A Benders decomposition approach for a real case supply chain network design with capacity acquisition and transporter planning: wheat distribution network

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Abstract

This paper considers a real case problem of supply chain network design inspired from a wheat distribution network in Iran. It generates a network with capacity acquisition and fleet management. The problem first is formulated as a mixed integer linear programming model. Then, a logic-based Benders decomposition algorithm is appropriately developed as the solution methodology. In the presented algorithm, the problem is decomposed into two models of master and subproblem. The master problem is improved by means of the preprocessing and valid inequalities. Moreover, three Benders cuts, one optimality and two feasibility cuts, are developed for the algorithm. The general and relative performance of the model and algorithm is experimentally evaluated. A wheat distribution system of Iran is considered here as the case study of this research. The model is developed based on Iran’s wheat distribution system. All the results show that the algorithm significantly outperforms the mathematical model of the case study. For example, the algorithm solves 95% of the tested instances to optimality, yet the model solves 29%.

Keywords: Supply chain network design; transporter planning; logic-based Benders decomposition algorithm; Benders cut; mathematical model.

1. Introduction

Supply chain management is viewed as coordinating the flow of goods, information, and finances from origin to consumption areas. To build practical supply chains, industries expect academia to investigate innovative extensions and to effectively solve them (Govindan et al., 2015). Supply chain network design (SCND) is to determine the physical configuration and infrastructure of the supply chain (Boloori and Farahani,
More precisely, SCND is the practice of locating facilities within the supply chain, determining the capacity of these facilities, and finally determining how to source customer demand through the network while satisfying the required service level of customers at the lowest cost.

The classical SCND assists companies in responding to two types of decisions: capital asset management (location problem) and distribution management (allocation problem). These two problems involve decisions of location and capacity of facilities to match the demands. Typically, the candidate locations for facilities are discrete. The selection of the sites where new facilities are to be established is restricted to a finite set of available candidate locations. Establishing a facility in each location brings a different setup cost to the chain. Moreover, the literature assumes either uncapacitated or capacitated facilities (Farahani et al., 2014). If the facilities are uncapacitated, each customer is allocated to the open facility that minimizes assignment cost. Yet, in the capacitated case, the closest-assignment rule is no longer applicable; multiple choices for capacity may be presumed. In other words, it is possible to extend the capacity of a facility by a higher setup cost (Verter and Dincer, 1995; Mazzola and Alan, 1999; Sadjady and Davoudpour, 2012).

Furthermore, fleet management is another closely related problem to these two decisions in SCND problems. It involves selecting transportation modes and capacity utilization to develop a cost-effective network (Cordeau et al., 2006; Kazami and Szmerekovsky, 2015). In this study, the aim of fleet management is to determine the minimum number of the transporters required to perform all transportations.

From a practical perspective, bread plays a vital role in the regular nutrition of many countries, especially in subsistence economies such as Iran, India, and so forth (Khera, 2011). Therefore, studying and improving the wheat collection and distribution systems is absolutely substantial and essential. However, this subject is neglected in the literature and to the best of our knowledge, no publication exists in the field of collection and distribution of wheat or wheat products. Consequently, in this study, the wheat distribution system is selected to be widely studied, formulated, and solved.

In terms of solution methodology, Benders decomposition algorithms (BDA), an exact solution method, are used to solve integer optimization problems. For instance, BDAs have been recently assigned to an orthogonal stock cutting problem by Delorme et
al. (2017), a charging station location problem by Arslan and Karaşan (2016), a multi-mode outpatient scheduling problem by Riise et al. (2016), an aggregate production planning problem by Makui et al. (2016), a fixed-charge multi commodity network design problem by Fakhri and Ghatee (2016), and a closed-loop supply chain network design by Keyvanshokooh et al. (2016). BDAs have been developed for SCND problems as well, including Ernesto et al. (2013) for reverse supply chain network design; Pishvaee et al. (2014) for sustainable supply chain network design under uncertainty; Khatami et al. (2015) for the closed loop supply chain network design with demand and return uncertainty; Shaw et al. (2016) for green SCND with carbon emission restriction; and Jeihoonian et al. (2016) for closed-loop supply chain network design with durable products of different quality levels. Finally, Marufuzzaman and DuniEkşioğlu (2017) utilized a hybrid rolling horizon and Benders decomposition for a multi-period production-distribution problem.

More recently, an extension of BDA, called logic-based Benders decomposition algorithm (LBBDA), has been introduced. This extension overcomes some limitations of conventional BDAs. For example, the BDA requires one of its decomposed problems (called subproblem) to be a linear program, while in LBBDA, the decomposed problems can be in any form of mathematical programs. LBBDAs have shown high performance in solving mixed integer programming problems. See, for example, the machine scheduling problem by Hooker (2007), capacity and distance constrained plant location problem by Zarandi and Beck (2011), the parallel machine scheduling by Tran et al. (2016), an inventory-location problem with service constraints by Wheatley et al. (2015), and the distributed operating room scheduling problem by Roshanaei et al. (2016).

The characteristics of the developed problem of this study are defined based on a real case of a wheat distribution network in Iran. This network comes out the problem of supply chain network design with transporter management where facilities have flexible capacity. Therefore, a mixed integer linear programming model is presented and then a logic-based Benders decomposition algorithm is developed in order to solve the problem. The problem is decomposed into two models of master and subproblem. The master problem is improved by means of the preprocessing and valid inequalities. Moreover, three Benders cuts, one optimality and two feasibility cuts, are developed for the algorithm.
The rest of the paper is organized as follows: The related literature review is analyzed in Section 2. Section 3 defines the real case of this study. Section 4 formulates the problem mathematically. Section 5 develops a logic-based Benders decomposition algorithm. Section 6.1 evaluates the performance of the algorithm. Further, Section 6.2 also considers the results of the real case study of a wheat network distribution system and, finally, Section 7 concludes the paper.

