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A new constant amplitude equivalent principal component analysis-based method for non-proportionality quantification of variable amplitude loaded welded joints in large-scale structures

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A B S T R A C T

The levels of non-proportionality in fatigue stress-time series are difficult to determine. Approaches from literature are not easy to interpret and predicts different levels for variable and constant amplitude stresses. In this paper, an extension to the PCA-based method is proposed in which variable amplitude stresses are transformed into equivalent constant amplitude stresses by scaling the stress reversal points. This ensures robust and easily interpretable results. The proposed method is compared with other approaches given in literature using published experiments and a designed offshore jacket structure. The results show that the proposed method can predict the levels of non-proportionality accurately.

1. Introduction

Fatigue is a failure mechanism that is relevant for welded joints exposed to cyclic loading. Many approaches for determining the fatigue strength or fatigue life of the joints have been proposed in the literature. In the governing guidelines for civil engineering and offshore structures such as the EN 1993-1-3 [1], the recommended practice from DNV [2] and the recommendations from International Institute of Welding (IIW) [3], detailed descriptions of the proposed fatigue methods and fatigue criteria are provided. Common for both the IIW recommendations and the DNV recommended practice is that when non-proportional stress states are observed, increased fatigue damages should be taken into account. This is taken into account by “penalty factors” which either increase the considered fatigue stresses [2] or decrease the allowed equivalent damage [3]. Different definitions for non-proportionality can be found in literature, however, the most common definition is based on the principal stress direction [4,5]. If the principal stress directions rotate during a loading, non-proportional stress states are present. Non-proportional stress states result in lower fatigue lives as compared to proportional stress states for various welded joints see for example the experiments performed by Yousefi et al. [6] Somsino [7] and Razmjoo [8] or the multiaxial fatigue evaluations performed by Pedersen [9].

In the recommendations by both IIW and DNV, no approaches for determining the level of non-proportionality in the stress-time series are provided. This means that as soon as non-proportionality is observed, the maximum penalty factor should be used in the fatigue calculations according to the guidelines. Non-proportional stress states, resulting in an change of principal stress direction, can be caused by several stress conditions. In general, two main types of non-proportional stress states can be defined, namely non-proportional in-phase and out-of-phase stress states. In Fig. 1, the different causes for principal stress rotations are summarized in accordance with [10] and examples of corresponding stress-time histories are shown. As observed from Fig. 1, stresses that are either having different forms or different amplitude-to-mean stress ratios will cause rotating principal stresses. The mean stress effect is, however, often considered by the guidelines and recommendations for welded structures by providing S–N curves generated under high mean stress levels [1–3]. Furthermore, it is well known that in welded joints in large-scale civil, offshore and railway structures large residual stresses may be expected, which causes the maximum stress to be close to the yield limit of the material [2,3,11].

Some efforts have been made to quantify the “level” of non-proportionality. These quantification approaches can be divided into two main methods, namely methods which determine the level of non-proportionality for a certain plane or methods which determine a level of non-proportionality which is independent of coordinate system. In accordance with [12], the first type of quantifier can be denoted as a critical plane type quantifier and the coordinate system independent...
of non-proportionality and only phase shifts does. Furthermore, the stress levels of the stress-time histories does not influence the level of non-proportionality in accordance with [10]. Fig. 1. Causes for non-proportionality in accordance with [10].

The approach is of the critical plane type, as only the plane normal to the surface and aligned with the weld toe is considered. The approach was found to accurately determine the level of non-proportionality for three constant amplitude (CA) fatigue test series, when assuming that 90 degrees phase shift results in the highest amount of non-proportionality. This approach makes it possible to automatically determine when a penalty factors for non-proportionality should be included in the fatigue criteria. As a result, more accurate fatigue life estimations can be predicted for non-proportionally loaded welded joints. This approach was compared to the quantifiers by Gaier et al. [13] and Bolchoun et al. [12] and was seen to perform better. However, the PCA-based approach cannot directly be used for variable amplitude (VA) fatigue tests, as the approach will result in different quantification values for different stress states. Thus, in this paper the PCA-based approach [18] is further extended to capture the effect of phase shift in VA stress states and the approach is validated against VA fatigue experiments performed by Witt [19]. The extended method is based on transformation of the VA stress states into equivalent CA stress states. As all stresses are normalized to CA stresses, the resulting principal component analysis results in equally scaled circles, which in turn ensures that a phase shift of 90° returns a NP level of 1 using the CA equivalent PCA-based approach. The extended CA equivalent PCA-based approach is thus very robust in predicting the NP levels. Furthermore, the approach is simple to implement as PCA is a well-known method. The PCA-based approach is compared to the methods from Bolchoun et al. [12] and Gaier et al. [13].

Finally, the three approaches are used in an effort to determine the level of non-proportionality in an offshore jacket foundation for
wind turbine generators designed by an engineering company, *Ramboll* Group A/S. Offshore jacket foundations are exposed to variable wind and wave loads which can be out-of-phase, resulting in non-proportional stress states. The level of non-proportionality in jacket structures are of interest as non-proportionality needs to be included by additional penalty factors [3] leading to the need of increased fatigue resistances and subsequently higher thicknesses and production costs. Thus, the considered jacket structure is examined for the level of non-proportionality in the welded joints. The jacket structure is designed by *Ramboll* and corresponds to a real manufactured design. A number of joints in the jacket structure is investigated for various load directions. The results show that low levels of non-proportionality is observed using all considered approaches in the case of joints located in the middle of the jacket, whereas joints located closer to very stiff connections (such as the top platform or the soil), observes higher non-proportionality levels. The extended PCA-based approach predicts higher levels of non-proportionality as compared to the other considered approaches from literature.

2. Non-proportionality quantification methods proposed in literature

Different approaches have been proposed in the literature to determine the level of non-proportionality in stress-time signals. Some of the most well-known approaches are developed by Chu et al. [16] and modified by Gaier et al. [13], which is also used in the commercial software FEMFAT [20,21]. Bolchoun et al. [22] likewise developed an approach for non-proportionality quantification. The Gaier et al. and the Bolchoun et al. approaches can be classified as non-proportionality quantifiers of the integral type according to Bolchoun et al. [12], whereas the Chu et al. quantifier is of the critical plane type. The methods described in [13,16,22] are all designed for the general stress state, thus do not take into account the actual structure considered in the non-proportional estimation. This means that the effect of mean stresses and stress amplitudes are accounted for in the quantifiers, leading to different values depending on the stress signal. In [18] the authors proposed an approach using the principal component analysis, denoted the PCA-based quantifier. The method standardizes the input stress signals as described below, in order to make the quantification values independent of mean stresses and amplitudes. The standardization is made to account for large-scale structures where high residual stresses are often present, making the mean stress effect less relevant [2,3,18,23]. The result of this standardization of the stress signals is an approach that always results in a value of 1 in the case of a phase shift between the shear and normal stresses of 90°, for CA stress states.

Common for the approaches by Gaier et al. [13], Bolchoun et al. [22] and the PCA-based approach proposed by the authors [18] is that they all return different quantification levels caused by VA loading, which are not easily interpreted.

In [12,15] detailed reviews of some of the proposed approaches in literature can be found.

2.1. Original principal component analysis-based non-proportionality quantifier

In [18], a principal component analysis-based measure for non-proportionality was developed with focus on large-scale welded structures. The approach consists of calculating the principal components (PCs) [24–26] for the normal stress \( \sigma_x \) and shear stress \( \tau_{xy} \) time signals. The stress signals \( \sigma_x \) and \( \tau_{xy} \) are parallel to the weld and the corresponding shear stress, respectively. This means the quantifier is a so-called critical plane quantifier as only that single plane is considered. In the PCA-based quantifier, the parallel-to-weld stresses are neglected as they are not considered in the multiaxial fatigue criteria from the overall guidelines and recommendations such as the IIW [3] and the DNV [2]. The definition of normal, parallel and shear stresses are shown in Figure 2.

