Citation:

Coordination by Quantity Flexibility Contract in a two-echelon Supply Chain System: Effect of Outsourcing Decisions

Abstract
This paper investigates a two-echelon supply chain (SC) with one product and two members, including one manufacturer and one retailer under stochastic demand. A quantity flexibility (QF) contract is applied where the retailer is allowed to update its primary order both downwards and upwards to share overstocking and overproduction risks between both channel members. In the proposed QF model, it is possible for the manufacturer to outsource production based on a reservation fee. The manufacturer’s decision variable is in-house/outsource amount while the retailer decides on order quantity. Numerical experiments and sensitivity analyses have been implemented on the contract parameters. The results indicate that outsourcing a fraction of production not only increases the profit of the supply chain but can also increase each member’s profit.

Keywords: Supply chain coordination; Quantity flexibility contract; outsourcing; stochastic demand

1. Introduction
Outsourcing strategy allows companies to focus on their main capabilities they have acquired over the years (Min, 2013). There are many situations that justify an outsourcing strategy in a generic SC; one such situation is when disruption events occur with a supplier. Disruption events in the upstream side of the SC may need a reliable backup supplier (Hou, Zeng and Sun, 2017). For example, the fire in the Philips’ microchip plant in Albuquerque,
New Mexico in 2000 forced Nokia to change the configuration of its products, and by doing so, to use outsourcing strategies to be responsive to customer demand during a supply disruption (Chen and Xiao 2015). Reducing operational costs and increasing agility are among other popular reasons for an SC manager to apply an outsourcing strategy. The HP Company is a good real-world example of such a situation. HP used a factory in Singapore to manufacture base stock of its products and also outsourced a portion of remaining products to a contract manufacturer in Malaysia to manage uncertain demand fluctuations (Chen and Xiao 2015).

Through outsourcing usual logistics activities, the SC members have the opportunity to focus on their core activities and reduce their costs. In the literature, this kind of logistics outsourcing is referred to as third-party logistics or 3PL. According to existing statistics, 70 percent of companies in the U.S. outsourced some of their logistics operations in the past two decades. Through modification and outsourcing logistics operations, logistics costs have decreased from 17.2% GDP in 1980 to 10.5% in 1997 and 7.5% in 2004 (Min 2013).

Due to the complexity and dynamism in SCs, disruption and/or demand fluctuations are considered to have a significant role in overstocking/understocking risks. When demand fluctuates and is less predictable, the retailer loses due to overstocking (in low-demand seasons) or understocking (in high-demand seasons) if its primary order is fixed. Contracts such as minimum order quantity (MOQ) are common practices imposed by the manufacturers to stabilize markets. However, MOQ-type contracts restrict the ordering flexibility of the retailer. If there is no commitment for the retailer to buy according to its primary approximate order and the retailer has the right of cancelling a considerable amount of its primary order, then the manufacturer may be affected due to overproduction. On the other
hand, late orders result in insufficient production time, which in turn increases production costs. Flexible decisions help SC managers adjust to uncertainties and shocks (Wallace and Choi, 2011). In such situations, a quantity flexibility (QF) contract may be helpful to satisfy both channel members. Under a QF contract, the retailer is allowed to update its primary order both downwards and upwards in a limited volume. That way, the retailer is committed to buying a minimum agreed amount while the manufacturer is committed to providing an extra amount if required.

There are many real cases in the industry which benefit from the application of QF contracts. Sun Microsystems uses QF contracts to buy from its work stations. Nippon Otis, a manufacturer of elevator equipment, uses a QF contract with the Tsuchiya factory which produces components and switches. QF contracts have also been used by other companies such as Toyota Motor Corporation, IBM, and Hewlett Packard (Tsay 1999).

Under a regular QF, the manufacturer is forced to produce all committed volume. However, the retailer calls for an extra amount only when the demand exceeds the primary estimation. Otherwise, there is no call for the extra amount and some of the manufacturer’s production batch will be scrapped. The manufacturer would like to operate in a more stable environment (i.e. less flexibility for the retailer to change its primary order) to avoid overstocking/understocking risks. These risks are common in classic QF contracts and impose extra costs to the manufacturer. To overcome this major shortcoming of a classic QF, in this study we develop a novel model that allows the manufacturer to reserve from an external supplier and outsource part of the production, if required. The manufacturer is required to pay a penalty fee to the external supplier per reserved but unpurchased item. It is the manufacturer's duty to provide committed amounts based on the agreed QF contract.
and therefore there is a need for an outsourcing agreement between the manufacturer and an external supplier. As mentioned earlier, outsourcing may be used for reducing the SC’s operational costs and increasing agility. In this study, we use outsourcing strategy in an SC under a QF contract to simultaneously overcome cost and agility issues of the manufacturer. Based on the above, the paper tries to answer the following questions: (1) Does considering outsource capability by the manufacturer under the QF contract increase the profit of the total SC? (2) What is the optimum level of outsourcing under the QF contract?

To answer these questions, we consider a two-echelon SC with one product under stochastic demand. In the proposed model, in order to entice the retailer to place a primary order before the beginning of the selling season, the manufacturer offers certain flexibility in order quantities. The retailer promises to order not less than a certain amount under the primary order when market demand is known. On the other hand, the manufacturer commits to keep a certain inventory ready to deliver more than the primary order quantity. If an item is produced by the manufacturer but it is not purchased by the retailer (due to low demand) then the cost of the remaining scrap items is imposed on the manufacturer. The manufacturer has an agreement with an external supplier to reserve/outsource part of the retailer order. Outsourcing production to the external supplier is expensive on its own and therefore the manufacturer has to solve the trade-off between outsourcing expenses and remaining scrap costs.

The main contribution of this paper is to consider a QF contract in a manufacturing SC where there is an opportunity for the manufacturer to reserve/outsource a part of production. Overstocking on the supply-side of an SC under a QF contract is common due to the flexibility of the retailer to change primary order and leads to extra costs for the manufacturer. The
model presented in this study removes this major shortcoming of classic QF contracts by applying an outsourcing strategy along with the QF contract for the first time. Although Hu, Lim and Lu (2013) studied the case of outsourcing, in their model the supplier is allowed to outsource only when he faces disruption and is not able to fulfil its commitments. What distinguishes our paper is that it does not consider disruption. In other words, in this paper the manufacturer is able to produce the retailer’s order completely and uses the outsourcing strategy only to reduce overproduction costs. As we discussed earlier, outsourcing may be applied in different situations (e.g. in disruption occasions or emergency supply cases) and has various benefits (e.g. allows the SC members to focus more on core activities, reduces operational costs in some cases, increases agility in some other cases).

To overcome the primary shortcomings of classic QF contracts, we combine them with an outsourcing strategy in this study. To the best of the authors’ knowledge, considering outsourcing along with QF contracts is not studied to date. From a practical point of view, using an outsourcing strategy in combination with a QF contract, the proposed model in this paper could significantly reduce the expected salvage amounts in the SC. As another contribution of this study, in the proposed model a closed-form relation between two parameters of the QF contract (i.e. downward adjustment and upward adjustment) is obtained so that for any value in the possible range of the downward adjustment parameter, we can obtain an optimal value for the upward adjustment parameter. Indeed, contrary to some previous works (e.g. Bicer and Hagspiel 2016; Kim, 2011; Kim et al. 2014), our model does not require a numerical algorithm to adjust QF parameters. Instead, by estimating one parameter (i.e. the downward adjustment parameter) our model is able to calculate the optimal value of the other parameter (i.e. the upward adjustment parameter). In addition, in
our model optimal values of both QF variables are dependent on each other. This feature of the proposed model is interesting from an applied point of view in situations where there is a restriction in tuning the parameters (e.g. the retailer requires at least a specific downward flexibility). In such situations, by fixing the restricted parameter, the model is able to calculate the optimal value of other parameter.