2. Literature review

The SCND problem, even in its simplest format, is NP-hard (Gourdin et al., 2000). Therefore, developing solution algorithms for different variants of the problem is always an interesting and challenging topic for researchers. Due to the difficulty of these problems, heuristic and metaheuristic algorithms are commonly used to solve the SCND and its extensions. Pirkul and Jayaraman (1998) study SCND by locating a given number of distribution centers. They mathematically formulate the problem as a mixed integer linear programming model and present a heuristic procedure based on Lagrangian relaxation. Amiri (2006) considers the SCND problem with multiple levels of capacities available to facilities and develops another heuristic based on Lagrangian relaxation algorithm. Poudel et al. (2016) study biomass co-firing supply chain network under feedstock supply uncertainty, and they present a hybrid algorithm based on sample average approximation and a progressive hedging algorithm. Ardalan et al. (2016) study the SCND problem with multi-mode demand policies and develop a Lagrangian relaxation algorithm for the problem.

Eskandarpour et al. (2016) define an SCND problem with several transportation modes and propose a large neighborhood search technique hybridized with a greedy heuristic. Zhang et al. (2016) propose an artificial bee colony metaheuristic for the SCND problem with multiple distribution channels. Genetic algorithms are also developed for different variants of SCND problems by Afrouzy et al. (2016), Huang et al. (2016), Dai and Zheng (2015), and Robles et al. (2016). For the closed loop SCND problem, Soleimani and Kannan (2015), Subramanian et al. (2013), and Devika et al. (2012) develop a hybrid particle swarm optimization, a simulated annealing, and an imperialist competitive algorithm, respectively. A Tabu search, a genetic algorithm, and an ant colony algorithm
are also proposed by Melo et al. (2012), Soleimani et al. (2016), and Zohal and Soleimani (2016), respectively, for redesigning a multi-echelon supply chain network.

Regarding fleet management in supply chain, Anily and Bramel (1999) review publications of integrated transporter routing and supply chain problems before 1999. Schmid et al. (2013) mathematically formulate different extensions of routing problems with different aspects of supply chain management. The extensions are lot-sizing, scheduling, packing, batching, inventory, and intermodality. Hajghasem and Abbasshojaie (2016) study fleet management in a supply chain where the location-allocation is known in advance. They assume there is a limited number of transporters available and rental transporters can be used if required. Marufuzzaman and DuniEkşioğlu (2017) consider a real case from the biomass supply chain network in USA with dynamic freight routing. Mostafa and Eltawil (2015) consider an inventory model in production-distribution problems with transporter planning. Zhalechian et al. (2016) study another inventory model in a sustainable fuzzy closed loop supply chain problem with routing aspects. Ultimately, it must be mentioned that a wheat distribution system has not yet been studied by operations research scholars.

For the wheat distribution network, Mogale et al. (2017) study a multi-period multi-modal bulk wheat transportation and storage problem in a two-stage supply chain network. They develop a mixed integer, non-linear programming mode and a metaheuristic called chemical reaction optimization. Gholamian and Taghazadeh (2017) consider a real case multi-period wheat distribution network. They develop a mathematical model and solve it by commercial optimization software. We generalize these two papers by considering the fleet management. We develop a mixed integer linear program as well as a logic-based Benders decomposition to solve the problem.

3. Wheat distribution network in Iran

Despite large domestic production of wheat, Iran usually comes out as one of the world’s biggest importers of wheat. Iran is the 8th biggest annual consumer and the 12th leading producer of wheat in the world. Moreover, Iran is among the world’s top four importers of wheat. Iran’s annual wheat consumption is 2.5 times more than that of the world average, according to the latest data published by the UN’s Food and Agriculture
Organization (FAO). These statistics validate how strategic and costly the wheat distribution network is in Iran.

On one hand, wheat cannot grow in all weather conditions. For example, it grows best when temperatures are warm (from 21° to 24° C) with a lot of sunshine (Hatfield and Prueger, 2015). Areas with low humidity are better since many wheat diseases thrive in damp weather. Wheat needs 31 to 38 centimeters of water to produce a good crop (Agriculture and Horticulture Development Board, 2018). On the other hand, Iran is a country with different climate types and large temperature variations. The weather can vary from hot to cold and from dry to even wet depending on the region and the time of the year. Thus, some regions of Iran are more suitable for wheat production. Clearly, wheat cannot evenly grow in all regions of Iran and in all the time periods of the year. Figure 1 Part (a) shows the regional crop production in Iran. Besides the weather variety, the population is not evenly spread throughout the country; some regions are more densely populated. Figure 1 Part (b) shows the regional population density in Iran.

Looking into the geographical crop production and the regional population density (shown in Figure 1), a large volume of wheat must move across Iran from production regions to consumption areas, and this requirement creates one of the largest distribution networks in the world. In this network, central silos play the role of distribution centers. There are some candidate locations to establish these central silos, and the silos can be of different sizes. Obviously, more capacity brings more setup cost. The problem is to determine the locations of central silos and their capacity mode along with the assignment of suppliers (production regions) and customers (consumption areas) to distribution centers. One integrated aspect needed to design this distribution network is the fleet management required, specifically the number of transporters needed to carry the wheat.

The wheat distribution network generates a supply chain network design with capacity acquisition and transporter planning. This problem holds the following features as well. The demand of each customer and the capacity of each supplier are known. There is a set of capacity modes for each candidate location for distribution centers. The network is single source, which means each customer can only be assigned to one single distribution center. Each transporter can be assigned to one distribution center and follows a full return trip from distribution centers to suppliers/customers. This type of transportation is called the full truckload (Brown and Graves, 1981; Gronalt et al., 2003).
a) Regional crops production  
b) Regional population density

Figure 1. Regional distribution of population and crops production in Iran
4. Problem formulation

Before presenting the mathematical formulation of the network, a schematic view of the wheat distribution system of Iran, considered in this study, is illustrated in Figure 2. The wheat crops produce wheat as the suppliers of the system; then, the harvested wheats are forwarded to the silos as the main distribution centers of the network. Finally, the appropriate fleets play the important role of supplying wheat flour manufacturers at the right time, quantity, and location. The following assumptions are established.