![Fig. 2. Example of structure with fillet weld and normal to weld stress \( \sigma_x \), parallel stresses \( \sigma_x \) and shear stresses \( \tau_{xy} \).](image)

By solving the eigenvalue problem defined in Eq. (1), the PCs are found as the corresponding eigenvalues \( \lambda \).

\[
(Cov - \mathbf{I}) \mathbf{V} = 0
\]  

\( (\mathbf{Cov} - \mathbf{I}) \mathbf{V} = 0 \)  

\( \mathbf{I} \) is the identity matrix and \( \mathbf{V} \) is the corresponding eigenvectors. \( \mathbf{Cov} \) is the covariance matrix of the parallel-to-weld stresses and shear stresses defined by Eqs. (2) and (3).

\[
\text{Cov}(\sigma_x, \tau_{xy}) = \frac{1}{n-1} \sum_{i=1}^{\infty} \left( \sigma_x - \bar{\sigma}_x \right) \left( \tau_{xy} - \bar{\tau}_{xy} \right)
\]  

\[
\text{Cov} = \begin{bmatrix}
\text{Cov}(\sigma_x, \sigma_x) & \text{Cov}(\sigma_x, \tau_{xy}) \\
\text{Cov}(\sigma_x, \tau_{xy}) & \text{Cov}(\tau_{xy}, \tau_{xy})
\end{bmatrix}
\]  

\( \text{Cov}(\sigma_x, \tau_{xy}) \) where \( \bar{\sigma}_x \) and \( \bar{\tau}_{xy} \) denote the normal and shear stress signals and \( \bar{\sigma}_x \) and \( \bar{\tau}_{xy} \) denotes the mean values of those signals. \( \text{Cov}(\sigma_x, \sigma_x) \) and \( \text{Cov}(\tau_{xy}, \tau_{xy}) \) can be found similarly.

The PCs \( (\lambda) \) describe how much information is present in each dimension of the data \( (\sigma_x \text{ and } \tau_{xy}) \). Thus, PCA corresponds (in the case of a plane stress state without parallel stresses), to plotting the normal and shear stresses in a \( \sigma_x - \tau_{xy} \) diagram and finding the axis showing the largest variation as exemplified in Fig. 3.

From the PCs, it is possible to determine the weight of each principal component \( W \) and the ratio between the smallest and largest weight provides a numerical measure for the level of non-proportionality \( N_{PCA} \), see Eq. (4).

\[
N_{PCA} = \min(W) / \max(W)
\]  

\( N_{PCA} = \min(W) / \max(W) \)

From Fig. 3 and Eq. (4), it can be observed that the method corresponds to assessing the axes of an ellipse. If the axes are equal in size, the ellipse will correspond to a circle and the considered signals will be 90 degrees phase shifted. Several examples of stress-time signals have been considered in [18]. The idea of using the axis of an ellipse (or ellipsoid) to describe the level of non-proportionality is not new, Chu et al. [16], Bruder et al. [27] and Gaier et al. [13] used similar approaches. However, in the proposed method based on PCA, the stress states are to be standardized in accordance to Eq. (5) before calculating the NP quantifier.

\[
Z_i = \frac{X_i - \bar{X}}{SD}
\]  

\( Z_i = \frac{X_i - \bar{X}}{SD} \)
where $Z_i$ is the standardized stress for the $i$th time-step, $X_i$ is the stress ($\sigma_i$ or $\tau_{xy}$) at time $i$, $X$ is the mean of the stress-time signal and $SD$ denotes the standard deviation of the stress-time signal. The standardization has two purposes. In the case of PCA, large signals tend to dominate the smaller signals. Thus, without standardization, a large normal stress signal would dominate a shear stress signal, not resulting in useful estimates. The second purpose, is to remove the effect of mean stresses and stress amplitudes in the quantification. This is done to account for the effects of high residual stresses in large welded structures [18].

As the PCA-based approach standardizes the signals, it means that small signals will appear to have a large weight in the PCA-estimation and thereby also on the non-proportionality factor. Thus, an additional check is made to ensure that small shear stresses are neglected in accordance to the recommendations by the IIW [3], where shear stress of less than 15% of the normal stresses can be neglected:

$$NP_{PCA} = \begin{cases} 0, & \text{if } \Delta \sigma_x > 0.15 > \Delta \tau_{xy} \\ \min(W)/\max(W), & \text{otherwise} \end{cases} \quad (6)$$

The check of shear stresses in Eq. (6) is based on the IIW guideline. The check can similarly be used to check if the normal stresses should be neglected if they are very small compared to the shear stresses.

The PCA-based approach is sensitive to the length of the time series considered. This is due to both the standardization of the signals in Eq. (5) and the principal component analysis itself. For very short time series where only a few stress cycles are considered, the PCA-based quantifier will not converge as even small variations in the number of stress cycles will result in changes in the PCA-based quantifier. In Fig. 4, an convergence study of the quantification value using the PCA-based approach is shown for a normal and shear stress-time series with a phase shift of 90°. As seen from the convergence study, the quantifier converges around 15 full stress cycles where only a 3% scatter is observed in the case of the constant amplitude stress signal with 90° phase shift.

2.2. Approaches by Bolchoun et al. and Gaier et al.

Different approaches have been proposed in literature for quantifying the level of non-proportionality. In this paper, the PCA-based approach is compared to the method proposed by Gaier et al. [13] and Bolchoun et al. [22]. The approaches has previously been compared to the PCA-based approach in [18] using CA stress-time histories and showed promising results.

The quantification approach proposed by Bolchoun et al. [22], consists of evaluating the correlation coefficients of the stress-time signals and integrating them over rotations on either the plane or the 3D stress state. The quantifier is expressed in Eq. (7):

$$NP_{Bolchoun} = 1 - M = 1 - \frac{1}{\pi} \int_0^\pi C^2(\sigma_x, \tau_{xy}) \, d\theta \quad (7)$$

where $C$ denotes the correlation coefficient between the normal stresses and shear stresses in accordance with Eq. (8), $\theta$ denotes the angular rotation of the considered plane around the surface-normal (z-axis in Fig. 2). Thus, the quantifier proposed by Bolchoun et al. is predicted as an average value over multiple planes and can therefore be considered an integral type quantifier.

$$C = \frac{\text{Cov}(\sigma_x, \tau_{xy})}{SD_{\sigma_x} SD_{\tau_{xy}}} \quad (8)$$

$SD_{\sigma_x}$ and $SD_{\tau_{xy}}$ denotes the standard deviation of the normal stress signal and shear stress signal, respectively. Bolchoun et al. also formulated the approach in the 3D stress case, however, in this paper only the 2D approach is used. The $NP_{Bolchoun}$ will result in a value of zero for proportional loading and a value of 1 in the case where the plane stress state is given in the following form:

$$\begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) \\ \tau_{xy}(t) & \sigma_y(t) \end{bmatrix} = a \begin{bmatrix} \cos(t) & \sin(t) \\ \sin(t) & -\cos(t) \end{bmatrix} \quad (9)$$

Bolchoun et al. [22] proposes that the approach is to be used with any shear based stress criteria, as it provides an integral measure of the distribution of the shear stress amplitudes over different planes. The approach was used on an example with variable amplitude loading and showed to get similar quantification values as in the case of constant amplitude loading [22]. The method, similarly to the PCA-based approach, only considers the normal and shear stresses of the signals and neglects the parallel stresses.