2. Literature review

The main focus of this paper is on two major topics: (1) outsourcing strategy, (2) coordination mechanisms and contracts in the SC through specific concentration on quantity flexibility contracts. We point out the previous studies on these two topics.

In the field of outsourcing, Benaroch, Webster, and Kazaz (2012) and also Alvarez and Stenbacka (2007) studied flexible sourcing models to find out the optimum expected time to change resourcing. They considered dynamic models in which the outsourcing cost per transaction is variable. Spinler and Huchzermeier (2006), and also Inderfurth and Kelle (2011), considered the outsourcing strategy when both demand and cost are uncertain. Liu and Nagurney (2013) suggested a model with a quick-response mechanism and global outsourcing. By considering uncertainty in demand and cost, vibrational inequality theory is used for analysis, in which some scenarios are analysed to take both production costs and demand into account. Chen and Xiao (2015) considered the outsourcing and coordination mechanism for two Stackelberg game models under various uncertainty parameters such as demand, capacity, and disruption risk. They pointed out that when disruption-risk/production-capacity is low/high enough, the manufacturer is not motivated to outsource. Nosoohi and Nookabadi (2016) developed an outsourcing model for a manufacturer to study optimal ordering policy under uncertainty of final processing costs
and customer demand. To neutralize the impact of uncertainty in cost parameters, they used different option contracts. Min (2013) studied the usual outsourcing methods of logistics operations in U.S. factories and identified the key factors of outsourcing in U.S. logistics operations. Zhao, Langendoen, and Fransoo (2012) considered a situation where a manufacturer outsources a part of its production to a supplier. In their model, the manufacturer has two options to outsource, namely slow transportation mode and fast transportation mode. In this way, they investigated ordering behavior of companies that outsource their production with long distances.

Supply chain coordination is another stream of literature which is closely related to this study. Coordination between SC members is a critical issue in better performance of an SC. Different coordination schemes are developed depending on factors such as product type, condition of business environment, economic factors, current business laws, consumer buying behaviours, and so on. Contracts are among the most applicable mechanisms for achieving SC coordination. There are various types of contracts, each of which are appropriate for a specific condition. Quantity discounts (Heydari & Norouzinasab, 2015; Venegas and Ventura, 2018) are among the well-known contracts that can be applied easily in many commercial SCs. Buyback (also known as return policy) is another contract that is common for recyclable or returned products (Xu et al., 2018). Delay in payments contracts (also known as credit options) are interesting for small and medium-sized enterprises and are also applicable in environments with high interest rates (Aljazzar et al., 2017; Heydari, 2015). Revenue sharing (Hu and Feng, 2017; Raza, 2018) can also coordinate SCs by sharing risks between SC members when the SC operates in an uncertain environment. Sales rebate contracts (Chiu et al., 2011) are another type of contract that can coordinate SCs and enhance
sales volumes when there is an opportunity in a business environment to effort to sell more products. Quantity flexibility, which is the main focus of this study, is one of the solutions when market demand is uncertain, the selling season is short, and production lead time is long, such that the orders should be placed a long time before the beginning of the selling season. Such long production/manufacturing lead times are common in various industries and previously emphasized by researchers (e.g. see Heydari 2014). In the following, we investigate the latest studies on developing QF contracts in the field of SC coordination.

As early works on QF contracts, Bassok, Bixby, Srinivasan, and Wiesel (1997) studied a series of supply contracts for electronics industries. In the mentioned research, optimum buying policy and the commitment strategy according to the conditions of the contract were specified. Unlike the current study, they used a supply contract problem with periodic commitments and update flexibility (PCUF) that has a different structure than the news-vendor problem. Tsay and Lovejoy (1999) presented a model for operations analysis and QF design. Tsay (1999) also studied a QF contract to share the risk of demand uncertainty between a supplier and a buyer to motivate channel members towards the system-wide optimal outcome. In the above study, the flexibility parameters are dependent on a fraction of the total system inventory for which the retailer is ultimately responsible while in the current study, flexibility parameters are independent of system inventory. These early works in this field showed the ability and applicability of QF contracts in the presence of production lead time and demand uncertainty.

More recently, Wang and Tsao (2006) formulated the QF contract from the buyer’s viewpoint and obtained a closed-form expression for the behaviour of the buyer when demand is uniformly distributed. Similar to the current study, they assumed a uniform distribution for
the market demand. Afterward, Miltenburg and Pong (2007) studied the problem of ordering policy for some special products in which the demand is unknown and two ordering opportunities exist. Similar to Miltenburg and Pong (2007), the current study also assumes two ordering opportunities for the retailer.

Also, there are some previous works in this area with different methodological approaches than the current study. For example, a model for a buyer's replenishment decision under a class of SC contracts called Rolling Horizon Flexibility (RHF) was presented by Bassok and Anupindi (2008). Their methodology is different from the applied methodology in this paper. In order to evaluate the effectiveness of their RHF contract (a type of QF contract), they proposed two heuristics and derived a lower bound while our paper follows the exact solution method. Lian and Deshmukh (2009) proposed a dynamic programming model to find the optimal solution for the replenishment strategy, as well as solution approaches which allow buyers to choose the order quantity that minimizes their expected total cost in each period. They assumed that the buyer has to pay a higher per-unit cost for the incremental units, but in the current study the buyer does not pay a higher per-unit cost for the extra units. Kim (2011) considered a four-level SC where each member orders based on the buyer's prediction. The results indicated that flexibility does not necessarily lead to better services for the customers. While in the current study we apply mathematical modelling approaches, Kim (2011) used a simulation methodology. Kim and Wu (2013) used a scenario aggregation approach to formulate and solve an SC contract using a two-stage stochastic programming model. Using a different methodology than the current study, they proved that the stability of customer demand would be beneficial for SC members. Furthermore, Kim, Park and Shin (2014) studied a QF contract in a multi-period system including several heterogeneous
suppliers. In their paper, a linear programming model is developed from the buyer's perspective and not from the whole channel view point. In the current study the news-vendor model is developed and a channel coordination model is considered.

As one of latest studies in developing QF contracts, Karakaya and Bakal (2013) considered a multi-product two-level supply chain including one retailer and one manufacturer, and concentrated on a joint QF contract which includes all products. They showed that joint a QF contract improves the retailer's profit substantially and the manufacturer benefits from this contract as well. Hu, Lim, and Lu (2013) studied a flexible ordering policy among a manufacturer and a supplier with random yield and demand uncertainty. In their model, although the flexible ordering policy can significantly reduce the yield uncertainty for the supplier, it may decrease the manufacturer's expected profit. While in current study, the proposed QF contract itself achieves coordination, they proposed a revenue sharing policy with an order penalty and rebate contract to fully coordinate the supply chain. A QF contract was considered by Mahajan (2014) in a case where demand is stochastic and also depends on price in which, similar to the current study, the aim is to achieve channel coordination solutions for both SC members. Chung, Talluri and Narasimhan (2014) designed a combinational contract named QFi, which is a combination of QF and price discount. They showed that this contract can effectively balance the inventory risk between the buyer and supplier. The effect of lead time on the profitability of the QF contract was investigated and it has been shown that lead time reduction increases the value of QF contracts (Bicer and Hagspiel 2016). A QF contract with optimal pricing strategies in a two-level SC in the cosmetic industry was developed based on a two-period dynamic model in order to calculate the
retailer’s optimal replenishment policy and the manufacturer’s optimal pricing policy (Li, Lian, Choong, and Liu 2016).