- The network includes suppliers, distributors, and customers.
- The demand and production capacity of each node is known.
- Each transporter is limited to a maximum travel distance.
- The distribution centers and transporters are capacitated.
- The establishment costs vary in different nodes.

Therefore, based on the wheat distribution system of Iran, presented in Figure 2, the mathematical model of the problem under consideration is as follows. Before presenting the model, the following notations are defined.

Parameters and indices:

\[ n \quad \text{The number of suppliers } i = \{1, 2, \ldots, n\} \]
\[ b \quad \text{The number of possible distribution centers } k = \{1, 2, \ldots, b\} \]
\[ m \quad \text{The number of customers } j = \{1, 2, \ldots, m\} \]
\[ v \quad \text{The number of transporters } l = \{1, 2, \ldots, v\} \]
\[ a_k \quad \text{The number of capacity modes for distribution centers } k, t = \{1, 2, \ldots, a_k\} \]
\( d_j \)  The demand of customer \( j \)

\( f_{k,t} \)  The fixed cost of distribution center \( k \) in mode \( t \).

\( s \)  The fixed cost of each transporter

\( g_i \)  The capacity of supplier \( i \)

\( e_{k,t} \)  The capacity of distribution center \( k \) in mode \( t \).

\( h \)  The maximum travel distance of each transporter

\( t_{i,k} \)  The distance between supplier \( i \) and distribution center \( k \)

\( p_{j,k} \)  The distance between distribution center \( k \) and customer \( j \)

\( c_{i,k} \)  The unit transportation cost of between supplier \( i \) and distribution center \( k \)

\( r_{j,k} \)  The unit transportation cost of between distribution center \( k \) and customer \( j \)

Decision variables:

\( X_{i,k,l} \)  Binary variable taking value 1 if supplier \( i \) is assigned to distribution center \( k \) by transporter \( l \); and 0 otherwise.

\( Y_{j,k,l} \)  Binary variable taking value 1 if customer \( j \) is assigned to distribution center \( k \) to by transporter \( l \); and 0 otherwise.

\( Z_{k,t} \)  Binary variable taking value 1 if distribution center \( k \) in mode \( t \) is open.

\( W_{k,l} \)  Binary variable taking value 1 if transporter \( l \) is assigned to distribution center \( k \).

\( H_{i,k} \)  Continuous variable for flow from supplier \( i \) to distribution center \( k \).

The mathematical model is as follows.

Objective:

\[
\text{Min } \sum_{k=1}^{a_k} \sum_{t=1}^{b_k} f_{k,t} Z_{k,t} + \sum_{i=1}^{b_i} \sum_{k=1}^{a_k} c_{i,k} H_{i,k} + \sum_{k=1}^{b_k} \sum_{j=1}^{a_j} \sum_{l=1}^{b_l} r_{j,k} d_{j} Y_{j,k,l} + \sum_{k=1}^{b_k} \sum_{l=1}^{b_l} s W_{k,l}
\]

Subject to:

\[
\sum_{t=1}^{a_k} Z_{k,t} \leq 1 \quad \forall k \quad (1)
\]

\[
\sum_{k=1}^{b_k} \sum_{l=1}^{b_l} Y_{j,k,l} = 1 \quad \forall j \quad (2)
\]

\[
\sum_{k=1}^{b_k} \sum_{l=1}^{b_l} X_{i,k,l} \leq 1 \quad \forall i \quad (3)
\]

\[
\sum_{j=1}^{a_j} \sum_{l=1}^{b_l} d_{j} Y_{j,k,l} \leq \sum_{t=1}^{a_k} e_{k,t} Z_{k,t} \quad \forall k \quad (4)
\]
\[
\sum_{i=1}^{m} \sum_{l=1}^{n} d_i Y_{i,j,k,l} \leq \sum_{l=1}^{n} H_{i,k} \quad \forall_k \\
H_{i,k} \leq \sum_{l=1}^{n} g_l X_{i,k,l} \quad \forall_{i,k} \\
\sum_{k=1}^{b} W_{k,l} \leq 1 \quad \forall_l \\
\sum_{l=1}^{n} t_{i,k} X_{i,k,l} + \sum_{j=1}^{m} p_{j,k} Y_{j,k,l} \leq h W_{k,l} \quad \forall_{k,l} \\
0 \leq H_{i,k} \leq g_l \quad \forall_{i,l,k} \\
X_{i,k,l}, Y_{j,k,l}, Z_{k,l}, W_{k,l} \in \{0, 1\}
\]

Objective is to minimize the total cost that includes setup cost of distribution centers, the assignment cost of both suppliers and customers to distribution centers, and the fixed cost of transporters. Constraint set (1) is to make sure at most one of the capacity modes for each candidate distribution center site is selected. Constraint sets (2) and (3) assures the single source assignment. That is, each customer and supplier is assigned to a distribution center and transporter. Constraint sets (4), (5) and (6) ensure the capacity limitation of distribution centers and suppliers are met. Constraint set (7) assigns the selected transporter to one distribution center. Constraint set (8) limits the total distance each transporter derives to a maximum travel distance. Constraint sets (9) and (10) define the decision variables of the model.