Gaier et al. [13] proposed an approach based on the method from Chu et al. [16] that is independent of the considered plane. The quantifier is given in Eq. (10) and depends on the principal moments of inertia $I$ found by the eigenvalue problem given in Eq. (11). The $I_i$ values are found using Eq. (12), where $S_1(t_i) = \sigma_x(t_i), S_2(t_i) = \sqrt{2} \tau_{xy}(t_i)$ and $S_3(t_i) = \sigma_y(t_i)$. $t_i$ is each time step in the discrete stress data and $N$ is the maximum number of time steps.

$$NP_{Gaier} = \sqrt{\frac{I_1}{I_2}} \quad (10)$$

$$\begin{vmatrix} I_1 - I_3 & -I_3 & -I_3 \\ -I_3 & I_2 - I_3 & -I_3 \\ -I_3 & -I_3 & I_3 - I_1 \end{vmatrix} = 0, \quad i = 1, 2, 3 \quad (11)$$
of the shear stress have been varied. The considered signals are given to a curve and shear stress sine curve, with a phase shift $\phi$. Furthermore, the amplitude of the normal stress and the mean stress levels, as the amplitudes do not influence the result.

In the original paper, the approach was only used to quantify the NP loading and $N_{NP}$ of coordinate system. The principal moments of inertia are used to produce a quantifier independent of coordinate system. However, the principal stress rotations are determined as the most reliable indicator of non-proportionality levels. Furthermore, as $\sigma_y$ in the example is assumed to be 0, this also influences the results.

2.3. Quantification of CA stress signals

The Gaier et al. [13], Bolchoun et al. [12] and the PCA-based approach [18] was compared using constant amplitude load signals in [18]. In that investigation it was found that only the PCA-based approach always converges towards 1 in the case of 90° phase shift whereas the other approaches are sensitive to mean stresses and stress levels on constant amplitude loading [13]. In the original paper, the approach was only used to quantify the NP levels on constant amplitude loading [13].

2.4. Quantification of VA stress signals

The Gaier et al. [13], Bolchoun et al. [22] and PCA-based [18] quantifiers have been calculated for variable amplitude stress signals in Fig. 6. The signals consists of a random normal stress signal and a random shear stress signal generated using uniformly distributed random variables between $-1$ and $1$ generated in MATLAB 2020a [28]. The PCA-based approach results in a value of 1 at 90° phase shift, whereas only the PCA-based quantifier converges between 0.45–0.55. As the random signals are generated using uniform random variables, the PCA-based approach results in a value of 1 at 90° phase shift, whereas $N_{NP}$ is not the case always, as described further below. The differences in the quantification values are caused by the definition of non-proportionality. The PCA-based approach and some extent the Bolchoun et al. approach, focuses on the phase shifts between signals, where the Gaier et al. approach calculates the level of NP which are independent of coordinate system. In the considered signal, because the shear stresses and normal stresses include different amplitudes and mean values, the principal stresses do rotate at 0° phase shift. Thus, the quantifiers have been evaluated for different phase shifts. The results are shown in Fig. 6. As observed from Fig. 6, the PCA-based quantifier $N_{NP}$, the Gaier et al. quantifier $N_{NP}$ and Bolchoun et al. quantifier $N_{NP}$ all start at NP levels higher than 0 at 0° phase shift. Only the PCA-based approach is capable of predicting a value of 1 at 90° phase shift, whereas $N_{NP}$ and $N_{NP}$ converges between 0.45–0.55. As the random signals are generated using uniform random variables, the PCA-based approach results in a value of 1 at 90° phase shift, however, this is not the case always, as described further below. The differences in the quantification values are caused by the definition of non-proportionality. The PCA-based approach and Bolchoun et al. approach, focuses on the phase shifts between signals, where the Gaier et al. approach calculates the level of NP which are independent of coordinate system. In the considered signal, because the shear stresses and normal stresses include different amplitudes and mean values, the principal stresses do rotate at 0° phase shift. The principal stress rotations are determined as the most reliable indicator for non-proportionality effects in the fatigue damage, the Gaier et al. approach may provide better results than the PCA-based approach and the method by Bolchoun et al. However, as seen from Fig. 6, nearly the same level of non-proportionality is determined for all phase shifts.
meaning that the Gaier et al. approach does not assess the signals differently.

From Fig. 6, it can be observed that the $NP_{PCA}$ is the only approach that in this case converges towards 1, however, as it does not predict a value of 0 at 0° phase shift, the PCA-based approach in its original form cannot be implemented directly on variable amplitude loading. The only approach which predicts a value of 0 at 0° phase shift and 1 at 90° phase shift, is the $NP_{CA,PCA}$ approach, which is further described below.

3. Extension of PCA-based approach for VA loading

The PCA-based approach has been extended to also provide robust results for variable amplitude loading. In the original formulation of the PCA-based approach by Larsen et al. [18], the mean stresses and stress amplitudes of the stress-time signals are neglected. In large-scale structures, high tensile residual stresses are expected near the welds. Thus, the effect of mean stresses is expected to not influence the level of non-proportionality in the quantifier. The same assumption is used for the extended PCA-based approach. In the proposed extension, the VA stress signals are transformed into equivalent CA signals before calculating the principal components. The transformation is performed by scaling each reversal point in the stress-time series to an arbitrarily chosen value and afterwards scaling the intermediate time steps accordingly. This ensures consistent and robust predictions of the NP levels considering phase shifts. Even though, standardization of the stress-time signals are performed in accordance with Eq. (5), the standardized signals will still be varying in amplitude. A simple example is shown in Fig. 7. In this example, the normal and shear stress-time histories provided in Fig. 7(a) is used to predict the NP level using the PCA-based approach. The signals have a stress ratio $R = 0$ and the normal stress is phase shifted with 90°.

Furthermore, the signals are equal in form, however, scaled differently as seen by the resulting mean stresses in Fig. 7(a). Performing PCA after standardization in accordance with Eq. (5) results in the standardized signals and principal components presented in Fig. 7(b). As observed from, Fig. 7(b), the stress-time signals results in circles with different diameters. The PCs are thus calculated to be oriented as shown in the figure and with different values. Performing the weighting and NP calculation in accordance with Eqs. (1) and (4), results in $NP_{PCA} = 0.51$. In the original definition of the PCA-based quantifier, a phase shift of 90° should result in a quantification value of 1. However, as observed from the simple example, this is not the case for VA loading. From Fig. 7, it can easily be observed that the reason for the lower NP quantification are the variable amplitudes, causing the $\sigma_x - \tau_{xy}$ path to be drawn with different radii which results in a averaged circumscribed ellipse (considering the PCA analogy), to resemble an ellipse more than a circle.

The main idea of the PCA-based approach is that a 90° phase shifted signals should result in a quantifier level of 1. To achieve this, an extension to the original PCA-based approach from [18] has been proposed. The solution consists of transforming the variable amplitude stress signals into equivalent constant amplitude signals. Transforming the VA signals into equivalent CA signals results in the circles which was shown in Fig. 7 to be drawn with equal radii and as a result return values which are not biased by the variable amplitude signals. An example is shown in Fig. 8, where the stress signals from Fig. 7(a), have been transformed into equivalent CA signals as shown in Fig. 8(a). As a result, the $\sigma - \tau$ path will result in perfect circles if the signals are phase shifted with 90° as shown in Fig. 8(b).

