Relative Performance Evaluation and Earnings Management*

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Abstract
Conventional agency theory suggests that firms should benchmark CEO compensation to absorb systemic risk and to more efficiently incentivize executives to work hard. Yet, empirical research has found only a modest use of benchmarking in CEO compensation contracts. In this paper, I highlight one weakness of relative performance evaluation (RPE). When earnings management is possible, benchmarking creates stronger incentives for misreporting performance measures compared to benchmark-independent pay. The optimal contract will depend less on a correlated benchmark (e.g. a stock market index) if it is easier for the manager to misreport performance. Thus, the model predicts that firms with weak internal controls and bad auditors are less likely to use RPE, offering a theoretical explanation for the empirically observed lack of RPE use.

Keywords: relative performance evaluation, earnings management, CEO compensation
1. Introduction

Incentivizing executives to work hard has been studied extensively in the literature. Holmstrom (1982) was one of the first to show that using the performance of the competition in determining compensation can be desirable if there are common shocks that influence output. Filtering this additional noise gives a better understanding of the executive’s effort. Positive covariance among firms’ performances seems to be descriptive of the real world, where industry-wide and economy-wide events regularly affect multiple firms at once. Hence, one would expect a widespread use of relative performance evaluation (RPE). Yet, empirical research has found only modest use. Bannister and Newman (2003) and Gong, Li, and Shin (2011) find that less than half of firms use RPE.\(^1\) A good example of this is the oil industry. In 2007, at the height of oil prices, executives received (compared to other industries) disproportionately high pay raises, four times as high as the average (Herbst 2008). A similar correlation was observed in 2015. When oil prices plummeted, so did the compensation of many executives. Median compensation fell, while it rose in other industries (Olsen 2016).

Bertrand and Mullainathan (2001) provide one potential explanation for the lack of benchmarked CEO compensation. They find that better governed firms use more RPE, and conclude that this is consistent with the hypothesis that CEOs have power over their boards and essentially set their own pay. In contrast, I show that the association of weaker corporate governance with less RPE use can also occur in an optimal contracting setting, in which the manager has no power over the compensa-

\(^1\)A current working paper by Bizjak et al. (2020) shows that RPE use has increased to 48% in 2015.
tion committee. An important feature in my model is that the CEO can manipulate the performance measure on which her compensation is based. I find that the optimal contract will make less use of RPE if the CEO can more easily misreport the performance measure. Thus, my paper provides guidance to future archival studies in identifying conditions that determines RPE use.

The intuition for my result is driven by an asymmetry which arises in the optimal contract in equilibrium. Consistent with empirical evidence (Bannister and Newman 2003, Garvey and Milbourn 2006), in my model the manager receives a large reward for outperforming the benchmark, but her compensation only decreases slightly for underperforming it. This causes two effects, both of which are disadvantages of RPE.

First, the asymmetry creates a difference in ex-post manipulation incentives. When the manager performs similar to the benchmark, ex-post incentives to manipulate will be higher, while they are lower when she underperforms the benchmark. If the manager’s performance were uncorrelated with the benchmark, then these forces would offset, and ex-ante expected manipulation would be unchanged. However, because the manager’s performance and the benchmark are positively correlated, the chance that the manager performs similar to the benchmark increases. Hence, she is more likely to be in a situation in which she has high incentives to manipulate. Without RPE, a contract could still be nonlinear. Yet, the lack of correlation due to the absence of a benchmark means that the manager is not more likely to be in a situation where her manipulation incentives are strong. Therefore, RPE in expectation incentivizes a higher level of manipulation compared with a lack of RPE use.

The second effect working against RPE is the observability of the benchmark.
The manager has to commit to her effort decision early in the year, well before the benchmark is realized. Ideally, she would prefer to work hard when the benchmark is easier to beat, and shirk when it is harder. However, she does not have to commit to an earnings management decision until very late in the fiscal year, possibly even after the fiscal year has ended. Real earnings management decisions such as accelerating next year’s sales (e.g. by giving discounts) or delaying R&D spending by a month can be made late in the fiscal year; and accrual-based earnings management can be executed even after the fiscal year has ended, when many major (discretionary) accounting decisions have to be made. At that point, she will have already observed the outcome of shocks, such as changes in economic conditions, industry demand, etc. Thus, the manager can condition the manipulation level on the realization of the benchmark. With nonlinear RPE, her manipulation decision will depend on her performance relative to the benchmark. This makes manipulation more efficient for the agent, hence reducing the incentive effect to actually work hard. Benchmark-independent pay does not have this disadvantage, because the information about the benchmark is useless to the agent. Her manipulation decision would be independent of economic conditions in that case.

The firm anticipates the two effects described above and proactively responds by reducing the weight of relative compensation in the contract structure. The firm trades off the benefit of using the more informative signal with the benefit of reducing incentives to manipulate. This result can explain why so few firms find it optimal to use RPE, despite its advantages. The result also leads to the empirical prediction that RPE will be used more in firms with better corporate governance,
tighter accounting standards, etc. that limit the CEO’s manipulation potential.\textsuperscript{2}

The remainder of the paper is organized as follows. Section 2 discusses related literature, section 3 describes the model, section 4 analyzes the scenario without earnings management, section 5 discusses the main results and also explores some comparative statics, and section 6 concludes. All proofs are in the appendix.

2. Related literature

Relative performance evaluation (RPE) has received significant attention in the academic literature. One of the earliest examples is Holmstrom (1979, 1982). He shows that the optimal compensation scheme of an agent depends on her performance alone, if, and only if, her performance is independent from anyone else’s; in such cases no information about the agent’s actions can be learned from comparing her output with that of others. Doing so would only add noise and thus be detrimental in the case of risk-aversion because the additional risk that is imposed on the agent has to be compensated. However, with dependence among performance outcomes, some information is embedded in the performance of others and this can be used to more efficiently incentivize effort. Under some conditions, a weighted average of all others’ performances and the agent’s output are sufficient for optimal incentive contracting.\textsuperscript{3}

Holmstrom’s results do not specify how the weighted average should be used in

\textsuperscript{2}This empirical prediction can be tested differently than the above mentioned one by Bertrand and Mullainathan (2001) about powerful CEOs influencing the pay-setting process. My prediction can use proxies such as distance between firm headquarters and the auditor’s office (Choi et al. 2012, Kubick et al. 2017), while a CEO’s power can be measured using CEO tenure, CEO-board duality, etc.

\textsuperscript{3}Similarly, Antle and Demski (1988) show that it can be optimal to evaluate a manager based on information which she cannot control.
the compensation scheme. This is an important question that is addressed in subsequent research. Banker and Datar (1989) find necessary and sufficient conditions when a linear aggregation of signals is optimal. Specifically, using profit is only optimal when revenue and costs are equally intense (sensitivity times precision) signals.\textsuperscript{4} Celentani and Loveira (2006, p. 525) find that "if the marginal return of effort depends on the aggregate state, optimal contracts are not monotonically decreasing in the performance benchmark" and claim that this may explain the lack of RPE use in the business world. Fleckinger (2012) generalizes these results by not imposing any restrictions on the covariance between a manager’s performance and the performance of a second manager. His results predict that RPE is most effective when this covariance is constant and positive.

