1388
10	0.5495	0.1282	0.1138
11	0.4284	0.1011	0.0912
12	0.3227	0.0770	0.0706
13	0.2311	0.0558	0.0521
14	0.1520	0.0371	0.0353
15	0.0841	0.0207	0.0201
16	0.0259	0.0064	0.0064

$$\frac{3}{2} \leq \mathcal{D}[\mathcal{B}] \leq \frac{5}{2}. \tag{8}$$

This implies, for models in which the baryonic operators are composites of three fermions [3,27–30], that the anomalous dimension should be

$$2 \leq \gamma^{\mathcal{B}} \leq 3. \tag{9}$$

Our results indicate that such large anomalous dimensions can hardly occur in the minimal template of an SU(3) model with n_f fundamental fermions.

Furthermore lattice computations for the anomalous dimension of the mass [13], find no solid evidence of large anomalous mass dimensions suggesting that these might not be generated at the lower boundary of the conformal window even for fermions in higher representations.

Partial compositeness also requires four-fermion operators to be less relevant with respect to baryonic operators. By estimating the anomalous dimension of four-fermion operators $\mathcal{D}[(\bar{\psi}\psi)^2] \approx 6 - 2\gamma_m$ [31]. Therefore if $\gamma^{\mathcal{B}} \leq \gamma_m$, as in the present case, four-fermion operators will always be more relevant than baryonic ones. This estimate becomes precise in the weakly coupled limit.

V. CONCLUSIONS

We determined the anomalous dimension of baryonic operators for the SU(3) gauge theory with n_f fundamental fermions inside the conformal window at the maximum known order in perturbation theory. Within the conformal window at $n_f \geq 12$ our results indicate that anomalous dimensions for baryons remain small $\gamma^{\mathcal{B}} \leq 0.07$, and substantially smaller, about a factor ≈ 4, than the mass anomalous dimension.

We have argued that the physical δ expansion gives a rapidly converging expansion, which allows one to obtain a reliable estimate for a number of flavors as low as $n_f = 6$. Also within the δ expansion, the anomalous dimensions of the baryons never exceed 0.3 for n_f as low as six.

Our results were obtained for the most minimal setup for partial compositeness. More involved constructions, requiring fermions in multiple representations, have been proposed. For example, we can consider composite baryons of an SU(4) gauge theory with five Majorana fermions in the two-index antisymmetric representation and three Dirac fundamental fermions [27]. In this model, the anomalous dimensions of composite baryon operators at one loop have recently been computed [32]. Comparing the one-loop coefficients of the baryon operators to the ones of the mass operators, we find that, in all cases $\gamma^{\mathcal{B}} \leq \gamma_m$.

These results challenge minimal models of partial compositeness featuring baryon operators built out of three fermions.

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