Entering a New Market:
Market Profitability and First-Mover Advantages

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Abstract We analyze a firm’s investment problem when it faces pre-emption risk and profits are convex in market profitability. In a setup where firms have asymmetric profit convexity, which we relate to firm quality, we show that this has interesting effects on valuation and the order of entry. The interplay between profit convexity and market growth impacts whether a high-quality or a low-quality firm is the first mover. We relate the first-mover advantage to patents; we find that patents expedite investments and increase the incentives for high-quality firms to become first movers. Furthermore, even with a persistent first-mover advantage we show that first-mover advantages in terms of firm value are either over- or underestimated. Thus, our model sheds light on why empirical studies find mixed support for the existence of a first-mover advantage.

Keywords: strategic real investment, asymmetric firms, market growth, patents, estimation issues

JEL subject codes: G31, D81
1 Introduction

Investments are decisive for a firm’s future. In particular, when an investment makes a firm enter a new market a specific concern is whether the firm should become a first mover. We set up a dynamic investment model with two asymmetric firms to investigate this. Each firm has an option to invest in a new market. The firms differ in their strength of using internal flexibility which implies a difference in their profit convexity. The key contribution of the paper is that the asymmetry of profit convexity results in interesting effects on the order of entry and a firm’s valuation. We relate this to the effect of patents as well as why empirical studies have difficulties in establishing whether a first-mover advantage exists or not.

Firms differ in their strength of using internal flexibility due to various reasons. For example, firms can have different abilities to leverage scale of their production in response to a higher demand, or they can differ in their cost or productivity of an adjustable input such as labor. For example, Toyota had significantly better production compared to U.S. and European car companies. Of course, other dimensions are in practice also part of an entry decision, but our point is that the seemingly best firm may not prefer to enter a new market first. As a specific example, Nokia used to be a strong mobile phone producer and with their ability to use scale to dominate the phone market they were reluctant to switch to smart phones as this was perceived as a low growth market. At that time, Apple had lower scale capabilities and they (successfully) decided to enter the smart phone market first. IBM’s reluctance to enter the PC market is another example. We consider a reduced form model with profits being convex in the profitability of the market and we interpret the convexity parameter as a firm’s ability to exploit internal flexibility. We will talk about the firm’s use of internal adjustment possibilities as the firm’s quality.
Several market characteristics impact the entry decision and valuation. Our analysis focuses on two elements. One element is the growth rate of a market profitability index. The other element is how profits depend on a firm’s status as a leader or a follower. This directly influences the incentive for a firm to become a first mover (Pawlina and Kort, 2006; Mason and Weeds, 2010; Huisman and Kort, 2015; Jiang et al., 2015). We consider a reduced form model of the latter by having a simple scaling of the firms’ profit. This allows us to have a tractable model focusing on the implications of firm quality.

In contrast to intuition, our model predicts that low-quality firms can become the first mover. This holds particularly in industries in which the growth rate is low. The intuition behind this hinges on the incentive to preempt. Competition leads a firm to invest earlier in the preemptive equilibrium relative to a monopolist. Thus, a firm has an incentive to postpone investment if the threat of preemption decreases. The convexity effect implies that a low-quality firm has a lower value of waiting implying that early investment is less costly. On the other hand, a higher growth rate in itself increases a firm’s incentive to invest, and thus it becomes more important to win the preemption game. Therefore, the growth rate and a firm’s quality jointly enter the entry decision. When the growth rate is low enough, the convexity effect dominates, so that it is too expensive for the high-quality firm to preempt the low-quality firm. Thus, our results support Huisman and Kort (2015), who show that a small firm can exercise an investment option earlier than a large firm. Our result also fits well with R&D-intensive technology industries in which less established firms are often the first ones to enter into new markets (Phillips and Zhdanov, 2013). Similarly, in a related setting Çolak and Günay (2011) and Xu (2017) also find that a high-quality firm not always moves first.

The joint effect of the firm’s quality and the growth rate has implications for
estimating first-mover advantages. Depending on which effect dominates empirical studies both risk rejecting first-mover advantages when they exist, and risk supporting the existence of first-mover advantages, even if they are not present. In line with Robinson et al. (1992) and Franco et al. (2009), we thus argue that first-mover advantages must be studied in the right context. Specifically, we provide a theoretical explanation for the fact that studies controlling for competitive strengths between firms are more likely to find support for a first-mover advantage (VanderWerf and Mahon, 1997). This also explains why empirical studies that use a comprehensive set of industries have provided very mixed results (Golder and Tellis, 1993; Suarez and Lanzolla, 2007), while studies focusing on specific industries have been able to find support for the existence of a first-mover advantage (Huff and Robinson, 1994; Shankar et al., 1998; Makadok, 1998; Coeurderoy and Durand, 2004).