To the best of the authors’ knowledge, there is no previous work that considers the impact of outsourcing under a QF contract. The most similar work is the Hu, Lim, and Lu (2013) study, which is different from the current study in several respects. The main difference in the current study from the Hu, Lim, and Lu (2013) study is about the nature of outsourcing. In Hu, Lim, and Lu (2013), outsourcing is allowed only when the manufacturer is not able to produce the buyer’s order in-house. However, in our study, the manufacturer has enough capacity to produce the entire retailer’s order, but the outsourcing is used as a strategy to reduce the overall SC costs. In addition, there are many technical differences in the mathematical model of this study from the Hu, Lim, and Lu (2013) study.

3. Problem description

A two-echelon single commodity SC including one retailer and one manufacturer is analysed. In the basic model (without outsourcing opportunity), the retailer decides on order quantity and then the manufacturer produces and delivers the retailer’s order. Market demand is assumed to be stochastic with uniform distribution. Due to the manufacturing lead time, it is necessary for the retailer to place its primary order before the beginning of the selling season based on its estimation of the market demand. Therefore, it is possible that the actual demand is higher/lower than the primary order of the retailer. In practice, having complete knowledge and perfect forecast of the market demand is impossible before the selling season and therefore significant overstocking/understocking may occur. Although some industries such as fast fashion brands plan to have stock-out, both stock-out and overstocking events are assumed harmful. In the basic model, the manufacturer, as the leader of the supply chain,
aims to maximize the whole supply chain’s profit by offering a QF contract. In this regard, the manufacturer offers the retailer the option of cancelling some amount of the initial order or placing additional orders for a limited amount if needed. The allowed amounts of cancelation/additional orders are limited. As a result, with this policy, the manufacturer encourages the retailer to jointly optimize the order quantity. In our model, the retailer is responsible for all of the stock-out costs. Unsold items have the same salvage value for both the retailer and the manufacturer. The retailer places a primary order based on market estimation and commits to buying at least a certain amount of the primary order when the actual market demand is revealed. On the other hand, the manufacturer commits to preparing a pre-specified amount more than the retailer’s primary order, if required. The investigated model aims to optimize the flexibility thresholds (both upward and downward thresholds) to globally optimize the retailer’s order quantity.

In the next stage, we add the reserve/outsourcing opportunity for the manufacturer. Under the outsourcing model, the manufacturer is not forced to produce all its committed volume but it is possible to utilize an outsourcing opportunity. In this way, the likelihood of remaining scrap items on the manufacturer’s side is reduced. However, reserving items from the external supplier is more costly than in-house production and therefore it is not economical for the manufacturer to outsource a large amount. Indeed, the manufacturer needs to solve the trade-off between the cost of remaining scrap items and outsourcing costs to decide on an appropriate outsourcing amount. In addition, the manufacturer must pay a penalty to the external supplier for each reserved but unpurchased item. It is worth noting that this paper does not consider the shipping lead time and shipping cost.

4. Mathematical model
The notations used in this paper are presented as follows:

**Models’ parameters**
- $x$: Random variable representing amount of market demand that follows uniform probability distribution $f(x) \sim U[0,T_2]$
- $p$: Retail price per unit
- $w$: Wholesale price per unit
- $c$: Production cost per unit on the manufacturer’s side
- $o$: Outsourcing price per reserved and purchased unit for the manufacturer
- $c_M$: Penalty per reserved but unpurchased unit paid by the manufacturer to the external supplier
- $s$: The salvage value per unit of remaining item at the end of the selling season (equal for the retailer and the manufacturer)
- $b$: The shortage penalty per unit on the retailer’s side

**Logical relationships:** $0 < s < c < o < w < p$; $c_M < o - s$

**SC members’ decision variables**
- $q$: The retailer’s order quantity
- $Q$: The manufacturer’s maximum commitment to deliver; $Q = (1+u)q$
- $M$: The manufacturer’s in-house production amount

**QF contract variables**
- $u$: Upward adjustment parameter in the QF contract; $u \geq 0$
- $d$: Downward adjustment parameter in the QF contract; $0 \leq d \leq 1$

The subscripts $R$, $M$, and $SC$ stand for “retailer,” “manufacturer,” and “supply chain,” respectively.

### 4.1. QF contract without outsourcing

Due to a long production lead time, the manufacturer needs to start the production process long before the beginning of the selling season. Therefore the manufacturer requires the retailer to place orders earlier. The retailer does not normally commits to purchase a certain amount long before the selling season because the market demand is uncertain and therefore such a commitment imposes high risks of overstocking for the retailer. Proposing the QF in such situations is a rational method to encourage the retailer to place the order earlier.

Without appropriate arrangements (like the QF contract), the retailer does not accept placing
such an order with such a high risk a long time before the selling season and maybe leaves the SC. According to Fig. 1, under the quantity flexibility contract, the retailer places an order of size \( q \) before the beginning of the selling season based on the forecast of the demand and is committed to purchasing not less than \((1-d)q\) units. On the other hand, the manufacturer is committed to preparing \((1+u)q\) units, if necessary. Therefore, the manufacturer has to produce \((1+u)q\) units. Since there is no shipping cost, the retailer's first order is equal to its minimum purchasing commitment, i.e., \((1-d)q\). If there is a remaining market demand, the retailer places a secondary order such that the sum of the first and second orders does not exceed \((1+u)q\). At the end of selling season, the manufacturer and the retailer salvage the remaining products (if any) with an equal salvage value.

The profit function of the SC under the QF contract can be formulated as:

\[
\Pi_{SC}(q, u) = \begin{cases} 
px + s((1+u)q - x) - c(1+u)q & 0 \leq x < (1+u)q \\
 p(1+u)q - c(1+u)q - b(x - (1+u)q) & (1+u)q \leq x < T_2 
\end{cases}
\]  

(1)
In Eq. (1), in the first condition (i.e., $0 \leq x < (1 + u)q$), the first term denotes the total revenue from selling $x$ units, the second term indicates the total income from salvaging unsold items, and the third term indicates the total cost of production for the manufacturer. Similarly, in the second condition (i.e., $(1 + u)q \leq x < T_2$), the first term denotes the total revenue from selling $(1 + u)q$ units, the second term denotes production costs, and the last term refers to the penalty for unsatisfied demand.