5. Benders decomposition algorithm

Benders decomposition is a multistage solution technique in mathematical programming to tackle large-scale problems (Benders, 1962). It divides the original problem into two smaller problems, called master problem (MP) and subproblem (SP). The MP is solved first, and the solution is given to the SP. SP is, in fact, the original problem with the MP variables fixed to values given by the MP solution. Then, SP returns its solution as a cut to the MP. The BDA iterates between the MP and SP until their solutions converge. Figure 3 shows the general scheme of Benders decomposition algorithm. The BDA suffers from some limitations. It needs the SP to be a linear programming model. But in the logic-based Benders decomposition algorithm (LBBDA) extension, the subproblem can be in any form of mathematical programs.
5.1. Proposed logic based Benders decomposition algorithm

The SCND problem under consideration includes three decision dimensions: location, assignment, and transporter decisions. The master problem includes the first two decisions, location and assignment. To speed up the convergence, the MP commonly includes a relaxation of the decisions in SP. There is one subproblem for each candidate distribution center site that is established in the MP solution, and it includes the transporter decisions. A subproblem can be optimal, suboptimal, or infeasible. If the transporter decision of a subproblem matches to its relaxation in the MP (the required number of transporters are the same in the MP and SP), then the subproblem is called optimal. If the required number of transporters for a subproblem is more than that of MP, then the subproblem is called suboptimal. Finally, a subproblem may also be infeasible.

If all subproblems are labeled as optimal, then the solution of the MP is optimal. Otherwise, the algorithm adds some Benders cuts to the MP and repeats. In the case of adding cuts, for each optimal or suboptimal subproblem, one optimality cut is added to the MP, and for each infeasible subproblem, one infeasibility cut (type 1) is added. If the total required number of transporters violates the maximum available transporters, then one feasibility cut (type 2) is added to the MP.

5.2. Master problem

In the master problem, two decisions of location and assignment are made. To develop the MP’s model, the following decision variables are defined.
Decision variables:

- $X_{i,k}$: Binary variable taking value 1 if supplier $i$ is assigned to distribution center $k$; and 0 otherwise.
- $Y_{j,k}$: Binary variable taking value 1 if customer $j$ is assigned to distribution center $k$; and 0 otherwise.
- $Z_{k,t}$: Binary variable taking value 1 if distribution center $k$ in mode $t$ is open.
- $H_{i,k}$: Continuous variable for flow from supplier $i$ to distribution center $k$.
- $U_k$: Integer variable for the number of transporters required by distribution center $k$.

Note that decision variable $U_k$ is to approximate the required number of transporters for distribution center $k$. The mathematical model of the master problem is as follows.

Objective:

$$\text{Min } \sum_{k=1}^{b} \sum_{t=1}^{a_k} f_{k,t} Z_{k,t} + \sum_{i=1}^{n} \sum_{k=1}^{b} c_{i,k} H_{i,k} + \sum_{k=1}^{b} \sum_{j=1}^{m} r_{j,k} Y_{j,k} + \sum_{k=1}^{b} s U_k$$

Subject to:

1. $\sum_{t=1}^{a_k} Z_{k,t} \leq 1 \quad \forall_k$ (11)
2. $\sum_{k=1}^{b} Y_{j,k} = 1 \quad \forall_j$ (12)
3. $\sum_{k=1}^{b} X_{i,k} \leq 1 \quad \forall_i$ (13)
4. $\sum_{j=1}^{m} d_{j} Y_{j,k} \leq \sum_{t=1}^{a_k} e_{k,t} Z_{k,t} \quad \forall_k$ (14)
5. $\sum_{j=1}^{m} d_{j} Y_{j,k} \leq \sum_{i=1}^{n} H_{i,k} \quad \forall_k$ (15)
6. $H_{i,k} \leq g_i X_{i,k} \quad \forall_{i,k}$ (16)
7. $\sum_{i=1}^{n} t_{i,k} X_{i,k} + \sum_{j=1}^{m} p_{j,k} Y_{j,k} \leq h U_k \quad \forall_k$ (17)
8. $H_{i,k} \geq 0$ (18)
9. $U_k \geq 0$ and integer (19)
10. $X_{i,k}, Y_{j,k}, Z_{k,t} \in \{0, 1\}$ (20)

The objective includes the location, assignment, and transporter cost. Constraint set (11) is to ensure at most one mode of candidate distribution centers. Constraint set (12) is...
to ensure each customer is served by one distribution center. Constraint set (13) assures each supplier is assigned to at most one distribution center. Constraint sets (14), (15), and (16) are capacity limitations of distribution centers and suppliers. Constraint set (17) is the relaxation of the number of transporters for each candidate distribution center site. Finally, Constraint sets (18), (19), and (20) define all decision variables.

To further restrict a mathematical model, there are two commonly used approaches, preprocessing and valid inequality. We use both to improve MP.

a) By a simple preprocessing of instances’ data, the infeasible assignment of suppliers and customers to distribution centers, regarding maximum possible travel limit of transporters, can be discarded. Hence, the number of decision variables can be further reduced. Thus, all variables $X_{i,k}$ with $t_{i,k} \geq h$ and $Y_{j,k}$ with $p_{j,k} \geq h$ are removed.

b) For location problems in the literature, there exists one valid inequality that avoids distribution center sites with no customers and suppliers to be established. The adaptation of this valid inequality for the problem under consideration is as follows.

$$\sum_{t=1}^{p} Z_{k,t} \leq \sum_{j=1}^{m} Y_{j,k} \quad \forall k$$

$$\sum_{t=1}^{p} Z_{k,t} \leq \sum_{i=1}^{n} X_{i,k} \quad \forall k$$

It is expected this preprocessing and decision variable reduction, along with the two above-mentioned valid inequalities, would cause faster convergence of MP.

5.3. Subproblem

For each established distribution center site in MP, one subproblem is defined. In each subproblem, the transporter assignment is determined. The following notations are defined.