RPE has, of course, also been studied in the empirical literature. Early studies (e.g. Janakiraman, Lambert, and Larcker 1992, Aggarwal and Samwick 1999) use indirect tests to study RPE use and do not find evidence that firms use RPE. SEC rule changes in 1992 mandated more extensive executive compensation disclosures, giving researchers access to better data (e.g. Bannister and Newman 2003, Gong et al. 2011), and results indicate that about a quarter of all firms use RPE. Bannister and Newman (2003) and Garvey and Milbourn (2006) find that firms use one-sided RPE, where executives get rewarded for outperforming the peer-group, but do not get punished for underperformance relative to their peers. This result is consistent with the theoretical results of Celentani and Loveira (2006) and Feriozzi (2011).

Prior analytical research proposes solutions for the relative performance evalu-

\textsuperscript{4}See also Şabac and Yoo (2018) on performance measure aggregation.
ation puzzle. Dye (1992) finds that the agent’s option to choose among different projects can negatively affect RPE. Gopalan et al. (2010) study a similar idea. The agent will pick a project (or industry) where her skill is relatively high compared to peers in that industry, but may be low in absolute terms. This is not in favor of the principal, who cares about absolute, rather than relative, profit. Also, Dikolli et al. (2013) hypothesize that the empirical literature may underestimate RPE use if researchers calculate peer performance differently than the firm does.\footnote{\emph{Other papers that discuss the RPE puzzle include Fershtman et al. (2003) and DeMarzo and Kaniel (2017), who analyze agents’ utility functions that depend on others’ pay.}}

The two most closely related studies to my analysis are Bagnoli and Watts (2000) and Balakrishnan et al. (2020). Bagnoli and Watts (2000) differ because they do not endogenize the agent’s compensation structure and there is no moral hazard, key aspects of my paper. Balakrishnan et al. (2020) study employee performance ranking systems and their susceptibility to performance manipulation. Their paper, though, is more closely related to the tournament theory literature, rather than the RPE literature.

3. Model

I consider a single-period game with a risk-neutral principal and a risk-neutral agent.

\textbf{Timing:} There are two dates, $t = \{0, 1\}$. At date 0, the principal hires an agent to be in charge of a project and offers her a compensation contract. After signing the contract, the agent can exert costly effort that increases the probability of success. At date 1, the outcome of a correlated benchmark is publicly observed and the agent
Figure 1  Timeline of the model

<table>
<thead>
<tr>
<th>t = 0</th>
<th>contract</th>
<th>effort</th>
<th>benchmark, project outcome</th>
<th>manipulation, report</th>
<th>payments</th>
</tr>
</thead>
</table>

privately learns whether her project succeeded or failed, $R \in \{S,F\}$.$^6$ Then, she issues a potentially manipulated report. Based on the report and the benchmark, the agent is paid and the game ends. Figure 1 depicts the timeline.

The timeline is a stylized depiction of the real world. In reality, information about the final outcome of the benchmark is learned gradually as the year progresses. The agent will use all available information when she makes manipulation decisions. The important part of the timeline is that the agent has more information about the benchmark available for her manipulation decision than she had for her effort decision. Most manipulation, especially accrual-based, typically occurs at the end of the fiscal year, while most effort is typically exerted before that time.

**Effort:** The agent makes a binary effort decision after being hired: $e \in \{e_l, e_h\}$. High effort, $e_h$, increases the chance that the project will be successful (details below). Effort imposes a disutility on the agent, $k(e)$, and without loss of generality, I assume that $k(e_h) = c$ and $k(e_l) = 0$. Furthermore, to avoid trivial solutions, I assume that the principal always wants to induce high effort.

**Benchmark:** There is an observable and contractible benchmark $Z \in \{G, B\}$

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$^6$An unobservable outcome might occur if the project is long-term, but the agent needs to be compensated before the end of the project.
TABLE 1
Joint probabilities

<table>
<thead>
<tr>
<th>$R \setminus Z$</th>
<th>$B$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>$\frac{1}{2}(1 - p_e + \gamma)$</td>
<td>$\frac{1}{2}(1 - p_e - \gamma)$</td>
</tr>
<tr>
<td>$S$</td>
<td>$\frac{1}{2}(p_e - \gamma)$</td>
<td>$\frac{1}{2}(p_e + \gamma)$</td>
</tr>
</tbody>
</table>

Notes: The agent’s project can succeed or fail, $R \in \{F, S\}$, the benchmark can be good or bad, $Z \in \{B, G\}$, $p_e$ depends on the agent’s effort choice, and $\gamma$ is a covariance parameter.

A pair of results can take four values, $Y \in \{SG, SB, FG, FB\}$. Let the probability of the good benchmark be independent of the agent’s effort decision, $\Pr(Z = G|e) = \Pr(Z = G) = q$, and without loss of generality $q = \frac{1}{2}$ for most of the remainder of the paper. However, this setup does not imply that the probabilities of project success and benchmark outcome are independent. There is a positive covariance $\gamma$, which alters the conditional success probabilities such that $\Pr(R = S|Z = G) = p_e + \gamma$ and $\Pr(R = S|Z = B) = p_e - \gamma$, where $p_e = p_h$ with high effort, and $p_e = p_l$ for low effort, and $p_h > p_l$. Let $\gamma$ be small enough such that all probabilities are between zero and one. This leads to the joint probabilities for the four possible outcomes in Table 1.

These probabilities capture, in the simplest form possible, the character of posi-

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A somewhat more endogenous choice would be the average performance of a peer group. I address this interpretation in section 5. However, I deliberately do not use a two agent model. The situation I analyze focuses on a CEO who does not have a comparable coworker within the same company.
tive correlation. For example, when the economy is doing well, a CEO’s projects are more likely to succeed. The assumption of a constant covariance imposes a restriction on the model to keep the focus on the effects of earnings management, and the exposition simple.

**Report:** At date 1, the agent observes the outcome of her project and the benchmark. She then issues a report to the principal, \( r \in \{r_S, r_F\} \); no other communication between the two parties is allowed.\(^8\) Let the combination of the report and the observable benchmark be denoted by \( y \in \{y_{SG}, y_{SB}, y_{FG}, y_{FB}\} \). The agent can take an unobservable, manipulative action \( m \in [0, 1] \) to issue a favorable report, even though the project has failed; \( m \) is the probability that a failed project will be misreported as a success, i.e. \( m = \Pr(r = r_S|R = F) \). The agent would never misreport good news because the optimal compensation is higher when the report is better. Generally, the manipulation level will take two different values, \( m_G \) and \( m_B \), depending on the benchmark, good and bad respectively.

The cost of manipulation is \( gm^2/2 \), where \( g \) is an exogenous parameter that captures how easily reports can be manipulated.\(^9\) For example, better auditors, tighter accounting standards, and a vigilant board would cause \( g \) to be higher. Furthermore, I assume that \( g > g \equiv \frac{2c}{p_h - p_l} \) to guarantee an interior solution, i.e. \( m < 1 \), in equilibrium.\(^{10}\)

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\(^8\)For this reason, the revelation principle does not apply in this model.