Firms vary with respect to their productivity both across and within industries (e.g., Peteraf, 1993; Syverson, 2004). Thus, the investment strategy of a firm, as well as the value prior to investment, depend on both the firm’s productivity and the productivity of its competitors (Hopenhayn, 1992). We show that the ex ante value of a firm does not need to decrease when the quality of its competitor increases. This stems from the fact that a higher quality of one firm impacts the other firm’s incentive to preempt. For example, Apple did not have to fear preemption from Nokia. Specifically, a higher quality of the follower is to some extent hedged through the investment timing channel. This result relates to Carlson et al. (2014), who show that competition partly reduces a firm’s incentives to undertake new investments when a positive shock occurs. Similarly, in line with the empirical study in Akdoğan and MacKay (2008), we find that a change in the quality of a competitor impacts the investment timing in a non-monotonic manner due to the trade-off between the value of waiting and the risk of preemption.
Patents impact the first-mover advantage, but it is unclear whether they in fact expedite entry into new markets or technologies (Sakakibara and Branstetter, 1999; Takalo and Kanniainen, 2000; Weeds, 2002; Kanwar and Evenson, 2003; Czarnitzki and Toole, 2011). We extend our model to consider patents; for tractability, the first mover loses its advantage according to a Poisson driven event. Our model predicts that patents mitigate an underinvestment problem since they expedite investments in new technology and markets. Additionally, we find that patents provide stronger incentives for high-quality firms to be first movers. Our results lead us to suggest that it is easier to find first-mover advantages in high-R&D industries with patents.

Our paper is related to the vast literature using a real options approach to analyze corporate investments (e.g., McDonald and Siegel, 1986). More specifically, our paper relates to the recent literature on strategic investment under uncertainty; for example, Lambrecht and Perraudin (2003), Boyer et al. (2004), Pawlina and Kort (2006), and Mason and Weeds (2010). These papers show that the risk of preemption speeds up investment and leads firms to overinvest. Huisman and Kort (2015) further show that firms may overinvest in capacity to deter entry. Jiang et al. (2015) find a positive relation between competition and corporate investment which is driven by a high growth rate.

The remainder of the paper is organized as follows. Section 2 sets up the model and we derive the preemption equilibrium in Section 3. Section 4 discusses the implications of the growth rate and quality on investment strategies. Section 5 discusses the effects of patents. Section 6 considers valuation implications. Finally, we conclude in Section 7. Proofs are postponed to the appendix.

1A standard reference is, for instance, Dixit and Pindyck (1994). Other applications than the firm’s investment problem are, for example, capital structure (Brennan and Schwartz, 1978; Hackbarth et al., 2007; Clausen and Flor, 2015; Shibata and Nishihara, 2018), mergers and acquisitions (Lambrecht and Myers, 2007; Morellec and Zhdanov, 2008), R&D (Berk et al., 2004), and corporate risk management (Boyle and Guthrie, 2005).
2 The model

Two firms are initially equipped with an irreversible option to enter a new market. The firms are risk neutral and value-maximizing, and they use the risk-free rate of interest, $r$, for discounting. Important elements of our model are that the profitability of the market depends on the general market profitability index and a firm’s ability to exploit internal adjustment possibilities which results in a convexity effect. A firm’s profits also depends on whether it is a leader or not on the market. The details of the model follow below.

2.1 The value of the new market as a monopolist.

A firm has an option to enter a new market by paying an investment cost, $I$. Once initiated, the firm receives a profit flow, $\pi$. In this section we set up some basic notation and derive the value of the profit flow. We later turn to the case with two asymmetric firms and the risk of preemption on the new market. The profit depends on a state variable, $X$, which we consider as a market profitability index, (e.g., Morellec and Zhdanov, 2008; Carlson et al., 2014). The profitability index fluctuates randomly over time due to variations in demand. Specifically, we assume that $X$ follows the geometric Brownian motion

$$dX_t = \mu X_t dt + \sigma X_t dZ_t,$$

where $\mu$ is the growth rate and $\sigma$ the volatility.

Due to internal adjustments in the firm, the firm’s profit depends on the state variable in a convex manner. We let $\gamma > 1$ be a measure of how well the firm can exploit the internal adjustment possibilities. A setting like this follows if the firm
can easily adjust an input. For example, one way to interpret $\gamma$ is to think of the productivity of labor or as a higher quality of the produced good; by producing a good of higher quality the firm is able to service a larger fraction of the market at the same price.\(^2\) Thus, we henceforth talk about the firm’s use of internal adjustment possibilities as the firm’s quality. We assume a reduced form of the firm’s profit related to its quality:

$$\pi(X) = wX^\gamma, \quad (2)$$

where $w > 0$ is a scaling factor. For tractability, we let $\gamma \in (1, ((\mu/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2)^{1/2} - \mu/\sigma^2 + \frac{1}{2})$.\(^3\) The present value of the profit (2) at time $t$ is

$$V(X_t) = E\left[\int_t^\infty e^{-r(s-t)} \pi(X_s)ds\right] = \frac{w}{r - \hat{\mu}} X_t^\gamma, \quad (3)$$

where

$$\hat{\mu} = \gamma\mu + \frac{1}{2}\sigma^2\gamma(\gamma - 1). \quad (4)$$

Our earlier assumption regarding $\gamma$ ensures that $\hat{\mu} < r$ and hence (3) is positive.