Decision variables:

$X_{i,l}$ Binary variable taking value 1 if supplier $i$ is assigned to transporter $l$; and 0 otherwise.

$Y_{j,l}$ Binary variable taking value 1 if customer $j$ is assigned to transporter $l$; and 0 otherwise.

$W_{l}$ Binary variable taking value 1 if transporter $l$ is used.

The mathematical model of subproblem for established distribution center $k$ is as follows.
Objective:
\[ \text{Min} \sum_{l=1}^{m} sW_l \]

Subject to:
\[ \sum_{l=1}^{n} Y_{j,l} = 1 \quad \forall j | Y_{j,k}^{MP} = 1 \] (23)
\[ \sum_{l=1}^{n} X_{i,l} = 1 \quad \forall i | X_{i,k}^{MP} = 1 \] (24)
\[ Y_{j,l} \leq W_l \quad \forall l, j | Y_{j,k}^{MP} = 1 \] (25)
\[ X_{i,l} \leq W_l \quad \forall l, i | X_{i,k}^{MP} = 1 \] (26)
\[ W_l \leq W_{l-1} \quad \forall l > 1 \] (27)
\[ \sum_{l=1}^{n} t_{i,k} X_{i,l} + \sum_{j=1}^{m} p_{j,k} Y_{j,l} \leq hW_l \quad \forall l \] (28)
\[ X_{i,l}, Y_{j,l}, W_l \in \{0, 1\} \] (29)

where \( X_{i,k}^{MP} \) and \( Y_{j,k}^{MP} \) are the values of the corresponding decision variables of \( MP \) in the current iteration. Constraint sets (23) and (24) assign transporters to the customers and suppliers, allocated to that distribution center, respectively. Constraint sets (25) and (26) determine the used transporters. Constraint set (27) is a valid inequality for symmetry breaking. Constraint set (28) assures the maximum total distance limit of each transporter. Constraint set (29) shows the decision variables of \( SP \).

5.4. Optimality and Benders cuts

If all the subproblems are optimally solved and the number of required transporters for all subproblems becomes equal to that of the master problem, then the solution is optimal and the algorithm stops. Otherwise, Benders cuts are added to the \( MP \). To add Benders cut, the following procedure is applied. If a subproblem is optimally solved, then one optimality cut is added to the \( MP \) for that \( SP \). Otherwise (i.e., the subproblem is infeasible), one feasibility cut type 1 is added to the \( MP \). If all \( SPs \) are optimally solved, yet the total number of required transporters violates the maximum number of transporters available, one feasibility cut type 2 is added to the \( MP \).

Let us partition the objective function of \( MP \) into terms, \( OF_{MP}^1 \) and \( OF_{MP}^2 \) as follows.

\[ OF_{MP}^1 = \sum_{k=1}^{q} \sum_{t=1}^{p} f_{k,t} Z_{k,t}^* + \sum_{l=1}^{n} \sum_{k=1}^{b} c_{i,k} X_{i,k}^* + \sum_{k=1}^{b} \sum_{j=1}^{m} r_{j,k} Y_{j,k}^* \]
\[ OF_{MP}^2 = \sum_{k=1}^{b} sU_k^* \]

where \( Z_{k,t}^*, X_{i,k}^*, Y_{j,k}^* \) and \( U_k^* \) are optimal value of variables Z, X, Y and U in the MP, respectively.

Let us also define \( OF_{SP} \) as follows.

\[ OF_{SP} = \sum_{k=1}^{b} sU_k^{SP} \]

where \( U_k^{SP} \) is the optimal number of transporters required for \( k \)th subproblem/distribution center. In other words, \( OF_{SP} \) is summation of the objective functions of all the subproblems. Note that if all subproblems are feasible, therefore, the values of \( U_k^{SP} \) can be obtained.

On one hand, we know that the objective function of MP is the lower bound (\( LB \)) for the problem.

\[ LB = OF_{MP}^1 + OF_{MP}^2 \]

On the other hand, the combination of solutions of MP and SP generates a feasible solution of the original problem (if, of course, all SPs are feasible), then the upper bound (\( UB \)) of the problem is as follows.

\[ UB = OF_{MP}^1 + OF_{SP} \]

In this case, the optimality interval (\( OI \)) is as such

\[ OI = UB - LB = OF_{SP} - OF_{MP}^2 \]

And optimality gap (OG) becomes

\[ OG = \frac{OF_{SP} - OF_{MP}^2}{OF_{MP}^1 + OF_{SP}} \]

### 5.4.1. Optimality cut

If a subproblem is optimally solved, then the following optimality cut is added to MP. This cut means if the same subsets of suppliers and customers are selected for the candidate distribution center site \( k \), the optimal number of required transporters is known (\( U_k^{SP} \)).
\[ U_k \geq U_k^{SP} \left( 1 - \sum_{i=1}^{n} (1 - X_{i,k}^*) - \sum_{j=1}^{m} (1 - Y_{j,k}^*) \right) \forall k \text{ infeasible} \quad (30) \]

where \( U_k^{MP} \) is the required number of transporters obtained by MP for distribution center \( k \).

**Theorem 1.** Inequality (30) is a valid Benders optimality cut.

**Proof.** A valid Benders optimality cut must hold two properties: removing the current suboptimal solution, and not cutting off any feasible solutions. Inequality (30) is a valid Benders cut since it does not remove any feasible binary solution and it does not restrict the binary variables from taking any special values. Let us establish that a feasible solution to the original problem is a set of values for the binary variables in MP and SP. The integer variable \( U_k \) in MP is defined to have a relaxation of the binary variables of SP into MP. Thus, any restriction to variable \( U_k \) does not cut off any feasible solution of original problem. Moreover, the cut ensures \( U_k \geq U_k^{SP} \); that is, the number of required machines for the incumbent solution of MP is not less than its optimal value in SP. Therefore, it only removes the current suboptimal solution of the MP.