\(^9\)One could assume that \( g \) depends on the realization of the benchmark. When conditions are bad, auditors may expect more manipulation, thus auditing more. The results of the model are qualitatively unchanged, as long as the two different \( g \)'s are close to each other.

\(^{10}\)These assumptions about the manager’s reporting have been used extensively in prior literature with binary models and earnings management (e.g. Dutta and Gigler 2002, Jongjaroenkamol and Laux 2017).
Contracting: The agent is risk-neutral and is thus maximizing her expected wage, $E[w_y]$. The only two variables that are available for contracting are the report and the benchmark. Thus, the agent is offered a contract $w = \{w_{SG}, w_{SB}, w_{FG}, w_{FB}\}$ that specifies four state-dependent payments. For example, the agent receives $w_{SB}$ when she reports a success and the benchmark is bad, $y = y_{SB}$. The agent is protected by limited liability, that is $w \geq 0$ for each payment. The reservation utility is set to zero. The participation constraint does not bind in equilibrium, and the first-best solution cannot be achieved.

An incentive scheme uses RPE, when $\Delta w \equiv (w_{SB} - w_{SG}) > 0$, joint performance evaluation (JPE), a higher reward for success when the benchmark is high as well, when $\Delta w < 0$, and independent performance evaluation (IPE), when $\Delta w = 0$. The greater the absolute value of $\Delta w$, the greater the extent of RPE/JPE. The other two payments, $w_{FG}$ and $w_{FB}$, are not part of this definition because in the optimal solution these are zero.

4. No manipulation

The no-manipulation setting is a special case of the general model described above, when $g \to \infty$ and therefore $m = 0$. This case demonstrates the main advantage of RPE.

The principal minimizes expected compensation cost

$$\min_{w_{SB}, w_{SG}, w_{FG}, w_{FB}} E[w_y] = \frac{p_h - \gamma}{2} w_{SB} + \frac{p_h + \gamma}{2} w_{SG} + \frac{1 - p_h - \gamma}{2} w_{FG} + \frac{1 - p_h + \gamma}{2} w_{FB},$$

(1)
subject to the agent’s incentive constraint

\[
\frac{p_h - \gamma}{2} w_{SB} + \frac{p_h + \gamma}{2} w_{SG} + \frac{1 - p_h - \gamma}{2} w_{FG} + \frac{1 - p_h + \gamma}{2} w_{FB} - c
\geq \frac{p_l - \gamma}{2} w_{SB} + \frac{p_l + \gamma}{2} w_{SG} + \frac{1 - p_l - \gamma}{2} w_{FG} + \frac{1 - p_l + \gamma}{2} w_{FB},
\]

which simplifies to

\[
w_{SB} - w_{FB} + w_{SG} - w_{FG} \geq \frac{2c}{p_h - p_l},
\]

and the limited liability constraint

\[
w_{SG}, w_{SB}, w_{FG}, w_{FB} \geq 0.
\]

Solving this problem leads to the following proposition.

**Proposition 1.** When earnings management is not possible \((g \to \infty)\), then the optimal contract satisfies

\[
w_{SB} > w_{SG} = w_{FG} = w_{FB} = 0.
\]

It is optimal to set \(w_{FG} = w_{FB} = 0\), because any positive payment for project failure would only make it more difficult to provide incentives for the agent to work hard. The reason for \(w_{SB} > w_{SG} = 0\) is more subtle. Holmstrom’s work (1979, 1982) shows that it is efficient to incentivize effort by linking pay to the signal that is most informative about effort. If an agent is successful, despite a low benchmark, then success is a very informative signal about the agent’s effort. However, high agent
performance in the presence of a good benchmark is a less informative signal about
effort. Formally, this can be expressed with likelihood ratios:

\[
\frac{\Pr(y = y_{SB} | e = e_h)}{\Pr(y = y_{SB} | e = e_l)} > \frac{\Pr(y = y_{SG} | e = e_h)}{\Pr(y = y_{SG} | e = e_l)} \quad \frac{p_h - \gamma}{p_l - \gamma} > \frac{p_h + \gamma}{p_l + \gamma}.
\]

(6)

These likelihood ratios measure how strongly \(y = y_{SB}\) and \(y = y_{SG}\), respectively, signal that the agent chose high rather than low effort. A high likelihood ratio indicates high effort; a value of one would indicate that nothing new is learned from the signal (Hart and Holmstrom 1987). The inequality in (6) shows that \(y = y_{SB}\) (agent is successful and benchmark is bad) is more informative than \(y = y_{SG}\).

The benchmark provides the firm useful knowledge about the effort decision of the manager. Thus, with risk neutrality and limited liability, a corner solution emerges.

However, if one were to introduce risk-aversion, then there is a trade-off. A large spread between \(w_{SB}\) and \(w_{SG}\) imposes risk on the agent, which has to be compensated in the form of a risk premium. The principal would have to strike a balance between the risk premium and putting more weight on the more informative signal. More risk-averse agents would also be offered a contract with RPE, but not as extreme as with risk-neutrallity.

The main results below are derived assuming risk neutrality, to keep the intuition simple. A numerical example with risk aversion as a robustness check is provided in the next section.
5. Manipulation

Main results

The no-manipulation setting above demonstrates the advantage of using RPE as it employs the most informative signal to efficiently incentivize effort. The following paragraphs will show the disadvantage: RPE induces more manipulation via two channels.

Solving the model backwards, I first analyze the manager’s level of manipulation, given a contract which satisfies the incentive constraint with equality.\textsuperscript{11} The goal of this exercise is to analyze how much manipulation a given contract induces. The contract does not need to be optimal from the principal’s perspective.

The agent’s conditional utility functions after project failure at the time of the manipulation decision are

\begin{align*}
U_{FB} &= m_B w_{SB} - c - \frac{1}{2} gm_B^2, \quad \text{and} \\
U_{FG} &= m_G w_{SG} - c - \frac{1}{2} gm_G^2,
\end{align*}

after observing a bad and good benchmark respectively.

Taking the first-order condition of (7) with respect to $m_B$ and of (8) with respect

\textsuperscript{11}The incentive constraint is always binding at the optimum, while the participation constraint is slack. Thus, once again we have $w_{FB} = w_{FG} = 0$, as these payments do not incentivize high effort.
to \( m_G \) yields

\[
m_B = \frac{w_{SB}}{g} \text{ and } m_G = \frac{w_{SG}}{g}.
\] (9)

Manipulation depends on the outcome of the benchmark, \( Z \), because the agent can observe \( Z \) before the manipulation decision. In contrast, effort cannot be conditioned on \( Z \) because the manager only observes \( Z \) after the effort decision.

The expected manipulation level \( E[m] \), in general terms, is

\[
E[m] = \frac{1}{2} (1 - p_e + \gamma) m_B + \frac{1}{2} (1 - p_e - \gamma) m_G.
\] (10)

For the expected manipulation level, the following proposition can be obtained.