The original eigenvalue problem defined in Eq. (1), should thus be calculated using the equivalent constant amplitude covariance matrix $\text{Cov}_{CA}$:

$$\text{(Cov}_{CA} - \lambda_{CA} I) \text{V}_{CA} = 0$$

where subscript $CA$ represents the CA equivalent values. Thus, $\lambda_{CA}$ are the principal components for the equivalent CA signal and the equivalent CA covariance matrix $\text{Cov}_{CA}$ is defined by:

$$\text{Cov}_{CA} = \begin{bmatrix} \text{Cov}_{CA}(\sigma_{x,CA}, \sigma_{x,CA}) & \text{Cov}_{CA}(\sigma_{x,CA}, \tau_{xy,CA}) \\ \text{Cov}_{CA}(\sigma_{x,CA}, \tau_{xy,CA}) & \text{Cov}_{CA}(\tau_{xy,CA}, \tau_{xy,CA}) \end{bmatrix}$$

and

$$\text{Cov}_{CA}(\sigma_{x,CA}, \tau_{xy,CA}) = \frac{1}{n-1} \sum_{i=1}^{n} (\sigma_{x,CA} - \bar{\sigma}_{x,CA})(\tau_{xy,CA} - \bar{\tau}_{xy,CA})$$

where $\sigma_{x,CA}$ and $\tau_{xy,CA}$ denote the equivalent constant amplitude stress signals and $\bar{\sigma}_{x,CA}$ and $\bar{\tau}_{xy,CA}$ denotes the mean values of those signals. The transformation of the VA stress signals into equivalent CA signals can be performed by scaling each reversal point in the stress-time history to some arbitrarily chosen peak and valley points. The peak and valley points can be chosen arbitrarily due to the standardization, as described below. When the peak and valley points are scaled, the intermediate data points can be scaled equally afterwards. An example is shown in Fig. 9, where the arbitrarily chosen maximum and minimum stresses $P_{max}$ and $P_{min}$ are used to scale the CA stress signal. The method for scaling the stress signals consists of first finding the stress reversal points $\sigma_{x,rev}$ and corresponding indices in the discrete time history $id_{xrev}$. By pairing each set of reversal points, a linear scaling can be carried out between the reversal points based on the chosen stresses $P_{max}$ and $P_{min}$.

The algorithm to perform the scaling is shown in the flowchart in Fig. 10. The algorithm is presented in coding form, making it easy to implement using Python, MATLAB or similar. Note, (·) denotes that the specified index should be extracted from the considered vector. In essence, the code uses the discrete VA normal stress $\sigma_x$ or shear stress $\tau_{xy}$ signals as input and returns the equivalent CA stress signal $\sigma_{x,CA}$. The desired peak and valley magnitudes is set using $P_{max}$ and $P_{min}$. The
Fig. 8. Example of PCA-based approach used on a VA time signal converted into CA that is 90° phase shifted.

Fig. 9. Transformation of VA stress signal $\sigma_x(t)$ to CA equivalent stress signal $\sigma_{x,CA}$.

scaling of the VA signal is performed by finding the indices of the peak and valley points $i_{dx_{rev}}$ and scaling the values in-between based on the set maximum and minimum value. The algorithm is shown for the case, where the first reversal point in the considered stress-time series is higher than the second reversal point. This takes into account if the considered signal starts with a peak point. If the signal starts with a valley point, the next check (if $k$ is odd), should be changed to check whether $k$ is even instead as shown in Fig. 10. Due to its simplicity, this is only stated and not shown in the flowchart.

The resulting equivalent CA stress signals $\sigma_{x,CA}$ and $\tau_{xy,CA}$ can be used in the proposed extension to the PCA-based non-proportionality quantifier.

From the equivalent principal components, the constant amplitude equivalent PCA-quantifier can be found as:

$$NP_{CA,PCA} = \min \left( \frac{W_{CA}}{\max(W_{CA})} \right), \quad W_{CA} = \sum \lambda_{CA}$$

It should be noted, that the standardization using Eq. (5) should still be performed before calculating the equivalent covariance matrix in Eq. (17). By applying the standardization on the CA equivalent stress-time signal, the size of $P_{\max}$ and $P_{\min}$ is unimportant for the remaining calculations and can therefore be chosen completely arbitrarily. In the remaining part of this paper, $P_{\max}$ and $P_{\min}$ are chosen as the maximum and minimum stress occurring in the full considered time series.

By using the extended formulations given in Eqs. (15), (17) and (18), the corresponding PCA-plot looks similar to the one shown in Fig. 8. As observed from Fig. 8(a), the input signals is transformed into an equivalent CA signals, which results in the PCA-plot shown in Fig. 8(b). As observed from the figure, full circles are now formed with the exception of the start and end points that are not full circles.

This results in the $NP_{CA,PCA} = 0.92$, which corresponds nearly to full non-proportionality. Due to the start and stop of the signal, the value does not fully reach 1, but for long time series this will be negligible. Thus, the extended PCA-based quantifier is capable of predicting the same non-proportionality levels considering both CA and VA stress-time signals, which makes it a robust approach.
quantification. Thus, the mean levels of the input stress signals are neglected in the PCA-based approach. By transforming the VA stresses into equivalent stresses in the extended approach, the mean stress levels can still be neglected as in the original approach [18].

In Fig. 6, the extended CA based PCA-based approach $NP_{CA,PCA}$ is calculated for different phase shifts. As seen from the figure, the approach does now start at 0° where 0° phase shift occurs and end at 1, where 90° phase shift occurs.

The extended PCA-based quantification approach is based on the VA stress signals being transformed into CA equivalent stress signals. This ensures that robust quantification values are predicted even for VA stress signals. The non-proportionality quantification value can be used to evaluate, for example, the CV factor in the IJ criterion in a way that is dependent on the non-proportionality quantifier. This dependency is, however, not in the scope of this paper. The fatigue lives can then be calculated using the fitted CV factor and the original VA stress-time signals. This means that for the actual fatigue life prediction, the original stress states should be considered and not the CA equivalent stress-time signals which are only used for non-proportionality quantification.

The extended PCA-based approach, which considers CA equivalent stress-time signals is robust and determines similar quantification values at various phase shifts for VA stress states and CA stress states. Furthermore, the interpretation of the quantifier is easy as it is directly based on the phase shift between the considered stress signals. In Table 2, the variations between the quantification approaches considered in this paper are schematically shown. The approaches have been evaluated based on type (critical plane or integral type), mean stress effects (changes value depending on mean stress levels) and consideration of stress amplitude (changes value depending on stress amplitudes). Furthermore, the computational speed, ease of interpretation and if the approach can be used on VA loading are evaluated. The final three rankings are based on the authors’ experience with the considered approaches and may depend on the implementation. For the PCA-based approaches, the mean stress effect and stress amplitudes are considered, however, in a simplified manner. In the PCA-based approaches, the magnitude of the mean stresses are neglected as high residual stresses in the structure are typically assumed in the design phase. The stress amplitudes are considered explicitly by checking if the shear stresses are below 15% of the normal stress in accordance with Eq. (6). The approach by Bolchoun et al. is observed in Fig. 5 to predict different values when the stress signals include different stress amplitudes, however, the quantification values do not change when mean stresses are included. The Gaier et al. approach predicts different values depending on both mean stress and stress amplitudes. The computational speed is the lowest for the Bolchoun et al. approach as it requires evaluation of the stress states over multiple planes in order to predict an average value. The original PCA-based approach and the Gaier et al. approach are the fastest methods and the extended PCA-based approach is nearly as fast but requires additional transformation of the VA stress signals to CA stress signals before predicting the quantification value. In general, the PCA-based methods are the easiest to interpret as they are developed to predict specific values at different phase shifts. The other two approaches are more difficult to interpret. All approaches are observed to be able to predict quantification values for VA stress signals. However, only the extended CA equivalent PCA-based approach can be easily interpreted.