### 5.4.2. Feasibility cut

When a subproblem is infeasible, the feasibility cut type 1 is added to MP. This cut avoids allocating the same subset of suppliers and customers to that distribution center.

Feasibility cut type 1:

\[ \sum_{i=1}^{n} (1 - X_{i,k}^*) + \sum_{j=1}^{m} (1 - Y_{j,k}^*) \geq 1 \quad \forall k \text{ infeasible} \quad (31) \]

**Theorem 2.** Inequality (31) is a valid Benders feasibility cut.

**Proof.** A valid Benders feasibility cut must satisfy two properties: removing the current infeasible solution, and not cutting off any other feasible solution. Unlike Inequality (30), Inequality (31) puts a restriction on binary variables in MP. More precisely, it restricts MP not to accept the same optimal solution for the established
facility $k$ again. The right part includes all binary variables that take value one in the incumbent solution of MP. So, the right part for the incumbent solution becomes zero. Note that any other binary solution of MP differs from this incumbent solution in at least one binary variable. Thus, the incumbent solution is the only solution with value of zero for the right part of Inequality (31). By restricting the right part to be greater than one, the MP is forced to change at least one binary variable to be zero. Therefore, it only removes the incumbent solution.

If all subproblems are optimally or sub-optimally solved, yet the total number of required transporters exceeds the maximum available transporters, then the feasibility cut type 2 is added to MP. This cut avoid the same allocation across all distribution centers.

Feasibility cut type 2:

$$\sum_{i=1}^{n} \sum_{k=1|X^*_{i,k}=1}^{b} (1 - X^*_{i,k}) + \sum_{j=1}^{m} \sum_{k=1|Y^*_{j,k}=1}^{b} (1 - Y^*_{j,k}) \geq 1 \quad \text{if } \sum_{k=1}^{b} U^{SP}_{k} > v$$  \hspace{1cm} (32)

**Theorem 3.** Inequality (32) is a valid Benders feasibility cut.

**Proof.** This cut is similar to the previous feasibility notation, yet it restricts the binary variables across all established facilities. Therefore, the proof is similar to Theorem 2.

6. Experimental evaluation

This paper evaluates the performance of the model and algorithm using numerical experiments. The model and algorithm are coded into C++ platform Visual Studio 2010 and IBM ILOG CPLEX 12.6. All the experiments are run on a computer with core™ i5-5200U CPU @ 2.2 GHz Intel processor and 4.0 GB RAM. First, a set of experimental instances are generated to evaluate the performance of the model and algorithm. Then, the data of the real case is presented and solved by the algorithm.

6.1. Experimental evaluation
A set of instances is generated based on the following 3 combinations for \((n, b, m, v)\):

\[(n, b, m, v) = \{(10,10,10,70), (20,20,20,100), (30,30,30,150)\}\]

We consider 3 levels for \(s = \{500,1000,2000\}\). Also, we have \(h = 100, u = 2\) for all candidate distribution centers, \(d_k = 5 * U[5,10]\), \(e = 5 * U[20,80]\) for the first mode and for the second mode, the capacity is increased by 40, \(f = e * U[11,25]\) for the first capacity level and for the second level, the cost is increased by 50\%, \(g = 5 * U[10,15]\), \(t_{i,k}\) and \(p_{j,k} = U[10,60]\). For parameters \(c_{i,k}\) and \(r_{j,k}\), we consider two types, correlated and uncorrelated with the distance. In the correlated case, we have \(c_{i,k} = 2 * t_{i,k} + U[-5,5]\) and \(r_{j,k} = 2 * p_{j,k} + U[-5,5]\). In the uncorrelated case, \(c_{i,k}\) and \(r_{j,k} = U[5,95]\). Finally, for each of 18 combinations, we generate 10 random instances, summing up to 180 instances. The time limit was set to 1500 seconds for instances with \(n = 10\) and 20, and 3000 seconds for \(n = 30\).

To evaluate the model and algorithm, the general and relative performances are gauged. The general performance is shown by optimality gap obtained, and the relative performance is shown by relative percentage deviation (RPD). The optimality gap is calculated as follows.

\[
Gap = \frac{UB - LB}{UB} \times 100
\]

where UB and LB are the upper bound and lower bound obtained by each method. If the MP phase of the LBBDA is not optimally solved within the given computational time, it cannot obtain LB for that instance. For UB, the incumbent solution of MP is given to SP phase to have a feasible solution. Therefore, it finds UB. In such a case, LBBDA has no LB for that instance. To calculation of optimality gap, we use LB of MILP model instead. Note that MILP model always ends up with both UB and LB.

The RPD is also calculated as follows.

\[
RPD = \frac{UB - Min}{Min} \times 100
\]

where Min is the minimum of UB obtained by the model and algorithm. Therefore, RPD is the relative gap between two methods.

Table 1 also shows the number of instances solved to optimality and average computation time required to obtain the optimal solutions by both model and algorithm. Table 2 shows the results (i.e., two measures of Gap and RPD) in different problem sizes.
As can be seen, LBBDA outperforms MILP model by far. LBBDA solves all 120 instances of sizes 10 and 20, 51 instances of size 30 while the model solves only 53 instances out of 60 instances of size 10, and no instance in larger sizes of 20 and 30. LBBDA solves instances of size 10 with average computational time 1.9 seconds while the model solves those 53 instances with average computational time 322.7 seconds. LBBDA solves instances of sizes 20 and 30 with average computational times of 27.25 and 316.6 seconds, respectively.