**Proposition 2.** The expected manipulation level \( E[m] \) is always higher for contracts that use RPE \((w_{SB} > w_{SG})\) than for benchmark-independent contracts \((w_{SB} = w_{SG})\), ceteris paribus. \( E[m] \) is minimized, when the contract uses JPE.

Solving for the contract that satisfies the incentive constraint (equation (27) in the appendix) with equality, substituting all results into (10), taking the derivative of \( E[m] \) with respect to \( w_{SB} \), and substituting \( w_{SG} \) back in for simplicity, gives the following result:\(^\text{12}\)

\[
\frac{dE[m]}{dw_{SB}} = \frac{1 - p_h + \gamma}{2g} - \frac{1 - p_h - \gamma}{2g} \frac{g - w_{SB}}{g - w_{SG}}.
\] (11)

\(^\text{12}\)Note that in a contract that satisfies the incentive constraint with equality, \( w_{SB} \) and \( w_{SG} \) are negatively related. When one payment increases, the other can be lowered. Thus, a change in \( w_{SB} \) is always in the same direction as a change in \( \Delta w \equiv (w_{SB} - w_{SG}) \).

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The first term in (11) is the direct effect of an increase in $w_{SB}$ on $m_B$. The manager will manipulate more when the benchmark is low ($m_B$); if the bonus payment she receives when she is successful and the benchmark is low, $w_{SB}$, is higher. The second term in (11) is the indirect effect of a simultaneous decrease in $w_{SG}$ and hence $m_G$.

When $w_{SB} > w_{SG}$ (RPE), then the derivative in (11) is positive, i.e. an increase in RPE always increases expected manipulation. This result can be traced back to the positive correlation between the manager’s performance and the benchmark, as well as the nonlinearity of the contract due to limited liability. RPE itself does not cause higher manipulation. In a LEN model, the incentive to manipulate is constant, even with RPE. Nonlinear RPE ($w_{SB} - w_{FB} > w_{SG} - w_{FG}$) creates incentives to manipulate more when the benchmark is bad. Manipulation allows the manager to report that she outperformed the benchmark and collect a large bonus. This scenario is more likely to occur when positive correlation is at play because the agent is likely to perform similar to the benchmark. Additionally, the optimal contract creates lower incentives to manipulate when the benchmark is good. Manipulation only allows the CEO to hide her underperformance relative to the benchmark, and avoid a small decrease in her compensation. This case is less likely to occur due to correlation because the agent’s performance is less likely to deviate from the benchmark. Since these forces flip when joint performance evaluation (JPE) is used, $w_{SB} < w_{SG}$, expected manipulation is lower with JPE.

Importantly, changes in the manipulation level and thus the cost of manipulation

\[^{13}\text{See e.g. Christensen et al. (2013), and Dikolli et al. (2018).}\]
are effectively always borne by the principal. The manager can always choose to not work and not manipulate, guaranteeing a slack participation constraint. That is why the principal will optimally trade off the advantage of RPE (use of the most informative signal) with the disadvantage of RPE (higher manipulation).

The second channel in this model concerns the observability of the benchmark. Because the benchmark is observable before the manipulation decision, the manipulation level when the benchmark is good will differ from the level when it is bad (as long as \( w_{SB} \neq w_{SG} \)). If the benchmark were unobservable, then by construction the agent would have to choose \( m_B = m_G \). The loss of a degree of freedom for the manager means that she will receive a lower utility to the benefit of the principal. He can pay out the larger bonus less often because the manager is less efficient in her manipulation decision. This leads to the following proposition.

**Proposition 3.** If compensation depends on the benchmark \( (w_{SB} \neq w_{SG}) \), then the agent’s utility is higher when she can observe the benchmark.

This result is a drawback for both RPE and JPE, and is caused by the asymmetric contract induced by limited liability, as with the previous channel. When the agent submits a low report, then compensation is always zero. If compensation were symmetric, such that \( w_{SB} - w_{SG} = w_{FB} - w_{FG} \), then manipulation incentives would be independent of the benchmark. In a LEN model, the linear restriction imposed on the contract means this effect would not be present. The asymmetric contract can arise in many situations, depending for example on how effort affects covariance or limited liability (as is the case in this study), and is consistent with evidence from archival studies (e.g. Bannister and Newman 2003).
In the following proposition, I turn to the firm’s problem of choosing the optimal contract.

**Proposition 4.** The optimal contract sets \( w_{FB} = w_{FG} = 0 \). Furthermore, there exist thresholds \( g < \hat{g} < g^* \), where \( g = \frac{2c}{p_h - p_l} \), and \( \hat{g} \equiv \frac{8c}{3(p_h - p_l)} \),\(^{14}\) such that:

(i) \( w_{SB} > w_{SG} = 0 \) (max RPE) when \( g \in [g^*, \infty) \),

(ii) \( w_{SB} > w_{SG} > 0 \) (RPE) when \( g \in (\hat{g}, g^*) \),

(iii) \( w_{SG} = w_{SB} \) (IPE) when \( g = \hat{g} \),

(iv) \( w_{SG} > w_{SB} > 0 \) (JPE) when \( g \in (g, \hat{g}) \),

(v) \( w_{SG} - w_{SB} \to 0 \) (asymptotically IPE) when \( g \to g^* \).

The possibility of report manipulation creates a trade-off in RPE usage. On the one hand, RPE is beneficial due to the positive correlation as seen in the benchmark scenario. On the other hand, RPE incentivizes higher manipulation. As a result, when the use of RPE causes a significantly higher manipulation level, the principal will be more inclined to abstain from benchmarking compensation. In fact, this effect can be high enough, such that the principal actually prefers joint performance evaluation (JPE), i.e. \( w_{SB} < w_{SG} \), for some levels of \( g \) that are close enough to \( g^* \), i.e. when manipulation is very cheap. This means that the agent actually earns a higher wage when the competition was also successful. The fact that the agent can observe the benchmark before her manipulation decision is also a disadvantage for JPE and thus favors independent compensation. However, the other key intuition that worked against RPE, works in favor of JPE. The agent wants to report a success when the benchmark is high. This outcome is more likely due to the positive correlation, thus...

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\(^{14}\)The threshold \( g^* \) is defined in the appendix, equations (38) and (39).
effort is more likely to achieve the desired result. On the other hand, manipulation is uncorrelated with the benchmark, and thus, manipulation is not as efficient in reaching the agent’s preferred outcome. Of course, JPE runs counter to using the most informative signal, but when the manager can easily manipulate ($g$ close to $\hat{g}$), then its advantages can outweigh the drawbacks. At $g = \hat{g}$, the forces balance exactly, and independent performance evaluation is optimal.