Accordingly, the SC’s expected profit is formulated as:

$$E[\Pi_{SC}(q, u)] = p \left( \int_0^{(1+u)q} xf(x) \, dx + \int_{(1+u)q}^{T_2} (1 + u)qf(x) \, dx \right)$$

$$+ s \int_0^{(1+u)q} ((1 + u)q - x)f(x) \, dx - b \int_{(1+u)q}^{T_2} (x - (1 + u)q)f(x) \, dx$$

$$- c(1 + u)q$$

(2)

As emphasized by Miltenburg and Pong (2007), common probability density functions for demand during the selling season are the normal and uniform distribution functions. On the other hand, the uniform distribution results in the most conservative estimate of uncertainty in demand (Cai et al., 2015) and should be considered a good choice for representing demand distribution during the selling season. Meanwhile, in our case, applying a normal distribution for demand requires us to implement numerical methods, as the closed-form solutions are unobtainable. Therefore, we apply a uniform distribution in our case, which enables us to provide good insight into the problem through an analytical approach. Therefore, in this study, it is assumed that the demand follows the continuous uniform distribution in the interval $[0, T_2]$, (the minimum market demand is equal to zero and maximum demand given by $T_2$), therefore:
Given that in the basic QF model, the manufacturer is required to produce as much of its commitment \((1+u)q\), the SC's expected profit, assuming continuous uniform distribution, is calculated as:

\[
E[\Pi_{SC}(q,u)] = \frac{Q^2s - b(Q - T_2)^2}{2T_2} - 2cQT_2 - p(Q^2 - 2QT_2)
\]

**Lemma 1.** The SC's expected profit function under the QF is concave in \(Q\) and the optimal \(Q\) is:

\[
Q^* = \frac{(b + p - c)T_2}{b + p - s}
\]

All proofs are given in the Appendix.

Given that \(Q=(1+u)q\), the optimum value for \(q_{SC}\) from Eq. (5) can be formulated as:

\[
q^*_{SC} = \frac{(b - c + p)T_2}{(b + p - s)(1 + u)}
\]

Although calculating \(q^*_{SC}\) from Eq. (6) maximizes the SC profit, it is the retailer's decision variable. The retailer makes the decision based on its own profit and if the calculated \(q^*_{SC}\) cannot maximize the retailer's profit, then the retailer refuses it.

To investigate the retailer's decision process, we can formulate the retailer's profit function under the QF contract as follows:

\[
\Pi_R(q, d, u) = \begin{cases} 
px - w(1-d)q + s((1-d)q - x) & 0 \leq x < (1-d)q \\
px - wx & (1-d)q \leq x < (1+u)q \\
px - w(1+u)q - b(x - (1+u)q) & (1+u)q \leq x < T_2
\end{cases}
\]

As can be seen in Eq. (7), when market demand \(x\) is less than \((1-d)q\), the retailer purchases its minimum committed amount, i.e. \((1-d)q\) units. In such situations, the retailer has to
salvage unsold units at the end of the selling season. On the other hand, if demand $x$ exceeds
$(1 + u)q$ units, then the retailer is confronted with a shortage cost.

The retailer’s expected profit can be formulated as:

$$E[\Pi_R(q, d, u)] = p \left( \int_{0}^{(1+u)q} xf(x) \, dx + \int_{(1+u)q}^{T_2} (1 + u)qf(x) \, dx \right)$$

$$- w \left( \int_{0}^{(1-d)q} (1 - d)qf(x) \, dx + \int_{(1-d)q}^{(1+u)q} xf(x) \, dx \right)$$

$$+ \int_{(1+u)q}^{T_2} (1 + u)qf(x) \, dx + s \int_{0}^{(1-d)q} ((1 - d)q - x)f(x) \, dx$$

$$- b \int_{(1+u)q}^{T_2} (x - (1 + u)q)f(x) \, dx$$

Assuming Eq. (3), the retailer’s expected profit can be rewritten as:

$$E[\Pi_R(q, d, u)] = \frac{(q^2s - pq^2 - 2dq^2s + d^2q^2s + 2pqT_2 - 2pq^2u + 2pqT_2u - pq^2u^2)}{2T_2}$$

$$- b (q - T_2 + qu)^2 - q \left( \frac{d^2q - 2dq}{(1 + u) - qu(2 + u)} \right)$$

Lemma 2. The retailer’s expected profit function under the QF is concave in $q$; the optimal $q$ is:

$$q_R^* = \frac{T_2(1 + u)(b + p - w)}{p - s + b(1 + u)^2 + u(2 + u)(p - w) - (-2 + d)d(s - w)}$$

The whole SC is optimized if the retailer decision $q_R^*$ becomes equal to $q_{SC}^*$. To convince the retailer to accept $q_{SC}^*$, the manufacturer adjusts the QF contract parameters $d$ and $u$. The manufacturer offers the QF contract not only to convince the retailer to place his order earlier, but also to coordinate the SC channel. Indeed, it is assumed that the manufacturer, as the contract initiator, has a holistic view and intends to optimize the total channel profit. This assumption is valid and confirmed by previous works in the literature of SC coordination (e.g.}
Lemma 3. The optimal $u$ that coordinates the SC under the QF contract without outsourcing is:

$$u^* = -1 + \max \{1, A\}$$

(11)

where

$$A = \left[ \frac{(b - c + p)(w - s)(d - 1)^2}{(c - s)(b + p - w)} \right]^{\frac{1}{2}}$$

(12)

Lemma 3 gives the optimal relationship between the two parameters of the QF contract $d$ and $u$ that guarantees the global optimum decision of the retailer when no outsourcing is allowed.

**Corollary 1.** A unique value of $u$ can be obtained so that the retailer’s decision on $q_R$ equals the global optimum decision $q_{SC}^*$. 

The manufacturer’s profit function under the QF contract is calculated as:

$$\Pi_M(q, d, u) =
\begin{cases}
  w(1 - d)q - c(1 + u)q + s((1 + u) - (1 - d))q & 0 \leq x < (1 - d)q \\
  wx - c(1 + u)q + s((1 + u)q - x) & (1 - d)q \leq x < (1 + u)q \\
  w(1 + u)q - c(1 + u)q & (1 + u)q \leq x < T_2
\end{cases}$$

(13)

Accordingly, the manufacturer’s expected profit function is formulated as:
\[ E[\prod_M(q, d, u)] \]
\[
= w \left( \int_0^{(1-d)q} (1 - d)q f(x)dx + \int_{(1-d)q}^{(1+u)q} xf(x)dx \right)
+ \int_{(1+d)q}^{T_2} (1 + u)q f(x)dx \\
+ s \left( \int_0^{(1-d)q} ((1 + u)q - (1 - d)q)f(x)dx \right)
+ \int_{(1-d)q}^{(1+u)q} ((1 + u)q - x)f(x)dx - c(1 + u)q
\] (14)

Assuming Eq. (3), we can rewrite Eq. (14) as:

\[
E[\prod_M(q, d, u)] = \frac{q \left( -2cT_2(1 + u) + 2dq(s - w) + 2T_2w + d^2q(-s + w) \right) + u(q(2 + u)(s - w) + 2T_2w)}{2T_2}
\] (15)

Although the manufacturer adjusts the QF contract parameters in order to coordinate the SC channel, such an adjustment cannot reduce the manufacturer’s profitability. Indeed, the QF contract parameters \(d\) and \(u\) control the overstocking and order cancellation amounts which are both unfavourable from the SC viewpoint. Therefore, one can conclude that the manufacturer and the SC interests are either aligned or they do not conflict. Therefore, we do not expect that such a decision making structure damages the manufacturer’s profitability.

4.2. QF contract with outsourcing

In this case, as shown in Fig. 2, the manufacturer has the option of outsourcing part of its committed amount to an external supplier. The manufacturer produces \(M\) units in-house and reserves \(((1 + u)q - M)\) units from the external supplier. Since in-house production is more economical than outsourcing (as a basic assumption \(o > c\)), it is obvious that \(M\) should be greater than \((1 - d)q\). The manufacturer pays price \(o\) per reserved and purchased item. In addition, the manufacturer must pay a penalty \(c_M\) per reserved but unpurchased item. Since
shipping costs are not considered in the model, the retailer’s order is divided into two parts. At first, the minimum allowable order quantity of the retailer (i.e. \((1-d)q\)) is delivered and then if there is a remaining market demand, the secondary order is delivered such that sum of the first and the second delivered orders does not exceed \((1+u)q\).