<table>
<thead>
<tr>
<th>Size</th>
<th>No. of optimal instances</th>
<th>Computational time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LBBDA</td>
<td>MILP</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>53</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>51</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. The optimal instances and computational time of the model and algorithm

<table>
<thead>
<tr>
<th>Size</th>
<th>Co/Unco</th>
<th>C</th>
<th>Gap</th>
<th>RPD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LBBDA</td>
<td>MILP</td>
<td>LBBDA</td>
</tr>
<tr>
<td>10</td>
<td>Co</td>
<td>500</td>
<td>0</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Unco</td>
<td>500</td>
<td>0</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>0</td>
<td>0.90</td>
</tr>
<tr>
<td>20</td>
<td>Co</td>
<td>500</td>
<td>0</td>
<td>5.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0</td>
<td>9.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>0</td>
<td>12.01</td>
</tr>
<tr>
<td></td>
<td>Unco</td>
<td>500</td>
<td>0</td>
<td>5.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0</td>
<td>8.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>0</td>
<td>15.10</td>
</tr>
<tr>
<td>30</td>
<td>Co</td>
<td>500</td>
<td>0</td>
<td>11.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0</td>
<td>14.78</td>
</tr>
<tr>
<td></td>
<td>Unco</td>
<td>500</td>
<td>0</td>
<td>22.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0</td>
<td>15.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>0.89</td>
<td>21.20</td>
</tr>
</tbody>
</table>

Table 2. The Gap and RPD obtained by the algorithm and model
Regarding RPD (i.e., comparing the final solution/upper bound obtained by the two tested methods), LBBDA obtains the minimum objective in all cases, and this difference becomes even more significant in larger sizes. For example, the average RPD of the model goes up from 5.3% for size 20 to 12.24% for size 30. This result shows a larger difference between the model and algorithm in larger sizes.

Now, we study the impact of the parameters on the performance of the model and algorithm. Figure 4 shows the average optimality gap of the model and algorithm versus the problem size (i.e., $n, b, v, m$). As expected, the optimality gap of the model highly depends on the problem sizes while the algorithm is less sensitive. Figure 5 shows the average optimality gap of both methods versus the transportation cost type, and it is clear that performance is independent of the cost type. Figure 6 also shows the average optimality gap versus the unit cost of transporter. As shown, the instances become harder to solve if the unit cost of transporters increases.

![Figure 4. The average optimality gap of the model and algorithm versus the problem size](image-url)
6.2. Wheat network results

We generate a real case problem given parameters from the wheat distribution network in Iran. Then, we solve the problem using the proposed Benders algorithm. Iran has 31 provinces. We define each province as both a demand node and a supplier. Hence, each has a fixed demand and production capacity. Each province is a candidate for a distribution center. There are 31 demand nodes and suppliers. We also assume the imported wheat as another supplier. Since the import of wheat is carried out in
Hormozgan province, the distance for this supplier from the others is assumed the same as Hormozgan. Hence, in this case, we have 32 suppliers and 31 demand nodes.

We consider three capacity modes of 1, 2, and 3 million tonnes for distribution centers. The associated establishment costs are 20, 30, and 40 million dollars, respectively. In most provinces, the establishment cost is the same. But, in some provinces such as Tehran, Alborz, and Esfahan, this cost rises due to the higher land prices. Table 3 illustrates the total wheat production in Iran for year 2014-2015 based on the official reports of Iran’s Ministry of Agriculture and the analysis of the wheat import situation. Based on this report, the total demand of all provinces was 13 million tonnes, while the total wheat production capacity is 16.5 million tonnes, 30% to 40% more than the current wheat production. We also assume that the maximum possible wheat import can be 5 million tonnes. The transportation cost is assumed 0.5 dollar per ton-kilometer. The fixed cost of each transporter package is 0.3 million dollars. The distance between elements of the network is the real distances among the primary city of each province.

Table 3. Total wheat production in Iran for year 2014-2015.

<table>
<thead>
<tr>
<th>The center of the province</th>
<th>Production capacity (tonnes)</th>
<th>The cultivated area of wheat (Hectares)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Arak</td>
<td>346,683</td>
</tr>
<tr>
<td>2</td>
<td>Ardabil</td>
<td>734,206</td>
</tr>
<tr>
<td>3</td>
<td>Urmia</td>
<td>612,171</td>
</tr>
<tr>
<td>4</td>
<td>Isfahan</td>
<td>240,117</td>
</tr>
<tr>
<td>5</td>
<td>Ahvaz</td>
<td>1,276,640</td>
</tr>
<tr>
<td>6</td>
<td>Ilam</td>
<td>197,964</td>
</tr>
<tr>
<td>7</td>
<td>Bojnourd</td>
<td>226,615</td>
</tr>
<tr>
<td>8</td>
<td>Bushehr</td>
<td>414,73</td>
</tr>
<tr>
<td>9</td>
<td>Bandar Abbas</td>
<td>47,341</td>
</tr>
<tr>
<td>10</td>
<td>Birjand</td>
<td>53,435</td>
</tr>
<tr>
<td>11</td>
<td>Tabriz</td>
<td>775,000</td>
</tr>
<tr>
<td>12</td>
<td>Karaj</td>
<td>62,147</td>
</tr>
<tr>
<td>13</td>
<td>Tehran</td>
<td>198,189</td>
</tr>
<tr>
<td>14</td>
<td>Khorramabad</td>
<td>381,327</td>
</tr>
<tr>
<td>15</td>
<td>Rasht</td>
<td>12,715</td>
</tr>
<tr>
<td>16</td>
<td>Zahedan</td>
<td>183,731</td>
</tr>
<tr>
<td>17</td>
<td>Zanjan</td>
<td>279,700</td>
</tr>
<tr>
<td>18</td>
<td>Sari</td>
<td>210,411</td>
</tr>
<tr>
<td>19</td>
<td>Semnan</td>
<td>85,707</td>
</tr>
<tr>
<td>20</td>
<td>Sanandaj</td>
<td>650,861</td>
</tr>
<tr>
<td>21</td>
<td>Shahrekord</td>
<td>117,314</td>
</tr>
</tbody>
</table>
Table 3. Total wheat production in Iran for year 2014-2015.