### Table 2
Comparison of quantification approaches.

<table>
<thead>
<tr>
<th>Method</th>
<th>Type</th>
<th>Includes mean stress effect</th>
<th>Account for stress amplitude</th>
<th>Computational speed</th>
<th>Interpretation</th>
<th>Use with VA loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original PCA approach</td>
<td>Critical plane</td>
<td>Simplified</td>
<td>Simplified</td>
<td>Fast</td>
<td>Easy</td>
<td>No</td>
</tr>
<tr>
<td>Extended PCA approach</td>
<td>Critical plane</td>
<td>Simplified</td>
<td>Simplified</td>
<td>Moderate</td>
<td>Easy</td>
<td>Yes</td>
</tr>
<tr>
<td>Bolchoun et al. approach</td>
<td>Integral</td>
<td>No</td>
<td>Yes</td>
<td>Slow</td>
<td>Moderate</td>
<td>Yes</td>
</tr>
<tr>
<td>Gaier et al. approach</td>
<td>Integral</td>
<td>Yes</td>
<td>Yes</td>
<td>Fast</td>
<td>Hard</td>
<td>x</td>
</tr>
</tbody>
</table>

*Not used on VA loading in original publication, however, VA stress series can be used as input.*

### Table 3
Test series with variable amplitude loading performed by Witt [19].

<table>
<thead>
<tr>
<th>Series</th>
<th>Load 1</th>
<th>Load 2</th>
<th>$R$</th>
<th>$f_s/f_a$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF1</td>
<td>Bending</td>
<td>–</td>
<td>–1</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>BF2</td>
<td>Torsion</td>
<td>–</td>
<td>–1</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>BF3</td>
<td>Bending</td>
<td>Torsion</td>
<td>–1</td>
<td>1</td>
<td>0°</td>
</tr>
<tr>
<td>BF4</td>
<td>Bending</td>
<td>Torsion</td>
<td>–1</td>
<td>1</td>
<td>90°</td>
</tr>
<tr>
<td>BF5</td>
<td>Bending</td>
<td>Torsion</td>
<td>–1</td>
<td>5/1</td>
<td>0°</td>
</tr>
<tr>
<td>BF7</td>
<td>Bending</td>
<td>–</td>
<td>0</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>BF9</td>
<td>Bending</td>
<td>Torsion</td>
<td>0</td>
<td>1</td>
<td>0°</td>
</tr>
<tr>
<td>BF10</td>
<td>Bending</td>
<td>Torsion</td>
<td>0</td>
<td>1</td>
<td>90°</td>
</tr>
</tbody>
</table>

4. Evaluation of experimental data

To experimentally examine the effect of non-proportionality in VA stress-time histories and to compare the PCA-based quantifier with the quantifiers from literature, the variable amplitude experiments performed by Witt [19] are examined in the following section. The constant amplitude experiments performed by Witt were examined by the authors in [18], where it was seen that the proportionality quantifiers, including the PCA-based quantifier lead to accurate quantification of the non-proportionality. The geometry of the experiments performed by Witt [19] is shown in Fig. 11. In Fig. 11(a), the 3D geometry of the tested specimens is shown and in Fig. 11(b) the overall dimensions are shown. The thickness of the pipe section was 8 mm and the attached plate had a thickness of 25 mm. A torsional load and a bending moment was applied independently of each other resulting in a setup where torsional loads (resulting mainly in shear loading) and bending loads (resulting mainly in bending stresses) could be varied to obtain non-proportional stress states.

Experiments were performed with both constant amplitude loading (CA) and variable amplitude loading (VA). For the VA experiments a total of 9 test series were performed. Three of these series were performed considering only a single load (thus uniaxially loaded) and the rest were performed under both bending and torsional loading (multiaxially loaded) [19]. The multiaxially loaded test series were varied considering the phase shift between the signals $\delta$ and the frequency ratio between the bending load $f_a$ and torsional load $f_s$ ($f_s/f_a$). Furthermore, variations of the R-ratio were considered. In Table 3, the experimental series are summarized including the parameter variations. Note, that BF6 is not included as this series consisted of uncorrelated combined loading, however, information about the correlation was not included in original literature [19], thus the stress-time series could not be replicated. Furthermore, the naming is consistent with the naming given in [19], which is the reason why BF8 is missing.

The variable amplitude loadings in [19] was generated using two load spectra, generated as Gaussian processes in accordance to [29]. The load spectra were generated with either $R = -1$ or $R = 0$. Both spectra includes approximately 50,000 load cycles. A section of the loads are shown in Fig. 12. For the multiaxial load cases, the same load spectra were used for both the bending and torsion loads. That means that, in the case of a fully proportional load, the signals from the bending and torsion loads were applied simultaneously.

It should be noted that all tests were performed at the same nominal normal stress to shear stress range ratio.

### Table 3
Test series with variable amplitude loading performed by Witt [19].

<table>
<thead>
<tr>
<th>Series</th>
<th>Load 1</th>
<th>Load 2</th>
<th>$R$</th>
<th>$f_s/f_a$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF1</td>
<td>Bending</td>
<td>–</td>
<td>–1</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>BF2</td>
<td>Torsion</td>
<td>–</td>
<td>–1</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>BF3</td>
<td>Bending</td>
<td>Torsion</td>
<td>–1</td>
<td>1</td>
<td>0°</td>
</tr>
<tr>
<td>BF4</td>
<td>Bending</td>
<td>Torsion</td>
<td>–1</td>
<td>1</td>
<td>90°</td>
</tr>
<tr>
<td>BF5</td>
<td>Bending</td>
<td>Torsion</td>
<td>–1</td>
<td>5/1</td>
<td>0°</td>
</tr>
<tr>
<td>BF7</td>
<td>Bending</td>
<td>–</td>
<td>0</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>BF9</td>
<td>Bending</td>
<td>Torsion</td>
<td>0</td>
<td>1</td>
<td>0°</td>
</tr>
<tr>
<td>BF10</td>
<td>Bending</td>
<td>Torsion</td>
<td>0</td>
<td>1</td>
<td>90°</td>
</tr>
</tbody>
</table>
4.1. Finite element model

In this paper, the investigations are carried out based on the notch stress approach [2,3,30] as this approach results in consistent fatigue relevant stress measures and was used in the previous work in [18]. Using the notch stress approach makes it possible compare the data to other experimental datasets as it is a well-known and accepted approach. Witt [19] provided the nominal stress ranges at the location of the weld toe. Using nominal stresses makes the fatigue assessment reliant on the S–N curve for that corresponding detail. Using the notch stress approach means that predefined S–N curves can be used, which are applicable over a large variety of details. The finite element model is shown in Fig. 13. In accordance with the notch stress approach, recommendations from the IIW [3] and DNV [2] an artificial rounding with \( r = 1 \text{ mm} \) is modelled at the weld toe. The mesh has been generated in accordance to the recommendations in [31] and a close-up of the notch mesh is also shown in Fig. 13.

The FE model has been fixed in all degrees of freedom in the bolt holes as shown in Fig. 13. This simulates the real boundary conditions very well. Furthermore, the loads are applied to the face of the end of the pipe, which corresponds to the load appliance in the real experiments.

As the specimens consists of a pipe welded to a base-plate, it is difficult to determine one critical FE node for all considered load cases. Thus, multiple nodes are examined in the notch of the FE model at the weld toe and the stress tensors are evaluated for each node. The considered nodes are located as shown in the cutout in Fig. 13 and corresponds the highest stressed zone observed in the FE model.