The SC’s profit function under the QF contract along with outsourcing can be formulated as:

\[
\Pi_{SC}(q,u,M) = \begin{cases} 
px + s(M - x) - cM - c_M((1 + u)q - M) & 0 \leq x < M \\
px - cM - o(x - M) - c_M((1 + u)q - x) & M \leq x < (1 + u)q \\
p(1 + u)q - cM - o((1 + u)q - M) - b(x - (1 + u)q) & (1 + u)q \leq x < T_2 
\end{cases} 
\tag{16}
\]

As can be seen in Eq. (16), when demand is observed, some surplus produced items may need to be salvaged (when \(0 \leq x < M\)) and also some reserved items may need to be cancelled (when \(0 \leq x < (1+u)q\)) at penalty cost \(c_M\). According to Eq. (16), if \(x < (1 + u)q\) then a penalty for reserve cancelation should be paid. If demand exceeds \(M\), then the manufacturer has to purchase part (or all) of its reserve amount.
Accordingly, the SC’s expected profit can be formulated as:

\[
E[\Pi_{SC}(q, u, M)] = p \left( \int_0^{(1+u)q} x f(x) \, dx + \int_{(1+u)q}^{T_2} (1+u)q f(x) \, dx \right) \\
+ s \left( \int_0^M (M-x) f(x) \, dx \right) - cM \\
- o \left( \int_{M}^{(1+u)q} (x-M) f(x) \, dx + \int_{(1+u)q}^{T_2} ((1+u)q-M) f(x) \, dx \right) \\
- b \int_{(1+u)q}^{T_2} (x-(1+u)q) f(x) \, dx \\
- c_M \left( \int_0^M ((1+u)q-M) f(x) \, dx + \int_{M}^{(1+u)q} ((1+u)q-x) f(x) \, dx \right) \\
(17)
\]

By considering a continuous uniform distribution and also presuming \( Q = (1+u)q \), the SC’s expected profit is rewritten as:

\[
E[\Pi_{SC}(q, u, M)] = - \frac{\left( bQ^2 - oQ^2 + pQ^2 + M^2(o-s) - c_M(M-Q)(M+Q) \right) - 2(b-o+p)QT_2 + bT_2^2 + 2M(cT_2 - oT_2)}{2T_2} \\
(18)
\]

**Theorem 1.** The SC’s expected profit function under the QF with outsourcing is concave in both \( Q \) and \( M \) simultaneously and the optimal \( Q \) and \( M \) are calculated as:

\[
Q^* = \frac{T_2(b-o+p)}{b + c_M - o + p} \\
M^* = \frac{(o-c)T_2}{o - c_M - s} \\
(19)
\]

Given that \( Q = (1 + u)q \), using Eq. (19), amount of \( q_{SC} \) under the QF with outsourcing is:

\[
q_{SC}^* = \frac{T_2(b-o+p)}{(b + c_M - o + p)(1+u)} \\
(21)
\]

The retailer’s profit function under the QF with outsourcing is identical to the case of the QF without outsourcing and therefore Eq. (10) also gives the optimal retailer’s order quantity in
this case. Similar to the case of the QF without outsourcing, in this case we can also entice the retailer to set its optimal order quantity \( q_R^* \) equal to \( q_{SC}^* \) in order to optimize the entire SC through appropriate adjustment of \( d \) and \( u \).

**Theorem 2.** The optimal \( u \) that coordinates the SC under the QF contract with outsourcing is:

\[
  u^* = -1 + \max\{1, B\}
\]

where

\[
  B = \left[ \frac{(b - o + p)(w - s)(d - 1)^2}{c_M(b + p - w)} \right]^{1/2}
\]

Theorem 2 gives the optimal relationship between the two parameters of the QF contract \( d \) and \( u \) that guarantees the global optimum decision of the retailer when outsourcing is allowed. Parameter \( d \) represents the percentage of order cancelation opportunity for the retailer and therefore values of \( d \) closer to 1 result in more order cancelation opportunity for the retailer, and at the same time, a larger \( u \) allows the retailer to place larger additional orders. Therefore, there are two sources of flexibility for the retailer: on the one hand, larger values of \( d \) result in more flexibility in cancelation and on the other hand larger values of \( u \) result in more flexibility in placing larger additional orders. We define \( u + d \) as the flexibility offered to the retailer in ordering (i.e. placing additional orders or cancelling a previous order) and we have Theorem 3.

**Theorem 3.** The flexibility offered to the retailer under the QF with outsourcing is more than the QF without outsourcing if

\[
  \frac{b - o + p}{c_M} > \frac{b - c + p}{c - s}.
\]

More flexibility is always in favour of the retailer (Wu, 2005) and creates more profit for the retailer.
Corollary 2: If $c_M$ becomes sufficiently smaller than $(c - s)$, then under the QF with outsourcing the retailer experiences a better position than its position under the QF without outsourcing.

Corollary 3: In the QF contract with outsourcing, if $c_M$ approaches 0 then the manufacturer’s maximum commitment amount $Q$ will be equal to the maximum realizable demand, i.e. $T_2$. At the same time $\frac{o - c}{o - s} \times 100\%$ of the committed amount is produced in-house.

The profit function of the manufacturer under the QF with outsourcing can be formulated as:

\[
\Pi_M(q, d, u, M) = \begin{cases} 
(1 - d)qw - cM + (M - (1 - d)q)s - c_M((1 + u)q - M) & 0 \leq x < (1 - d)q \\
xw - cM + (M - x)s - c_M((1 + u)q - M) & (1 - d)q \leq x < M \\
xw - cM - (x - M)o - c_M((1 + u)q - x) & M \leq x < (1 + u)q \\
(1 + u)qw - cM - ((1 + u)q - M)o & (1 + u)q \leq x < T_2 
\end{cases} 
\]  

By assuming a continuous uniform distribution in the interval $[0, T_2]$ for the demand, we can restate the manufacturer’s expected profit function as:

\[
E[\Pi_M(q, d, u, M)] = w \left( \int_0^{(1-d)q} (1 - d)q f(x)dx + \int_{(1-d)q}^{(1+u)q} xf(x)dx \right) + \int_{(1+u)q}^{T_2} (1 + u)q f(x)dx - cM \\
+ s \left( \int_0^{(1-d)q} (M - (1 - d)q) f(x)dx + \int_{(1-d)q}^{M} (M - x) f(x)dx \right) - \alpha \left( \int_0^{M} (x - M) f(x)dx + \int_{M}^{T_2} ((1 + u)q - M) f(x)dx \right) \\
- c_M \left( \int_0^{M} ((1 + u)q - M) f(x)dx + \int_{M}^{(1+u)q} ((1 + u)q - x) f(x)dx \right) 
\]
Since $M$ has no role in the retailer’s expected profit function, calculating $M$ from the manufacturer’s expected profit function results in a similar solution as the one obtained from the SC’s expected profit function. Therefore, Eq. (20) gives the optimal $M$ from the viewpoint of both the manufacturer and the entire SC.