The likelihood ratios show that the firm uses JPE, even though $y = y_{SB}$ is the most informative signal:

$$\frac{(p_h - \gamma) + (1 - p_h + \gamma)m_B}{(p_l - \gamma) + (1 - p_l + \gamma)m_B} > \frac{(p_h + \gamma) + (1 - p_h - \gamma)m_G}{(p_l + \gamma) + (1 - p_l - \gamma)m_G}. \quad (12)$$

Intuitively, the above equation is true, because when JPE is used, then the signal $y = y_{SG}$ becomes even less informative about effort. The manager manipulates more when the benchmark is good, which results in a steeper decrease in informativeness for $y_{SG}$ compared to $y_{SB}$. The firm nonetheless optimally chooses JPE because it incentivizes less manipulation.

Part (v) of Proposition 4 follows because the principal’s concerns about manipulation start to dominate, and a high use of either RPE or JPE incentivizes heavy manipulation because the agent can wait and observe the outcome of the benchmark. For example, for $w_{SB} > 0$ and $w_{SG} = 0$, she would manipulate heavily when the benchmark is low and not at all when the benchmark is high. Given the convex manipulation cost function, the agent has to be compensated for this manipulation (otherwise she shirks) with a very high payoff that can be avoided by smoothing compensation across states. As a result, in addition to the forces described in the
Notes: The parameters are $p_h = 0.55$, $p_l = 0.45$, $\gamma = 0.4$, $c = 2$, and $q = 0.5$. For these parameters, $g = 40$, $\hat{g} \approx 53$, and $g^* \approx 156$.

previous section, RPE now has an additional disadvantage when the agent can wait and observe the outcome of the benchmark. The agent is now in a better position to efficiently exploit RPE because she will not waste her manipulation effort when it is not as beneficial to her. Figure 2 illustrates the above described observations.

In the figure, $g^* \approx 156$ is the point where $w_{SG}$ becomes zero. For $g \geq g^*$ (not in the figure), when manipulation is very costly, the agent only receives compensation if she reports success and the benchmark is low. The graph shows the main relationship.
As earnings management becomes less costly (moving from right to left on the graph), the firm has to reduce the importance of the benchmark by decreasing the difference between the two wage payments $w_{SB}$ and $w_{SG}$. At the very left end of the graph (when $g$ approaches $g = 40$), incentivizing the agent to work hard becomes tough enough that both wage payments have to increase to accomplish this. For $g < g$, no solution exists. There is also a range, $g \in (40, \frac{160}{3})$, where joint performance evaluation is optimal, i.e. $w_{SG} > w_{SB}$. At $g = \frac{160}{3}$, the two wage payments are identical.

**Comparative statics**

I now turn the analysis to some comparative statics. Less RPE is used when either the range “max RPE” in part (i) of Proposition 4 shrinks (i.e. $g^*$ increases) or the range “RPE” in part (ii) of Proposition 4 gets smaller (i.e. $\hat{g}$ increases). If $g^*$ increases, this means that the firm finds it optimal to keep $w_{SG} > 0$ for an increased interval, thus not using the maximum possible RPE as previously with the lower $g^*$.

**Corollary 1.** RPE is used less as

(i) the covariance $\gamma$ decreases,

(ii) the probability of success $p_h$ decreases, and

(iii) the cost of effort $c$ increases.

A change in correlation has two effects. First, when correlation increases, the advantage of RPE increases, as seen in the proof of the no manipulation scenario (see appendix). The likelihood ratios diverge and it is increasingly easier to incentivize effort using RPE. Second, an increase in correlation increases the difference in
manipulation levels that RPE and independent compensation cause. While earnings management does not change if benchmark-independent pay is used, it does with RPE. This is again owed to the fact that an increase in correlation makes it harder to outperform the competition by regular means, which is why the promised compensation has to increase. This increased compensation in turn causes the increase in manipulation. However, the second effect is lower than the main effect on the informativeness of the signal. Additionally, note that a change in $\gamma$ does not affect the firm’s choice between JPE and RPE. The threshold $\hat{g}$ is independent of $\gamma$.

An increase in $p_h$ is beneficial for RPE because it alleviates the incentive problem. The agent is more likely to succeed if she puts in the effort, and effort matters more because $p_l$ is kept constant, increasing $(p_h - p_l)$. Thus, the principal does not need to offer as much compensation which of course also decreases the manipulation incentives. Also, as already established, a principal that is less concerned about manipulation, will use RPE more heavily. The effect of $c$ can best be understood when the ratio $c/g$ is viewed as the relative cost of effort in relation to the cost of manipulation. Characteristics such as optimal use of RPE and the equilibrium manipulation level are unaffected, as long as $c/g$ stays constant. It is thus not surprising that an increase in $c$ causes a proportional increase in $g^*$.

**Corollary 2.** As manipulation becomes more difficult ($g$ increases),

(i) the expected manipulation goes down,

(ii) the expected wage decreases,

(iii) the agent’s utility decreases.

Part (i) of the corollary can be explained as follows. When manipulation is
more costly to the agent, holding everything else equal, she will manipulate less. A counteracting force here is the fact that with less manipulation, the principal will use more RPE. However, this force is weaker than the first one. If the principal increases RPE too much and causes more, not less, manipulation, then he would again suffer too much from the negative consequences of RPE, and compensation would not be optimal. Hence, RPE will only increase slowly and expected manipulation will decrease.

Part (ii) of the corollary follows because less manipulation makes it easier to incentivize effort. As noted above, the ratio \( c/g \) is important as the agent always weighs her options to achieve the desired result. So when manipulation is harder, the bonus necessary to induce effort declines.

Part (iii) follows because even though the expected wage decreases, it is not immediately obvious that this implies that the agent’s utility also decreases. After all, manipulation decreases and consequently so does the cost of manipulation as well. Why is it not possible that the second effect outweighs the first one? It is instructive to note that the agent can freely choose the manipulation level. When \( g \) decreases, she has the option to maintain the original manipulation levels, \( m_B \) and \( m_G \). This implies that she could keep the manipulation cost down while simply enjoying the increased wage. The only reason why the agent increases manipulation, is because it increases her overall utility.

\textit{Model robustness}

In this section, I discuss the model’s robustness.
A manipulated benchmark

Thus far I have assumed that the benchmark is completely exogenous. However, a reasonable argument can be made that executives at other companies can manage earnings as well. In this subsection I analyze how the optimal contract and the agent’s behavior changes in response to a change in $q$, the probability that the benchmark is good. While this does not endogenize the benchmark completely, it is a reasonable approximation in situations when each individual firm is only a small part of the peer-group. When that is the case, each individual manager’s actions will have negligible effects on other firms’ decisions. A higher $q$ can be a stand-in for other firms also manipulating their earnings, thus causing the benchmark to be higher more frequently.

The most direct and obvious effect of an increase in $q$ (when other firms manipulate more) is that it becomes harder to outperform the benchmark. The probability that the agent is successful while the benchmark is low decreases. This in turn requires an increased bonus, if one wants to use RPE, to compensate for the decreased likelihood of achieving the bonus. That bigger bonus, however, incentivizes more manipulation, and the board then uses less RPE.