4.2. Input S–N curves

In [18], the uniaxial S–N curves that relates to the notch stresses for the CA uniaxial fatigue tests from [19] were determined. It is well-known that VA loading causes more damage than CA loading when damage accumulation is performed. This is also included in the recommendations by the IIW [3], which states that the specified damage sum \( D_{spec} \) varies between 0.2 and 0.5 in Eqs. (20) and (21). Thus, using the S–N curves determined for the CA fatigue tests in [18], would result in very non-conservative results. As this paper focuses on the effect of non-proportionality it has therefore been chosen to use the input S–N curves suggested in literature. In accordance with the IIW [3] and the DNV [2], the normal stress FAT-225 S–N curve is used. For shear stresses the FAT-160 curve as proposed by Sonsino [32] is used. These curves are recommended for the use with the notch stress approach. The details of the S–N curves are provided in Table 4, for reference. The cycles to failure at the knee-point is defined \( N_k \).

4.3. Fatigue criterion

In [18], the CA data from Witt was examined considering both the IIW criterion [3], the DNV criterion [2] and the well-known Findley method [33]. The results showed that the IIW criterion in combination...
with the NP quantifiers provided accurate fatigue life estimations. Thus, in this paper the IIW criterion is used for further analysis.

In the IIW recommendation [3], the multiaxial fatigue criterion known as the Gough–Pollard (GP) [34] is proposed. The GP equation for CA stresses can be expressed as:

\[
\left( \frac{\Delta \sigma}{\Delta \sigma_R} \right)^2 + \left( \frac{\Delta \tau}{\Delta \tau_R} \right)^2 \leq CV
\]

(19)

where \(\Delta \sigma\) and \(\Delta \tau\) are the normal stress and shear stress respectively. \(\Delta \sigma_R\) and \(\Delta \tau_R\) are the fatigue strengths of the normal stress and shear stress, respectively. The fatigue strengths are given at a certain number of life time cycles, thus in case of fatigue calculations where the number of cycles to failure is unknown an iterative solution approach is required. \(CV\) is the comparison value that depends on the loading type. For proportional stress states \(CV = 1\) and for non-proportional stress states \(CV = 0.5\) in accordance with the IIW [3]. The IIW recommendations do not distinguish between different levels of non-proportionality, however, it is stated that if the shear stress range is less than 15% of the normal stress range, it can be neglected.

In the case of variable amplitude loading, \(\Delta \sigma_x\) and \(\Delta \tau_y\) should be replaced with equivalent amplitude stress ranges given as:

\[
\Delta \sigma_{eq} = \frac{1}{D_{spec}} \sum (n_i \cdot \Delta \sigma_i^{n_i}) + \sum (n_j \cdot \Delta \sigma_j^{n_j})
\]

\[
\Delta \tau_{eq} = \frac{1}{D_{spec}} \sum (m_i \cdot \Delta \tau_i^{m_i}) + \sum (m_j \cdot \Delta \tau_j^{m_j})
\]

(20)

where \(\Delta \sigma\) and \(\Delta \tau\) are the stress ranges above and below the S–N curve knee-point, respectively. \(n_i\) and \(n_j\) are the corresponding number of applied cycles at these stress ranges. The equation is valid for \(\Delta \sigma_{eq} > \Delta \sigma_{L}\), where \(\Delta \sigma_{L}\) is the fatigue strength at the knee-point of the relevant S–N curve. If \(\Delta \sigma_{eq} < \Delta \sigma_{L}\) the following expression should be used instead:

\[
\Delta \sigma_{eq} = \sqrt[2]{\sum (n_i \cdot \Delta \sigma_i^{n_i}) + \sum (n_j \cdot \Delta \sigma_j^{n_j})}
\]

\[
\Delta \tau_{eq} = \sqrt[2]{\sum (m_i \cdot \Delta \tau_i^{m_i}) + \sum (m_j \cdot \Delta \tau_j^{m_j})}
\]

(21)

In Eqs. (20) and (21), \(D_{spec}\) is the specified damage sum, which varies between 0.2 and 1 in accordance to the IIW [3] depending on the load case. If variable amplitude loading is observed with fluctuating mean stresses, \(D_{spec} = 0.2\) is recommended. In this paper, \(D_{spec}\) is fixed at 0.5, for simplicity and in order to compare the results. Note, that the \(\Delta \sigma_{eq}\) and \(\Delta \tau_{eq}\) is calculated using the two equations above, with the only difference being the input stress ranges being either \(\sigma\) or \(\tau\).

4.4. Fatigue lives

The experimentally determined fatigue lives are shown in Figs. 14(a) and 14(b), in S–N type diagrams. As the fatigue tests are conducted under variable amplitude loading, regular S–N curves which show the relationship between a single cyclic stress range and the corresponding fatigue life, cannot directly be plotted. Different methods can, however, be employed for showing VA results in S–N curve type diagrams see for example [35,36]. In this paper, the fatigue tests are compared based on the maximum amplitude of the load spectrum as these have been found to give the most meaningful interpretations when comparing variable amplitude fatigue tests [35]. These types of curves are also denoted Gassner curves [37]. The maximum stress amplitudes shown on the y-axis are taken directly from the publication by Witt [19] and corresponds to the nominal stresses found at the weld toe for the normal stress perpendicular to the welds in the case of pure bending tests and the nominal shear stress for the pure torsional experiments. For the multiaxial fatigue tests, the maximum normal stress is used for comparison. As Witt performed all tests at the same normal to shear stress ratio, this is deemed acceptable and will make direct comparisons possible.

From the uniaxial experiments plotted in Fig. 14(a) the following observations can be made:

- The pure bending load series with \(R = -1\), shows higher fatigue lives as compared to the bending series with \(R = 0\) (BF7). This is to be expected due to the mean stress effect.
- The pure torsional fatigue experiments (series BF2) show similar fatigue lives as compared to those from bending with \(R = 0\). However, it should be noted that the Gassner curves are plotted against nominal stresses in Fig. 14(a), thus not taking into account the stress concentrations occurring at the weld.

From the multiaxial experiments plotted in Fig. 14(b), the following observations can be made:

- Similar to the uniaxial experiments investigated, a reduction in fatigue life can be expected with increased mean stress.
- A phase shift of 90° results in a clear reduction in fatigue life compared to a proportional stress state (\(\delta = 0^\circ\)).
- Increasing the frequency of the shear stress signal for BF5 does not seem to reduce the fatigue life considerably compared to the proportional data in BF3. A slight decrease is, however, observed.

Thus, a clear effect of non-proportionality in the fatigue curves can be observed, when introducing a phase shift between shear and normal stress of \(\delta = 90^\circ\). Interestingly, for the experiments with increased shear stress frequency (BF5), the fatigue lives are observed to differ only slightly from same experiments performed with equal normal to shear stress frequency (BF3). The results show, that a degrading effect of non-proportionality in the stress states on the fatigue lives occurs.

4.5. IIW estimated fatigue lives

To evaluate the IIW criterion and the comparison value \(CV\), the experimental fatigue lives have been compared to the predicted fatigue lives using the IIW criterion. This has been done using the so-called prediction ratio which has been calculated for all experiments. The prediction ratio is similar to that employed by Bruun and Härkegård [38] and Pedersen [9]. The prediction ratio has previously been used for analysing the Witt data in [18]. The prediction ratio makes it possible to compare the calculated fatigue life based on equivalent stresses to the experimentally determined fatigue life. The prediction ratio has been shown by Bibbo et al. [39], to correspond to:

\[
p = \left( \frac{N_{exp}}{N_{calc}} \right)^{1/m}
\]

(22)

where \(N_{exp}\) and \(N_{calc}\) is the experimentally obtained fatigue life in cycles and calculated fatigue life in cycles, respectively. \(m\) is the slope of the S–N curve. In this paper, a slope of \(m = 3\) has been used since the count is lower than the knee-point at \(10^7\) cycles. The prediction ratios will be \(p > 1\) for conservative results where the experimental fatigue life is higher than that of the predicted fatigue live. Similarly, the prediction ratios will be \(p < 1\) for non-conservative results. The benefit of using the prediction ratio as compared to directly comparing fatigue life cycles \(N\), is that the numbers are scaled by the power of \(1/m\), which results in more reasonable numbers.