Under the QF with outsourcing, the manufacturer may benefit from both increased sales volume and decreased salvage amount. On the other hand, the manufacturer’s cost under outsourcing is composed of extra costs paid for purchasing from the external supplier and also the cost of cancelling reserved units. If the above benefits exceed the costs, then the manufacturer under the QF with outsourcing experiences a better position.

On the other hand, the external supplier earns revenue from both reserved and purchased $\min\{\max\{x - M, 0\}, (1 + u)q - M\}$ units by rate $o$. Further, the supplier charges the manufacturer for $\left((1 + u)q - M\right) - \min\{\max\{x - M, 0\}, (1 + u)q - M\}$ cancelled units by rate $c_M$. Since the external supplier trades in a spot market, it is possible to sell each remaining item there by employing a specific spot rate. It is rational to assume that the spot rate is equal to the outsourcing price $o$. In addition, we assume the same production cost for the external supplier as the one of the manufacturer. In this way, it will be possible to derive the external supplier’s expected profit and to investigate its position and profitability in the developed contract.

5. Numerical experiments
To verify the applicability and performance of the proposed models, three examples are examined. Datasets for the investigated numerical examples are shown in Table 1. All datasets satisfy the model’s assumptions and requirements.

<table>
<thead>
<tr>
<th>Table 1. Three investigated data sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
</tr>
<tr>
<td>$T_2$</td>
</tr>
<tr>
<td>$p$</td>
</tr>
<tr>
<td>$c$</td>
</tr>
<tr>
<td>$o$</td>
</tr>
<tr>
<td>$s$</td>
</tr>
<tr>
<td>$w$</td>
</tr>
<tr>
<td>$b$</td>
</tr>
<tr>
<td>$c_M$</td>
</tr>
</tbody>
</table>

By applying two proposed models on datasets, the decision variables are calculated for both the QF without outsourcing and the QF with outsourcing models. Under the QF with outsourcing, the manufacturer’s decision variable is in-house/outsource amounts. In both models, the QF’s downward and upward adjustment parameters $d$ and $u$ should also be determined based on Eqs. (11 & 22). As we know, in a SC, as a cooperative system, not only the overall system efficiency but also all SC members’ efficiency is vital. On this basis, as obtained in Eqs. (11) and (22), the QF model gives combinations of $d$ and $u$ which convince the retailer to globally optimize its order quantity. Each combination that satisfies the channel members’ preferences could be selected. In our numerical examples, we consider $d = 0.2$ which implies that the retailer has the authority to cancel 20% of its primary order. However, a sensitivity analysis on $d$ is also conducted. Table 2 presents the numerical results for the QF model without outsourcing.
Table 2. Value of decision variables, profit functions, and performance measures under the QF contract without outsourcing for $d=0.2$

<table>
<thead>
<tr>
<th></th>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^*$</td>
<td>0.4967</td>
<td>0.6395</td>
<td>0.0525</td>
</tr>
<tr>
<td>$q_R = q_{SC}$</td>
<td>25.98</td>
<td>35.58</td>
<td>67.51</td>
</tr>
<tr>
<td>$Q^* = (1 + u)q$</td>
<td>38.88</td>
<td>58.33</td>
<td>71.05</td>
</tr>
<tr>
<td>$E[\Pi_M(q, d, u)]$</td>
<td>194.44</td>
<td>583.33</td>
<td>355.26</td>
</tr>
<tr>
<td>$E[\Pi_R(q, d, u)]$</td>
<td>150.0</td>
<td>187.50</td>
<td>473.68</td>
</tr>
<tr>
<td>$E[\Pi_{SC}(q, u)]$</td>
<td>344.44</td>
<td>770.83</td>
<td>828.94</td>
</tr>
<tr>
<td>Expected sales (Retailer)</td>
<td>23.76</td>
<td>41.32</td>
<td>54.22</td>
</tr>
<tr>
<td>Expected purchase (Retailer)</td>
<td>28.09</td>
<td>45.37</td>
<td>63.95</td>
</tr>
<tr>
<td>Expected shortage (Retailer)</td>
<td>1.23</td>
<td>8.68</td>
<td>20.77</td>
</tr>
<tr>
<td>Expected salvage (Retailer)</td>
<td>4.32</td>
<td>4.05</td>
<td>9.72</td>
</tr>
<tr>
<td>Expected salvage (Manufacturer)</td>
<td>10.80</td>
<td>12.96</td>
<td>7.11</td>
</tr>
</tbody>
</table>

Some performance measures such as expected sales, purchase, shortage, and salvage amounts are also presented in Table 2.

Table 3 shows the value of decision variables and SC performance measures for the QF contract with outsourcing. Comparing Tables 2 and 3 reveals that the QF contract with outsourcing results in more profit for both SC members in comparison with the QF contract without outsourcing. In addition, all performance measures under the QF with outsourcing are improved. By comparing values in Tables 2 and 3, it is revealed that the proposed QF contract with outsourcing recommends lower order quantity $q_R$ and instead, to avoid shortages, increases the manufacturer's commitment amount. The total committed amount remains less/near to $T_2$, which is rational. The retailer will be in a better position when ordering less but receiving more, if requested. In addition, to avoid high salvage costs on the manufacturer's side, most of the manufacturer's commitment amount (i.e. $Q$) is supplied by outsourcing. In this way, at first the QF with outsourcing avoids high salvage amounts on the manufacturer's side and on the other hand avoids high shortages for the retailer by recommending high reservation volume.
Table 3. Value of decision variables, profit functions, and performance measures under the QF contract with outsourcing for $d=0.2$

<table>
<thead>
<tr>
<th>Decision variables and profits</th>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^*$</td>
<td>3.0398</td>
<td>3.1569</td>
<td>5.0967</td>
</tr>
<tr>
<td>$q_R = q_{SC}$</td>
<td>11.91</td>
<td>21.65</td>
<td>23.81</td>
</tr>
<tr>
<td>$M^*$</td>
<td>13.15</td>
<td>26.67</td>
<td>19.42</td>
</tr>
<tr>
<td>$Q^* = (1 + u)q$</td>
<td>48.11</td>
<td>89.99</td>
<td>145.19</td>
</tr>
<tr>
<td>$E[\Pi_M(q,d,u,M)]$</td>
<td>196.87</td>
<td>646.66</td>
<td>361.22</td>
</tr>
<tr>
<td>$E[\Pi_R(q,d,u)]$</td>
<td>233.02</td>
<td>425.0</td>
<td>1437.5</td>
</tr>
<tr>
<td>$E[\Pi_{SC}(q,u,M)]$</td>
<td>429.89</td>
<td>1071.66</td>
<td>1798.72</td>
</tr>
<tr>
<td>External supplier expected profit</td>
<td>108.80</td>
<td>617.50</td>
<td>798.49</td>
</tr>
</tbody>
</table>

Retailer’s metrics

| Expected sales                | 24.96     | 49.50     | 74.92     |
| Expected purchase             | 25.87     | 51.0      | 76.13     |
| Expected shortage             | 0.0356    | 0.5000    | 0.0770    |
| Expected salvage              | 0.9078    | 1.5000    | 1.2099    |

Manufacturer’s metrics

| Expected reserved and purchased | 13.54     | 26.39     | 56.76     |
| Expected reserved but not purchased | 21.41 | 36.93     | 69.01     |
| Expected salvage               | 0.8235    | 2.0556    | 0.0471    |
| Outsource/in-house production (%) | 72.7%/27.3% | 70.4%/29.6% | 86.6%/13.4% |

External supplier’s metrics

| Expected revenue from sold items | 169.22    | 1002.78   | 3167.2    |
| Expected revenue from cancellation charges | 21.41 | 110.83    | 69.01     |
| Expected revenue from spot market | 267.71   | 1403.88   | 3850.9    |

According to the results of the outsourcing model, although the purchase cost from the external supplier is more than the in-house production cost for all investigated examples, the manufacturer still outsources and all members benefit from the outsourcing. Therefore, we infer that the QF contract with outsourcing outperforms the QF model without outsourcing. We use the QF contract with outsourcing for further numerical investigations. Although the external supplier is assumed as an exogenous element of the SC, it is interesting to numerically investigate its position and performance criteria in the investigated test problems. As can be seen in Table 3, there are three sources of revenue for the external supplier. According to obtained results, the external supplier not only helps the manufacturer/retailer to avoid high salvage/shortage costs, but also can generate revenue
from (1) selling items to the manufacturer, (2) obtaining cancellation charges, and (3) selling remaining items in a spot market.