**Corollary 3.** RPE is used less ($g^*$ increases) as the probability of a high benchmark, $q$, increases.

Risk aversion and timing

Suppose now that the manager is risk-averse and has to make the manipulation decision $m$ before observing the benchmark, i.e. $m = m_B = m_G$. Note that I
keep the limited liability constraint. Numerical analysis in the appendix shows that interior solutions similar to the main findings in the paper prevail again. When the cost of manipulation is above a certain threshold, then RPE is optimal, and JPE is optimal when $g$ is below that threshold. The changed model differs from the main result at the extremes. IPE does not become optimal as $g \to g^*$, and we do not observe “max RPE”, i.e. $w_{SG} = 0$, when $g$ grows large. This is driven by the manager’s risk-aversion.

6. Conclusion

This paper analyzes the effect of earnings management on the desirability of using benchmarks such as a stock market index to determine a CEO’s compensation. When the CEO cannot misreport, using RPE is optimal because it filters industry- and economy-wide shocks and allows for a more accurate measure of the CEO’s performance. High firm performance when the market performed poorly is a more informative signal than good performance when the economy is running hot. This insight would predict that RPE should be widespread in the business world. However, empirical research shows that this is not the case. I show that one potential explanation for this can be earnings management. Manipulation makes it easier for a CEO to report success when economic conditions are poor. Additionally, when the CEO can observe the benchmark before her earnings management decision, then RPE increases manipulation incentives even further because the CEO will manipulate more when it is more advantageous for her. This gives the CEO an efficient way to boost her expected compensation, and thus makes it harder to incentivize effort.
Ultimately, the firm will anticipate the CEO’s incentives to manipulate earnings and will switch to a compensation scheme that depends less on an external benchmark in an effort to curb manipulation. Hence, the firm trades off the benefit of rewarding the CEO for the most informative signal (i.e. project success and low benchmark) with the drawback of incentivizing manipulation. This result can explain why RPE is not as heavily used as predicted. For some companies, the benefit of benchmarking the company’s performance (e.g., against a stock market index) will not be large enough to justify the expected increase in earnings management.

The paper has several limitations which provide opportunities for possible future research. First, I only provide a numerical example to show the effects of risk aversion and a changed timeline on the optimal contract. A more extensive, thorough analysis could provide further insights. Second, my model only has a single period. A multi-period setting could conceivably constrain the CEO’s earnings management. Managing earnings upward for multiple periods is difficult when there is an economic downturn. This could allow for more RPE use. Finally, the possibility of earnings management is of course just one of several factors that a compensation committee has to consider when constructing a contract. The analysis in this paper is silent on which factors dominate in practice.
Appendix

Proof for Proposition 1

The agent’s effort incentive constraint is

\[
(p_h + \gamma)q w_{SG} + (p_h - \gamma)(1 - q)w_{SB} + (1 - p_h - \gamma)q w_{FG} \\
+ (1 - p_h + \gamma)(1 - q)w_{FB} - c \geq (p_l + \gamma)q w_{SG} + (p_l - \gamma)(1 - q)w_{SB} \\
+ (1 - p_l - \gamma)q w_{FG} + (1 - p_l + \gamma)(1 - q)w_{FB}.
\] (13)

This can be rearranged to

\[
q(w_{SG} - w_{FG}) + (1 - q)(w_{SB} - w_{FB}) \geq \frac{c}{p_h - p_l}.
\] (14)

From this expression, it is clear that \(w_{FG} = w_{FB} = 0\) is optimal. As a result, the principal solves the following simplified problem:

\[
\min_{w_{SB}, w_{SG}} (p_h - \gamma)(1 - q)w_{SB} + (p_h + \gamma)q w_{SG}
\] (15)

s.t.
\[
q w_{SG} + (1 - q)w_{SB} = \frac{c}{p_h - p_l}.
\] (16)

Since both the objective function and the constraint are linear in the arguments, the solution will be a corner solution. Hence, I simply examine both possible solutions.

Case 1: \(w_{SG} = 0\)

\[
w_{SB} = \frac{c}{(1 - q)(p_h - p_l)},
\] (17)
\[ E[w] = \frac{(p_h - \gamma)c}{p_h - p_l}. \] (18)

**Case 2:** \( w_{SB} = 0 \)

\[ w_{SG} = \frac{c}{q(p_h - p_l)}. \] (19)

\[ E[w] = \frac{(p_h + \gamma)c}{p_h - p_l}. \] (20)

Comparing the two expressions for the expected wage shows

\[ \frac{(p_h - \gamma)c}{(p_h - p_l)} < \frac{(p_h + \gamma)c}{(p_h - p_l)}, \] (21)

because \( \gamma > 0 \) and therefore \( w_{SB} > w_{SG} \) is optimal, proving Proposition 1.

I now briefly describe the proof for the risk-aversion case, which shows the trade-off that I mentioned in the body of the paper. The principal’s problem can be stated in the following Lagrange form.

\[ \begin{align*}
\max_{w_{SB}, w_{SG}} L &= -(p_h - \gamma)(1 - q)w_{SB} - (p_h + \gamma)qw_{SG} \\
&\quad + \lambda \left( qu(w_{SG}) + (1 - q)u(w_{SB}) - \frac{c}{p_h - p_l} \right)
\end{align*} \] (22)

Taking the derivatives with respect to the arguments yields

\[ \frac{dL}{dw_{SB}} = -(p_h - \gamma)(1 - q) + \lambda(1 - q)u'(w_{SB}) = 0 \] (23)

\[ \frac{dL}{dw_{SG}} = -(p_h + \gamma)q + \lambda qu'(w_{SG}) = 0. \] (24)
After solving both for $\lambda$ and using algebra, I arrive at the following expression:

\[
\frac{u'(w_{SG})}{u'(w_{SB})} = \frac{p_h + \gamma}{p_h - \gamma} > 1.
\] (25)

This fraction shows that $w_{SB} > w_{SG}$ given the standard features of a risk-averse utility function.

**Proof of Proposition 2**

The agent’s utility function for high effort is

\[
U = (p_h + \gamma)qw_{SG} + (p_h - \gamma)(1 - q)w_{SB} + (1 - p_h - \gamma)q(m_G w_{SG} + (1 - m_G)w_{FG}) \\
+ (1 - p_h + \gamma)(1 - q)(m_B w_{SB} + (1 - m_B)w_{FB}) - c \\
- (1 - p_h - \gamma)q\frac{1}{2}gm_G^2 - (1 - p_h + \gamma)(1 - q)\frac{1}{2}gm_B^2.
\] (26)

This, together with the utility for low effort, can be used to construct the incentive constraint:

\[
q \left( (1 - m_G) (w_{SG} - w_{FG}) + \frac{1}{2}gm_G^2 \right) \\
+ (1 - q) \left( (1 - m_B) (w_{SB} - w_{FB}) + \frac{1}{2}gm_B^2 \right) \geq \frac{c}{p_h - p_l}.
\] (27)

Since $w_{FG}$ and $w_{FB}$ will not incentivize effort, they will be zero. This simplifies the above equation.