The prediction ratios have been calculated for all multiaxial VA experiments from Witt and the mean values are plotted in Fig. 15. Error bars corresponding to the standard deviations have similarly been plotted. The experiments with \(R = -1\) (BF3 to BF 5) and the experiments with \(R = 0\) (BF9 and BF10) have been grouped.
As seen from Fig. 15, there is a clear effect of non-proportionality, when the non-proportionality in the stress states are caused by phase shifts. In the case of non-proportionality caused by frequency shifts, a higher damaging effect of non-proportionality is less clear. The mean prediction ratios of the proportional experiments and are likewise shown in the figure \( \beta_p = 5 \). When the ratios are below 1, they indicate a reduction in fatigue lives caused by non-proportionality. For the frequency shifted data BF5, the ratio between the prediction ratios are calculated to 0.98 indicating nearly no difference in predictions. However, it should be noted that the standard deviation is much higher for the proportional experiments as compared to the frequency shifted experiments. No clear reason for these differences in standard deviations have been found, however, it is well known that fatigue tests will show considerably scatter.

From the ratios between the calculated prediction ratios, it is possible to obtain more fitting CV values. As the GP equation in Eq. (19), contains the normal and shear fatigue strengths, the ratios provided in Fig. 15, cannot directly be used to determine the CV factor to be used in the GP equation. In the previous work [18], it was found that with other fatigue criteria such as the Findley [33] or DNV [2] criteria, the prediction ratios are proportional to the respective penalty factors. In the case of the IIW GP criterion, an iterative approach for finding the mean CV factor is necessary.

From Table 5, it can be observed that the experimentally obtained comparison values are higher than the one proposed by the IIW of 0.5. Thus, the IIW recommendations are for these experiments conservative. It should be noted that a \( D_{\text{spec}} \) of 0.5 was used for all calculations. The experimental CV value for the experiments with \( R = -1 \) is 7% higher than the experiments performed at \( R = 0 \) (for the phase shifted data). This is a relatively small difference. Furthermore, it can be observed from Table 5, that the estimated CV factor for the data with frequency difference between normal and shear stresses is approximately 1. This means, even though the frequency difference results in rotating principal stresses, no decrease in fatigue lives is observed. Thus, the results indicate that increased frequency of shear stresses compared to normal stresses does not influence the fatigue life considerably. However, only a single test series with frequency differences and VA loading has been documented by Witt [19], thus more experimental data is required for concluding on this effect. This is out of the scope of this paper.

### 4.6. Non-proportionality quantifiers

In Table 6, the non-proportionality quantifiers estimated for each of the non-proportional test series are provided using the CA equivalent PCA-based approach, the Gaier et al. [13] and Bolchoun et al. [22] approach. As the VA experiments from Witt were performed with a fixed bending to torsion load ratio, all NP quantifiers are nearly identical for the individual tests in each test series. Thus, one NP quantifier value can be provided for each series. In the case of the proportional data, both uniaxial and multiaxial, all approaches predicted 0, thus they are not included in Table 6. As observed from Table 6, all approaches predict NP quantifiers above 0, indicating some level of non-proportionality. The CA equivalent PCA-based approach where the stress-time signals are transformed into a CA signal before calculating the quantifier returns nearly 1 for both the data with \( R = -1 \) and \( R = 0 \) with a phase shift of 90° (BF4 and BF6). The quantifier do not completely reach 1. This is due to the considered plane for the damage estimation (normal to the surface and aligned with the weld toe) and the load appliance are rotated compared to each other. Thus, a 90° phase shift in loading does not exactly result in a phase shift of 90° in the stress state. Thus, a lower value of 0.87 to 0.91 is observed. For the frequency shifted data the \( N_p = 0.67 \) only predicts a value of 0.51, however, as the PCA-based and CA equivalent PCA-based approach are developed to take into account phase shifts this is deemed acceptable. The Bolchoun et al. and Gaier et al. quantifiers both return values below 1 for the 90° phase shifted data. However, both approaches predict somewhat similar levels of NP levels. This is also to be expected, as very specific conditions needs to be fulfilled before the Bolchoun et al. and Gaier et al. quantifier returns a value of 1. From the investigation of the experiments, it is clear that a higher damaging effect of non-proportionality is present in the fatigue lives as seen from the experimental CV values in Table 5. As observed from Table 6, the PCA-based approach is capable of automatically determine this if the stress-time series are non-proportional. Thus, the results indicate that the PCA-based approach can be used to determine automatically and with confidence, when additional penalty factors need to be applied in the fatigue criteria.

As non-proportionality is normally defined by the rotation of the principal stresses, the three test series have been further examined.
Table 6
Non-proportionality quantifiers predicted for the non-proportional experiments from Witt.

<table>
<thead>
<tr>
<th>Test series</th>
<th>( N_{CA_{PCA}} )</th>
<th>( N_{CA_{total}} )</th>
<th>( N_{Bolchou} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF4, ( R = -1 ), ( \delta = 90^\circ )</td>
<td>0.91</td>
<td>0.69</td>
<td>0.59</td>
</tr>
<tr>
<td>BF5, ( R = -1 ), ( f_t/f_s = 5 )</td>
<td>0.51</td>
<td>0.74</td>
<td>0.61</td>
</tr>
<tr>
<td>BF10, ( R = 0 ), ( \delta = 90^\circ )</td>
<td>0.87</td>
<td>0.53</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Fig. 16. Rotation of principal stresses for a single experiment in each considered test series from Witt.

for predicting the principal stress states. On the surface of the weld, for which a plane stress condition is occurring, the principal stress rotation is found for a single experiment in each test series. The results are shown in Fig. 16. As observed from Fig. 16, large principal stress rotations are observed for all three test series. Furthermore, the principal stress rotations are seen to be comparable in size, meaning that in all series the same maximum and minimum rotations (±45°) are encountered. Using the rotations of the principal stresses for determining the level of non-proportionality is therefore difficult as it fluctuates considerably for the considered stress-time series. The frequency of changes in the principal stress rotations may influence the non-proportionality levels, however, none of the considered quantification approaches explicitly considers this. To the authors knowledge, no approaches focusing on the rotations of the principal stresses as shown in Fig. 16, has been published. Furthermore, the use of the principal stress rotations as a function of time requires intensive calculations as the full stress-time series should be evaluated.

The proposed CA equivalent PCA-based quantifier is an pragmatic approach which is robust, easy to implement and results in quantification values that are easy to interpret. As the PCA-based approach is based on the well-known principal component analysis, the base code for finding the principal components are included in many programming software. Furthermore, the interpretation is easy, as a specific NP level corresponds to a certain phase shift (for purely phase shifted data).

5. Case study

The PCA-based NP quantifier and the CA equivalent PCA-based NP quantifier are both developed with focus on large scale-structures such as offshore jacket foundations for wind turbine generators. Offshore structures are exposed to wind and wave loading which can result in stress-time signals with phase shifts. Furthermore, the loads are variable in amplitude, resulting in VA stress states. Welded offshore joints are also often exhibiting high residual stresses and are less exposed to high parallel stresses. This makes the PCA-based approaches highly suitable for assessing the non-proportionality of these kind of structures.