Figure 3 illustrates the interaction between two QF parameters. As shown in Figure 3, by increasing downward adjustment parameter $d$, the upward adjustment parameter $u$ decreases. This is a rational behavior, because by increasing $d$, the retailer can cancel more units from its primary order and as a direct result, the manufacturer’s costs increase due to high cancelation volume, which in turn leads to higher overstocking risk for the manufacturer. To neutralize this effect, the manufacturer reduces its committed volume and therefore the upward adjustment parameter $u$ decreases. In Figure 3, if $d > 0.8$ then $u = 0$ (in example 2) and the manufacturer does not commit to extra delivery. Indeed, by increasing $d$ to more than 0.8, $u$ must be set to zero and therefore Eqs. (22) could not be held anymore and therefore $q_R$ is not equal to $q_{SC}$. In such a situation, the retailer’s order $q_R$ could not be equal to $q_{SC}$ and as a result, the SC’s profit decreases.

![Fig.3 Effect of $d$ on the upward adjustment parameter $u$](image)
Figure 4 reveals that by increasing $d$, the retailer’s primary order quantity increases. Indeed, if there is an opportunity to cancel large amounts of the primary order, the retailer prefers to place a larger primary order. According to Figure 4, when $d$ rises close to 1, the retailer can cancel its complete order and as a result the retailer’s order quantity approaches the maximum possible demand, i.e. $T_2$.

To investigate the capabilities of the proposed QF contract with outsourcing, a set of sensitivity analyses are performed. Figure 5 shows the expected shortage on the retailer’s side as well as expected salvage on both the retailer and manufacturer’s sides. As shown in Figure 5, by increasing $d$ from a threshold (i.e. $d > 0.8$), both shortage and salvage volumes on the retailer’s side are decreased. Indeed, by increasing $d$, the retailer is authorized to cancel most of its order and therefore its order size increases (see Figure 4) and as a result, shortages are reduced. On the other hand, since the retailer can cancel its primary order, it is
expected that the salvage volume decreases. Instead, the manufacturer should salvage all cancelled orders and therefore, salvage amount on the manufacturer’s side increases.

![Graph](image)

**Fig.5** Effect of $d$ on the shortage and salvage amounts (data from example 2)

Given that one of the most important issues in classic QF contracts is salvage amounts on the manufacturer's side, we compare the classic QF without the outsourcing option with the proposed QF with the outsourcing contract. As can be seen in Figures 6-8, by reducing the salvage value, the overstocking risk increases and as a result outsourcing strategy along with the QF becomes more interesting for both SC members as well as the entire SC.
Fig. 6 Effect of salvage value $s$ on the retailer’s profit in QFs both with and without outsourcing (data from example 2)

Fig. 7 Effect of salvage value $s$ on the manufacturer’s profit in QFs both with and without outsourcing (data from example 2)
Fig. 8 Effect of salvage value \( s \) on the entire SC’s profit in QFs both with and without outsourcing 
(data from example 2)

Figure 9 compares the flexibility offered by the manufacturer to the retailer under QFs with and without outsourcing. According to Figure 9, the QF contract with outsourcing results in more flexibility for the retailer compared to the QF contracts without outsourcing.

Fig. 9 Effect of \( d \) on upward adjustment parameter \( u \) (data from example 2)
From a managerial perspective, the proposed model in this study advises managers to merge their outsourcing strategy with their flexible ordering policy. When the downstream expects a flexibility in ordering, the upstream undergoes the risk of overstocking to ensure sufficient inventory to meet the downstream demand. In such situations, combining the outsourcing strategy with the flexible ordering policy can significantly compensate for the upstream overstocking risk.

6. Conclusions and managerial implications

In a manufacturer-retailer system, the market demand cannot be accurately estimated prior to the beginning of the selling season. However, due to reasons such as long manufacturing/transportation lead times, the retailer has to place its order long before the selling season. This may cause an overstocking/understocking risk for the retailer. In this paper, using quantity flexibility (QF) contracts, we try to balance the imposed risks between two channel members in a two-echelon SC including one manufacturer and one retailer. Two models are developed: (1) the QF contract without outsourcing opportunity, and (2) the QF with outsourcing opportunity for the manufacturer.

Under the QF contract, the retailer has the authority to cancel its primary order in a limited amount. Also, the manufacturer provides a limited amount above the primary order to avoid understocking in a high season. Closed-form relations between the two parameters of QF contracts (downward and upward adjustment parameters) are calculated so that the entire SC is optimized. Since in the classic QF contract the manufacturer is committed to deliver more than the primary placed order, there is a high risk of overstocking on the manufacturer's side. The high risk of overstocking for the manufacturer is a major shortcoming of classic QF
contracts, which is the main concern of this study. This study solves this problem by proposing a “QF contract with outsourcing”. Comparing the “QF contract with outsourcing” with the “QF without outsourcing” revealed that considering outsourcing opportunity can significantly improve the SC’s profit as well as the channel members’ profit. Numerical investigation shows that outsourcing increases expected sales of the SC while decreasing shortages and at the same time diminishes overstocking risk in both the retailer and manufacturer warehouses. Furthermore, outsourcing strategy compensates for overproduction in the manufacturer’s production line.

Flexible ordering policy is common in today’s competitive business environments. Such flexibility may cause huge overstocking costs if the manufacturer decides to produce all received orders in-house. As a profit-oriented business, each manager would like to know how it is possible to minimize such overstocking costs. The results of this study advise the managers to partially outsource production, even with higher prices, to avoid overstocking costs. In this way, not only the overstocking costs are secured, but also by adopting a right outsourcing strategy it may be possible to offer further flexibility to buyers. In summary, the proposed models in this study advise managers to merge their outsourcing strategy with their flexible ordering policy to avoid extra costs.

This paper contributes to the literature by overcoming a major shortcoming of classic QF models by taking outsourcing strategy into account. Under a classic QF, the manufacturer is forced to produce and hold all committed amounts. However, the retailer calls for extra amounts only when the demand exceeds the primary estimation. Therefore, there is an overstocking risk on the manufacturer’s side which imposes costs of salvaging at low prices. Using outsourcing strategy in combination with the QF contract, the proposed model in this
paper could solve the problem of high salvage amounts on the manufacturer’s side and significantly reduce the salvage amounts in the SC. This way, the manufacturer’s costs are compensated and it will be possible to offer more flexibility to the retailer.