<table>
<thead>
<tr>
<th>The center of the province</th>
<th>Production capacity (tonnes)</th>
<th>The cultivated area of wheat (Hectares)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 Shiraz</td>
<td>1,173,709</td>
<td>355,529</td>
</tr>
<tr>
<td>23 Qazvin</td>
<td>315,280</td>
<td>156,999</td>
</tr>
<tr>
<td>24 Qom</td>
<td>33,457</td>
<td>9,100</td>
</tr>
<tr>
<td>25 Kerman</td>
<td>285,218</td>
<td>84,572</td>
</tr>
<tr>
<td>26 Kermanshah</td>
<td>630,835</td>
<td>390,495</td>
</tr>
<tr>
<td>27 Gorgan</td>
<td>1,044,712</td>
<td>399,166</td>
</tr>
<tr>
<td>28 Mashhad</td>
<td>741,207</td>
<td>339,904</td>
</tr>
<tr>
<td>29 Hamadan</td>
<td>426,060</td>
<td>278,811</td>
</tr>
<tr>
<td>30 Yasuj</td>
<td>97,871</td>
<td>107,000</td>
</tr>
<tr>
<td>31 Yazd</td>
<td>40,219</td>
<td>13,325</td>
</tr>
<tr>
<td>32 Import point</td>
<td>Up to 5,000,000</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>16522315</strong></td>
<td><strong>5715616</strong></td>
</tr>
</tbody>
</table>

According to the data of Table 3 and the abovementioned data, the model is solved by the proposed Benders algorithm. Although the final model is absolutely large as a real case study at the level of a country, the proposed Benders could be able to solve the wheat distribution network in just 4 minutes. It should be mentioned that the case could not be solved by regular CPLEX modeling approach; only the Benders approach could solve it. The optimal objective (minimum cost of wheat distribution in Iran) becomes 3643 million dollars where the transportation cost constitutes 88% of this cost. It establishes 15 distribution centers (15 wheat hubs in Iran), 8 ones in mode 1 of capacities, 4 ones in mode 2, and 3 ones in mode 3. In the optimal solution, the production capacity of 22 suppliers is fully used. There are 5 suppliers with more than 50% usage and 0.5 million tons of wheat is also imported which could be an achievement in the wheat distribution network in Iran.

The sensitivity of the optimal solution towards the increase in transportation cost and establishment cost of distribution centers is analyzed. In this regard, the model is solved assuming 5%, 10%, 15%, and 20% increases in the transportation cost and the fixed cost separately. Figure 7 shows the increase in the objective function.

Table 4. The increase in the objective versus the increase in the transportation and establishment cost.

<table>
<thead>
<tr>
<th>Increase in the transportation cost</th>
<th>Increase in objective</th>
<th>Increase in the establishment cost</th>
<th>Increase in objective</th>
</tr>
</thead>
</table>
According to the results of Table 4, 10% increase in the transportation cost rises the optimal objective by 8.85%. The objective is more sensitive towards the transportation cost. This result is expected since the transportation cost constitutes the largest part of the total cost. The optimal solution remains unchanged in 5% and 10% increases. However, in 15% increase in the transportation cost, the optimal solution changes and the number of established distribution centers goes up to 21 centers from 15 ones in order to decrease the required transportation. That may not be beneficial since the fixed costs of establishing distribution centers may not be desirable for governmental decision makers.

7. Conclusion and future research

In the supply chain network design (SCND), it is commonly assumed that the capacity of a candidate facility is fixed. Yet, in practice, it is usually possible to increase the capacity to some extent. Another closely related issue to SCND is the fleet management or transporter planning. In this paper, the problem of wheat SCND with capacity acquisition and transporter planning is studied. The problem characteristics were defined according a real case of wheat distribution network in Iran. Indeed, 31 provinces and 1 import point are considered as potential distribution centers in Iran. Real datasets of wheat production are regarded in the model.

The mathematical formulation of the problem is developed. This model uses a mixed integer linear program. To effectively solve the problem, a logic-based Benders decomposition algorithm is also developed. This algorithm decomposes the original model into two models of master problem and subproblem. The master problem includes the location and assignment decisions while the subproblem covers the transporter planning. The master problem is improved by means of preprocessing and valid inequalities. Moreover, three Benders cuts, one optimality and two feasibility cuts, are also defined to pass the information from subproblem to master problem.

To evaluate the model and algorithm, 180 experimental instances are solved in different sizes. The algorithm solves 95% of these instances while the model only solves
29%. The average computational time of the model for those instances it optimally solves is 322 seconds while the algorithm solves the same size of instances in 1.9 seconds. All the results show that the proposed algorithm is very effective to solve the problem under consideration.

In terms of wheat distribution system, the proposed Benders could successfully solve the wheat distribution network in just 4 minutes while the regular CPLEX optimizer could not reach an acceptable solution even in more than 24 hours. The minimum cost of wheat distribution in Iran becomes 3643 million dollars by establishing 15 hubs of distribution centers (8 ones in mode 1, 4 ones in mode 2, and 3 ones in mode 3 of capacity). Besides, the sensitivity analysis of the optimal solution is undertaken for transportation cost and fixed establishment cost.

As an interesting future research direction, researchers can work on developing logic-based Benders’ decomposition for other supply chain network design problems. The structure of supply chain network design problems are decomposable and the approach developed in this paper can be generalized to other problems. Another future direction of this research would be to utilize the presented model with real studies of other grains. Furthermore, the model can be extended to consider sustainability aspects (Kannan, 2018), multi resourcing and multi-mode transportation options.

Acknowledgement

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References