To evaluate the actual levels of non-proportionality that can be expected in a offshore jacket structure, in this section, five welded joints in an offshore jacket foundation modelled by Ramboll Group A/S has been examined. The welded joints between the trusses are denoted nodes and their names are determined by their configuration (Y- K- or X-nodes). The considered nodes and overall jacket design are shown in Fig. 17. The jacket is a traditional three-legged jacket support structure with five levels of joints. The detailed information about the jacket size including thickness, outer diameter of the pipes and lengths are not included due to non-disclosure. However, the dimensions of each pipe are less relevant in this investigation, as any scaling of the dimensions will only result in scaling of the magnitude of the occurring stresses and not changes in the overall non-proportionality.

5.1. Considered nodes, locations and load cases

In total five nodes have been examined for a total of 12 different load cases. Each load case corresponds to a load direction as shown in Figure Fig. 18 and a mean wind speed of 11 m/s. The load direction corresponds to the direction in which the wind and wave loading primarily is aligned and is measured from north clockwise using the angle \( \theta \) as shown in Fig. 18. For each node, two critical points have been examined corresponding to one saddle point and one crown
point. The crown points are marked with blue and the saddle points are marked with red in Fig. 17. In total, 36 critical points are often examined on the circumference of the nodes during design, however, in this examination, only two points are considered for each node. The considered critical points correspond to the highest damage locations for the crown and saddle point, respectively. The nodes have been chosen based on their severity in the overall fatigue design and to examine the effects on different types of nodes (X-nodes, K-nodes or Y-nodes).

The stress-time series considered are based on the hot-spot stress approach for tubular joints as described in DNV-RP-C203 [2]. Each stress-time series consists of 30,000 time steps and has been extracted on the surface of the structure, thus being in plane stress conditions.

5.2. NP quantification

The NP quantifier for using the CA equivalent PCA-based approach has been compared to the approaches from Gaier et al. [13] and Bolchoun et al. [22]. The results are shown in Fig. 19, where the y-axis shows the NP quantification level and the x-axis shows the considered load direction. It should be noted that for the considered comparisons, the additional check performed in Eq. (6) which ensures that in the case of very low shear stresses, the NP returns zero, has been neglected. As the other considered approaches from literature, does not include this check, it has not been performed to make the results comparable. However, the cases in which this check is relevant, a black cross has been plotted as seen for the upper K-node, lower K-node and lower Y-node. Especially, the case of upper and lower Y-nodes are interesting as great differences in the NP levels for the saddle and crown points are observed. This is especially observed for the upper Y-node in Fig. 19(a), where the crown point sees nearly no non-proportionality and the saddle points sees much more.

The approach from Gaier et al. [13] is also observed for nearly all nodes and load directions, to provide very low estimates of non-proportionality. This is most likely caused by the fact that this approach utilizes the principal moments of inertia instead of the correlation between different stress signals. However, the Bolchoun et al. [22] approach results in much higher estimations, even though this is also an integral approach considering all planes.

The extended CA equivalent PCA-based approach which transforms the VA stress signals into CA signals are seen to follow the same trends for nearly all cases. However, relatively different quantification values can be observed.

In order to quantify the overall level of non-proportionality in all nodes, all data has been combined and binned. For each bin, the percentage occurrence of a specific level of non-proportionality has been observed from Fig. 19, this is only the case on a handful of locations and nodes. Furthermore, Eq. (6) is highly dependent on the stress counting between different stress signals. However, the Bolchoun et al. [22] approach results in much higher estimations, even though this is also an integral approach considering all planes.

In general only relatively low levels of non-proportionality with only a few exceptions. In general only relatively low levels of non-proportionality is determined by the different quantifiers, with only a few occurrences close to 1.

The extended CA equivalent PCA-based approach is the only approach showing NP levels close to 1. The saddle and crown critical points show similar levels of non-proportionality except for the upper Y-node and the lower Y-node where great differences are seen.

A handful of data points can be excluded when considering Eq. (6).
indicates that using the penalty factors from the guidelines to account for non-proportionality in all cases and for all joints, could be highly conservative.

It should be noted, that the investigation of the jacket nodes are carried out using hot-spot stresses as opposed to the investigation of the experiments described by Witt [19], which was performed using the notch stress approach. This means, that some discrepancies between the investigation of experimental data and simulated jacket data are expected. However, the PCA-based quantifier is less influenced by the change of stress estimation approach as the stress-time signals are standardized before these signals are used for non-proportionality quantification. The Gaier et al. and Bolchoun et al. methods are expected to be more prone to changes in the quantification values, as they are influenced by mean stresses and amplitudes.

6. Conclusions

In this paper, a constant amplitude equivalent principal component analysis-based approach for determining the level of non-proportionality in stress-time histories has been proposed and evaluated on variable amplitude signals. The approach is an extension to the original PCA-based approach previously described by the authors. In the method, the variable stress-time histories are transformed into constant amplitude equivalent signals before calculating the non-proportionality values to ensure reliable results. The proposed approach can be implemented easily in different programming software and is robust in its predictions.

The proposed extended method is evaluated with two other approaches from literature on experimental fatigue data from literature performed with variable amplitude loading. The results show, that the PCA-based requires only a low numerical calculation effort and that it is the easiest approach to interpret. Furthermore, the non-proportionality levels at the welds in an offshore jacket structure design are evaluated. The results show that in general only low levels of non-proportionality are observed.

The following overall conclusions can be drawn from the paper:

- The newly proposed extension to the PCA-based non-proportionality quantification approach results in robust non-proportionality predictions for phase shifted variable amplitude stress signals.
- The approach is easy to interpret as it predicts a value of 0 at 0 degrees phase shift and predicts a value of 1 at 90 degrees phase shift.
- The CV penalty factor proposed by the International Institute of Welding is conservative based on the investigated experimental data.
- Using the non-proportionality estimators on the welded joints of an offshore jacket structure indicates only low levels of non-proportionality.

For future work, the effect of different phase shifts in the fatigue damage should be investigated. Currently, only limited amount of fatigue tests performed on welded specimens with other phase shifts than 0 and 90° between the normal and shear stresses have been performed. With more experimental data, it can be possible to determine if correlation between phase shift and damage is present and possibly use the PCA-based NP quantifier for determining more relevant penalty factors such as the CV factor.

### Table 7

<table>
<thead>
<tr>
<th>NP Level</th>
<th>0.0–0.2</th>
<th>0.2–0.4</th>
<th>0.4–0.6</th>
<th>0.6–0.8</th>
<th>0.8–1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{P_{CA,CA}}$</td>
<td>51%</td>
<td>17%</td>
<td>13%</td>
<td>14%</td>
<td>5%</td>
</tr>
<tr>
<td>$N_{P_{CA}}$</td>
<td>71%</td>
<td>14%</td>
<td>10%</td>
<td>4%</td>
<td>1%</td>
</tr>
<tr>
<td>$N_{P_{CV}}$</td>
<td>85%</td>
<td>11%</td>
<td>4%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>$N_{P_{Gaier}}$</td>
<td>20%</td>
<td>50%</td>
<td>18%</td>
<td>12%</td>
<td>0%</td>
</tr>
</tbody>
</table>

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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### References

[32] Sonsino CM. A consideration of allowable equivalent stresses for fatigue design of welded joints according to the notch stress concept with reference radii rreff = 1.00 and 0.05 mm. Welding World 2009;53:64–75.