As a future study, it is possible to consider uncertainty in the production capacity of the manufacturer and assess the impact of multiple sources of uncertainty on achieving channel coordination. In addition, in this study we assume the external supplier is an exogenous element of the SC, therefore another interesting idea for further investigation is assuming the external supplier to be an endogenous element of the SC and optimizing its profit. Taking transportation costs and lead time considerations into account is also another interesting opportunity for further investigation.

Acknowledgements

The authors are thankful to the two anonymous reviewers for all their kind, insightful, and constructive comments.

Appendix

Proof of Lemma 1. To prove concavity of the SC’s expected profit function in $Q$, the second order derivative of the SC’s profit function, Eq. (4), can be calculated as:

$$
\frac{\partial^2 \Pi_{SC}(q,u)}{\partial Q^2} = - \frac{b + p - s}{T_2} < 0 \quad (A.1)
$$

Given that $p > s$, Eq. (A.1) is always negative; therefore, the SC’s profit function is strictly concave in $Q$.

When the second-order optimality condition is met for $Q$, we have:

$$
\frac{\partial \Pi_{SC}(q,u)}{\partial Q} = \frac{(b - c + p)T_2 - (b + p - s)Q}{T_2} = 0
$$

$$
Q^* = \frac{(b + p - c)T_2}{b + p - s} \quad (A.2)
$$
Proof of Lemma 2. The second-order derivative of the retailer’s expected profit function in $q$ is calculated as:

$$\frac{\partial^2 \Pi_R(q, d, u)}{\partial q^2} = -\frac{p - s + b(1 + u)^2 + u(2 + u)(p - w) - (-2 + d)d(s - w)}{T_2} \tag{A.3}$$

Eq. (A.3) can be rewritten as Eq. (A.4) through some simplifications:

$$\frac{\partial^2 \Pi_R(q, d, u)}{\partial q^2} = -\frac{p - w + b + 2bu + bu^2 + 2u(p - w) + u^2(p - w) + (w - s)(d - 1)^2}{T_2} < 0 \tag{A.4}$$

Given that $p > w > s$, Eq. (A.4) is always negative; therefore, the retailer expected profit function is strictly concave in $q$. Therefore, we have:

$$\frac{\partial \Pi_R(q, d, u)}{\partial q} = 0$$

$$q^*_R = \frac{T_2(1 + u)(b + p - w)}{p - s + b(1 + u)^2 + u(2 + u)(p - w) - (-2 + d)d(s - w)} \tag{A.5}$$

The numerator of Eq. (A.5) is obviously positive. To show the obtained value $q^*_R$ from Eq. (A.5) is positive, it is enough to show the denominator of Eq. (A.5) is also positive. For this purpose, we replace $p$ with $w$ in the denominator of Eq. (A.5) and the result will be less than the original denominator. If we show the revised denominator is positive, then the original denominator is also positive. The revised denominator will be $\left(\frac{w - s + b(1 + u)^2 + u(2 + u)(p - w)}{-(-2 + d)d(s - w)}\right)$. By some simplifications we obtain $\left(\frac{b(1 + u)^2 + u(2 + u)(p - w)}{+(w - s)(d - 1)^2}\right)$ which is obviously positive and therefore the denominator of Eq. (A.5) is also positive. It is proved that the obtained value $q^*_R$ from Eq. (A.5) is positive.
**Proof of Lemma 3.** By setting $q^*_R = q^*_SC$, it will be guaranteed that the retailer's order quantity becomes equal to the optimal value of the order quantity from the perspective of the entire SC, which means achieving coordination. Using Eqs. (6) and (10) and setting $q^*_R = q^*_SC$, we have $u^* = -1 \pm A$. Given $s < c$ and $w < p$, which are logical assumptions, term $(-1-A)$ is always negative and thus $(-1+A)$ is accepted. To avoid negative values for $u$, which is meaningless, we revise the formula as $u^* = -1 + \max\{1,A\}$

**Proof of Corollary 1.** According to Eq. (11) in Lemma 3, since $u$ depends on $d$ for each acceptable value of $d$ (i.e. $0 \leq d \leq 1$), using Lemma 3, it is straightforward to calculate a unique value of $u$ if we have $q^*_R = q^*_SC$.

**Proof of Theorem 1.** To prove concavity of the SC’s expected profit function in $Q$ and $M$, the Hessian matrix is calculated as:

$$
H[\Pi_{SC}(q,u,M)] = 
\begin{bmatrix}
\frac{-b + c_M + p - o}{T_2} & 0 \\
0 & \frac{(c_M - o + s)}{T_2}
\end{bmatrix}
$$

(A.6)

Given that $p > o$, the first principal minor is always negative. Since we have a logical relationship $c_M < o - s$, $H_{22}$ is negative and therefore the second principal minor is positive. Thus, the Hessian matrix is negative definite and the SC’s expected profit function is concave in both $Q$ and $M$ simultaneously.

Using the first-order optimality condition we have:

$$
\frac{\partial \Pi_{SC}(q,u,M)}{\partial Q} = \frac{- (b + c_M - o + p) Q - (b - o + p) T_2}{T_2} = 0
$$

$$
Q^* = \frac{T_2 (b - o + p)}{b + c_M - o + p}
$$

(A.7)
\[
\frac{\partial \Pi_{SC}(q, u, M)}{\partial M} = -\frac{M(o - s - c_M) + (c - o)T_2}{T_2} = 0
\]
\[
M^* = \frac{(o - c)T_2}{o - c_M - s}
\]

As mentioned previously, we have \( c_M < o - s \) to make it unprofitable for the manufacturer to purchase products from the external supplier by the aim of salvaging them.

**Proof of Theorem 2.** By setting \( q^*_R = q^*_{SC} \), we have
\[
\frac{T_2(b-o+p)}{(b+c_M-o+p)(1+u)} \text{ which results in } u^* = -1 \pm B. \text{ Given } w < p, \text{ term } (-1 - B) \text{ is always negative and } (-1 + B) \text{ is accepted. To avoid negative values for } u, \text{ which is meaningless, we revise the formula as } u^* = -1 + \max\{1, B\}
\]

**Proof of Theorem 3.** If Eq. (22) results in more value than Eq. (11) for a fixed \( d \), then it is proved that the QF with outsourcing creates more flexibility for the retailer than the QF without outsourcing. Since both numerators and denominators are positive in Eqs. (12) and (23), it is obvious that Eq. (23) results in a higher value for \( u \) than Eq. (12) if:
\[
\frac{b - o + p}{c_M} > \frac{b - c + p}{c - s}
\]

(A.9)

**Proof of Corollary 2:** According to Theorem 3, we know that the flexibility offered to the retailer under the QF with outsourcing will be greater than the flexibility offered under the QF without outsourcing if term \( \frac{b-o+p}{c_M} \) is greater than \( \frac{b-c+p}{c-s} \). We know that always \( o > c \), therefore if \( c_M \) becomes sufficiently smaller than \( (c - s) \) the condition will be held and as a result, flexibility offered to the retailer under the QF with outsourcing is increased, which in turn causes a better position for the retailer.

**Proof of Corollary 3:** According to Eq. (19), by setting \( c_M = 0 \) we have \( Q^* = T_2 \) which means the maximum commitment amount is equal to the maximum realizable demand. Meanwhile,
using Eq. (20) by setting $c_M = 0$, we have $M^* = \frac{o-c}{o-s} T_2$ which means $\frac{o-c}{o-s}$ share of $T_2$ is produced in-house.

References