To get the manipulation decision, I take the first derivative of $U$ with respect to
\[ m_B \text{ and } m_G \text{ respectively:}^{15} \]

\[ \frac{dU}{dm_B} = w_{SB} - g m_B = 0, \quad (28) \]

\[ \frac{d}{dm_G} = w_{SG} - g m_G = 0. \quad (29) \]

Substituting (28) and (29) into (27), and solving for \( w_{SG} \) yields

\[ w_{SG} = g(1 - \sqrt{1 - \frac{2c}{gq} \left(\frac{c}{ph - pl} - (1 - q)((1 - \frac{w_{SB}}{g})w_{SB} + \frac{w_{SB}^2}{2g})\right)}). \quad (30) \]

Substituting all of the above generated insights into (10) gives

\[ E[m] = \frac{w_{SB}}{g}(1 - ph + \gamma)(1 - q) \]
\[ + (1 - \sqrt{1 - \frac{2c}{gq} \left(\frac{c}{ph - pl} - (1 - q)((1 - \frac{w_{SB}}{g})w_{SB} + \frac{w_{SB}^2}{2g})\right)})(1 - ph - \gamma)q. \quad (31) \]

Taking the derivative of \( E[m] \) with respect to \( w_{SB} \), and substituting (30) back in for simplicity results in

\[ \frac{dE[m]}{dw_{SB}} = \frac{1}{g}(1 - ph + \gamma)(1 - q) - \frac{1}{g}(1 - ph - \gamma)\frac{g - w_{SB}}{g - w_{SG}}(1 - q). \quad (32) \]

If \( w_{SB} = w_{SG} \), then the equation simplifies to \( \gamma > 0 \). If \( w_{SB} > w_{SG} \), then the derivative is even greater, hence proving the first part of Proposition 2.

To solve for the minimum, I set the equation equal to zero. To check, whether

\[ ^{15}\text{Taking the first-order condition of the agent’s utility function at the time of the manipulation decision yields the same manipulation constraints.} \]
the solution is a corner solution, I set $w_{SB} = 0$, and solve for $w_{SG}$. I eventually arrive at the following condition for a corner solution

$$g > g_T = \frac{c(1 - p_h + \gamma)^2}{2 \gamma q (p_h - p_l) (1 - p_h)},$$

proving the second part of Proposition 2.

**Proof of Proposition 3**

When $w_{SB} \neq w_{SG}$, then (28) and (29) show that when the benchmark is observable, the optimal manipulation choice is $m_B \neq m_G$. Hence, the restriction imposed on the manipulation decision by an unobservable benchmark that $m_B = m_G$ must decrease the agent’s utility.

**Proof of Proposition 4**

The principal’s problem can be stated in Lagrange form:

$$\max_{w_{SG}, w_{SB}, m_G, m_B} L = -((p_h + \gamma)q w_{SG} + (p_h - \gamma)(1 - q)w_{SB} + (1 - p_h - \gamma)q m_G w_{SG}$$

$$+ (1 - p_h + \gamma)(1 - q)m_B w_{SB}) + \lambda((1 - m_G)w_{SG} + \frac{g m_G^2}{2})$$

$$+ (1 - q)((1 - m_B)w_{SB} + \frac{g m_B^2}{2} - \frac{c}{(p_h - p_l)}) + \mu(w_{SB} - g m_B)$$

$$+ \nu(w_{SG} - g m_G).$$

(34)
The principal maximizes his profit subject to the agent’s IC constraint (Lagrange multiplier \( \lambda \)), and the agent’s two manipulation decision constraints (Lagrange multipliers \( \mu \) and \( \nu \)). The individual rationality constraint is slack. The agent’s outside option is zero (by assumption), which she is guaranteed to achieve inside the company simply by not working hard and not engaging in manipulative activities.

**Proof of part (i)**

Assume that the solution will be \( w_{SB} > 0 \) and \( w_{SG} = 0 \). Using this, (27), and (28), I get

\[
2(1 - m_B)gm_B + gm_B^2 = \frac{2c}{(1 - q)(p_h - p_l)} \quad \text{and} \quad (35)
\]

\[
w_{SB} = (g - \sqrt{\frac{g}{(1 - q)(p_h - p_l)} (g(1 - q)(p_h - p_l) - 2c)}).
\]

These equations can be used to solve for the Lagrangian multipliers. Importantly,

\[
\lambda = \frac{(p_h - \gamma)g + 2(1 - p_h + \gamma)w_{SB}}{g - w_{SB}} \quad \text{and} \quad \nu = 0. \quad (37)
\]

Then, I check whether \( dL/dw_{SG} < 0 \) is indeed satisfied: \( \lambda + \nu < p_h + \gamma \) reduces after algebra to

\[
g > g^* \equiv \frac{2c}{(1 - q)(p_h - p_l)(1 - b^2)},
\]

where \( b = 1 - \frac{2\gamma}{2 - p_h + 3\gamma} < 1 \),

thus proving part (i) of Proposition 4.
Proof of parts (ii), (iii), and (iv)

Assume that the solution will be \( w_{SB} = w_{SG} = w \). Using simplified notation \((m_B = m_G = m)\), I get the following results:

\[
w = gm \quad \text{and} \quad m = 1 - \sqrt{1 - \frac{2c}{g(p_h - p_l)}}. \quad (40)
\]

Again, I check whether \( dL/dw_{SG} = dL/dw_{SB} = 0 \) is indeed satisfied. The four Lagrangian derivatives are

\[
\frac{dL}{dw_{SB}} = -(1-q)(m_B(1-p_h+\gamma)+p_h-\gamma)+\lambda(1-m_B)(1-q)+\mu = 0, \quad (41)
\]

\[
\frac{dL}{dw_{SG}} = -q(m_G(1-p_h-\gamma)+p_h+\gamma)+\lambda q(1-m_G)+\nu = 0, \quad (42)
\]

\[
\frac{dL}{dm_B} = -w_{SB}(1-q)(1-p_h+\gamma)-\lambda(w_{SB}-gm_B)(1-q) - g\mu = 0, \quad (43)
\]

\[
\frac{dL}{dm_G} = -qw_{SG}(1-p_h-\gamma)-q\lambda(w_{SG}-gm_G)-g\nu = 0. \quad (44)
\]

Solving for the Lagrangian multipliers \( \nu \) and \( \mu \) reduces the system to two equations:

\[
m_B(1-p_h+\gamma)+p_h-\gamma-\lambda(1-m_B) = (-w_{SB}(1-p_h+\gamma)-\lambda(w_{SB}-gm_B))\frac{1}{g}, \quad (45)
\]

\[
m_G(1-p_h-\gamma)+p_h+\gamma-\lambda(1-m_G) = (-w_{SG}(1-p_h-\gamma)-\lambda(w_{SG}-gm_G))\frac{1}{g}. \quad (46)
\]
Substituting the above expressions for wage and manipulation, solving for $\lambda$, and reducing the system to one equation, yields

$$m = \frac{1}{2} \text{ and } g = \hat{g} \equiv \frac{8c}{3(p_h - p_l)},$$  

proving part (iii) of Proposition 4.

Additionally, one can show that for $g > \hat{g}$, it is the case that with benchmark-independent compensation, $dL/dw_{SB} > 0$, and $dL/dw_{SG} < 0$. And if $g < \hat{g}$, then vice versa. This shows that for $g > \hat{g}$, RPE is optimal, while for $g < \hat{g}$, JPE is optimal, proving parts (ii) and (iv) of Proposition 4. The proof can also be obtained by invoking the intermediate value theorem, because the functions are continuous.

**Proof of part (v)**

For this proof, I will show that when $g \equiv 2c/(p_h - p_l)$, the only (asymptotically) feasible compensation structure is IPE. Assume that the (only feasible) solution is $w_{SG} = w_{SB} = g = 2c/(p_h - p_l)$. Substituting these values into equation (30) shows that this contract satisfies the incentive constraint with equality.

Substituting the above values into equations (28) and (29) yields $m_B = m_G = 1$. Due to the negative relationship between $w_{SG}$ and $w_{SB}$, any deviation from IPE would result in either the invalid $m_B > 1$ or the invalid $m_G > 1$, proving part (v) of the proposition that IPE is the only feasible solution.
**Proof of Corollary 1**

The use of RPE can be analyzed with the cutoff values $\hat{g}$ and $g^*$. An increase in $g^*$ means that RPE is used less because the range “max RPE” (part (v) in Proposition 4) gets smaller. An increase in $\hat{g}$ means that RPE is also used less because the range “RPE” (part (iv) in Proposition 4) gets smaller.

Take the above derived formula for $g^*$,

$$g^* = \frac{2c}{(1-q)(p_h - p_l)(1-b^2)},$$

where $b = 1 - \frac{2\gamma}{2 - p_h + 3\gamma} < 1$. 

It is immediately clear that an increase in $b$ causes an increase in $g^*$. Thus, it is sufficient to take the derivative of $b$ with respect to $\gamma$:

$$\frac{db}{d\gamma} = -\frac{2(2-p_h)}{(3\gamma - p_h + 2)^2} < 0.$$  

Because $\hat{g}$ is not affected by a change in $\gamma$, this completes the proof of part (i).

I can do the same for $p_h$:

$$\frac{db}{dp_h} = -\frac{2\gamma}{(3\gamma - p_h + 2)^2} < 0.$$  

There is another $p_h$ term in the formula for $g^*$, but the effect of a change there has the same direction as the effect via $b$ (that is, increasing the denominator), and hence the inverse relationship holds; $\hat{g}$ and $p_h$ are also inversely related, thus completing
the proof of part (ii).

The third part of the corollary can be shown directly:

\[
\frac{dg^*}{dc} = \frac{2}{(1 - q)(p_h - p_l)(1 - b^2)} > 0. \tag{52}
\]

An increase in \(c\) also causes \(\dot{g}\) to increase, thus the effect is again unambiguous, completing the proof of part (iii) of Corollary 1.

**Proof of Corollary 2**

Substituting (28) and (29) into the IC constraint (27) gives the following constraint:

\[
q((1 - m_G)gm_G + \frac{1}{2}gm^2_G) + (1 - q)((1 - m_B)gm_B + \frac{1}{2}gm^2_B) = \frac{c}{(p_h - p_l)}. \tag{53}
\]

Since this constraint must hold with equality, there must be an inverse relationship between \(m\) and \(g\).

To show (ii), assume by contradiction that the agent will ex-post decide not to manipulate. Proving that expected wage declines in this case, is sufficient proof, because manipulation is declining in \(g\) as well. Thus, I want to show that

\[
E[w] = (p_h - \gamma)(1 - q)w_{SB} + (p_h + \gamma)qw_{SG} \tag{54}
\]

is declining in \(g\). Solving (30) for \(w_{SB}\) and substituting it into (54) expresses expected
wage as a function of $w_{SG}$ alone.

$$E[w] = (p_h - \gamma)(1 - q)g \left(1 - \sqrt{\frac{1}{1-q} \left(1 - \frac{2c}{g(p_h - p_l)} - q \left(1 - \frac{w_{SG}}{g} \right)^2\right)}\right)$$

$$+ (p_h + \gamma)qw_{SG}$$

(55)

Since $dw_{SG}/dg < 0$, it is sufficient to show that $dE[w]/dw_{SG} < 0$ to prove part (ii) of the corollary.

The proof of (iii) is explained in the text below the proposition. The agent could keep her manipulation decision constant and simply enjoy a higher rent. Any change in manipulation level must be to the benefit of the agent.

**Proof of Corollary 3**

Taking the derivative of (48) with respect to $q$ yields

$$\frac{dg^*}{dq} = \frac{2c}{(p_h - p_l)(1 - b^2)(1 - q)^2} > 0,$$

(56)

proving the corollary ($\hat{g}$ is unaffected by a change in $q$).

**Numerical example for risk aversion and alternate timeline**

Let the manager’s utility function be $U = 1 - e^{-w}$, and assume she has to provide effort and manipulate simultaneously. The manager’s utility function for high effort
and risk aversion is now

\[ U = (p_h + \gamma)q (1 - e^{-w_{SG}}) + (p_h - \gamma)(1 - q) (1 - e^{-w_{SB}}) \]

\[ + (1 - p_h - \gamma)qm (1 - e^{-w_{SG}}) + (1 - p_h - \gamma) q (1 - m) (1 - e^{-w_{FG}}) \]

\[ + (1 - p_h + \gamma) (1 - q)m (1 - e^{-w_{SB}}) - c \]

\[ + (1 - p_h + \gamma) (1 - q) (1 - m) (1 - e^{-w_{FB}}) - (1 - p_h + \gamma (1 - 2q)) \frac{1}{2} gm^2. \]

Setting up the IC constraint and simplifying yields:

\[ q (1 - m) (e^{-w_{FG}} - e^{-w_{SG}}) \]

\[ + (1 - q) (1 - m) (e^{-w_{FB}} - e^{-w_{SB}}) + \frac{1}{2} gm^2 \geq \frac{c}{(p_h - p_l)}. \]

Thus, we can observe that it is still optimal to choose \( w_{FG} = w_{FB} = 0 \).

The manipulation constraint yields

\[ m = \frac{(1 - p_h - \gamma)q (1 - e^{-w_{SG}}) + (1 - p_h + \gamma) (1 - q)(1 - e^{-w_{SB}})}{(1 - p_h + \gamma (1 - 2q)) g}, \]

and the expected wage \( E[w] \) is

\[ E[w] = (p_h + \gamma)qw_{SG} + (p_h - \gamma)(1 - q)w_{SB} \]

\[ + (1 - p_h - \gamma)qm w_{SG} + (1 - p_h + \gamma)(1 - q)m w_{SB}. \]

From here, I proceed with a numerical example illustrating the new optimal contract. For the parameters \( c = 0.05, p_h = 0.6, p_l = 0.1, q = 0.5, \gamma = 0.01 \), the optimal contract, manipulation level, and expected wage cost for different levels of
the cost of manipulation, \( g \), are illustrated in Table 2.

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<th>( w_{SB} )</th>
<th>( w_{SG} )</th>
<th>( m )</th>
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References