\(^2\)We illustrate in the appendix that our reduced form in (2) follows from a setting in which the output or the cost of labor is related to $\gamma$. The convexity effect is also seen in, for example, Dixit and Pindyck (1994). Alternatively, if the firm’s profit is piecewise linear with an increasing slope, our setup can be considered as an approximation of this setting.

\(^3\)This condition ensures that the perpetual value in (3) is finite. Alternatively, the condition can be written as $\mu < r/\gamma - (\gamma - 1)\sigma^2/2$ with $\gamma > 1$. 

6
3 Investment strategies under preemption

We augment the model to include two asymmetric firms. The difference between the firms stems from their ability to exploit internal adjustment possibilities, that is, $\gamma$. The high-quality firm has quality parameter $\gamma_H$, the low-quality firm has quality parameter $\gamma_L < \gamma_H$. The firms consider whether to enter the new market first. The winner obtains the position as first mover (leader), while the other firm retains the option to enter the market at a later point. We define the firm’s profit in the new market, so that the first mover is able to extract a high profit prior to the entry of the follower. Thus, we let the first mover’s profit be as in the monopoly case. However, once the follower enters, the first-mover is no longer able to extract the high profit but it still enjoys the benefit of being the leader. To model this we let each firm’s profit be a scaling of the profit in (2). Specifically, after the follower’s entry, the leader’s profit is scaled by $\delta_L$, whereas the follower’s profit is scaled by $\delta_F \leq \delta_L \leq 1$. We are interested in understanding how the investment decision is affected by a persistent first-mover advantage (and the convexity effect). Having $\delta_L - \delta_F > 0$ allows us to control for this.\textsuperscript{4}

3.1 The follower

Assume that the high-quality firm is preassigned as the follower; the analysis follows similar steps if instead the low-quality firm is preassigned as the follower. The

\textsuperscript{4}Similar to Pawlina and Kort (2006) we think of this reduced form modeling as capturing the outcome of a Cournot duopoly game ($\delta_L = \delta_F$) as well as the advantage of being the market leader in a Stackelberg duopoly game ($\delta_L > \delta_F$). Alternatively, we could let the firms face the same market price, $X$, but that being the first mover allows this firm to serve a larger fraction of the market as the leader. This is the outcome if one introduces a market demand function and allows for Stackelberg duopoly competition. In a related setting we can think of the first mover as a firm being an early mover in IPO waves. Banerjee et al. (2016) show that leaders’ profitability remain higher than followers’ profitability. We later consider the possibility that the first-mover advantage is lost after a period of time.
follower chooses the optimal investment strategy to maximize its value. We denote the value of the high-quality firm as \( V_{H,F}(X) \). The first subscript indicates the firm’s quality and the second subscript denotes the position in the new market. That is, at time \( t \) the high-quality firm solves the following problem:

\[
V_{H,F}(X_t) = \sup_{\tau_F} E \left[ e^{-r(\tau_F-t)} \left( \int_{\tau_F}^{\infty} e^{-r(s-\tau_F)} \pi_{H,F}(X_s) ds - I \right) \right],
\]

where \( \tau_F \) denotes the time of investment of the follower. We follow the usual steps in analyzing real option problems (e.g., Dixit and Pindyck, 1994; Hackbarth et al., 2007). That is, we transform the problem to a solution of a differential equation and require value matching and smooth pasting at the investment threshold; details are postponed to the appendix. From this we obtain that the follower’s value before investment is

\[
V_{H,F}(X) = \left( \frac{w\delta_F}{r - \hat{\mu}_H} X_{H,F}^{\gamma_H} - I \right) \left( \frac{X}{X_{H,F}} \right)^{\nu},
\]

where \( \nu = \frac{1}{2} - \frac{\mu}{\sigma^2} + \frac{\sqrt{(\sigma^2-2\mu)^2+8r\sigma^2}}{2\sigma^2} \), and the optimal investment threshold is

\[
X_{H,F} = \left( \frac{\nu}{\nu - \gamma_H} r - \hat{\mu}_H \right) \left( \frac{w\delta_F}{r - \hat{\mu}_H} \right)^{\frac{1}{\gamma_H}}.
\]

The value of the high-quality firm as a follower after investing is

\[
V_{H,F}^+(X) = \frac{w\delta_F}{r - \hat{\mu}_H} X^{\gamma_H}.
\]
3.2 The first mover

Suppose instead that the high-quality firm is preassigned the role of first mover. The first mover chooses its optimal investment strategy, taking into account the future entry of its competitor. That is, at time $t$ the firm solves

$$
V_{H,FM}(X) = \sup_{\tau_{FM}} \mathbb{E} \left[ e^{-r(\tau_{FM} - t)} \left( \int_{\tau_{FM}}^{\tau_F} e^{-r(s-\tau_{FM})} \pi_{H,M}(X_s) ds - I \right) + e^{-r(\tau_F - t)} \int_{\tau_F}^{\infty} e^{-r(s-\tau_F)} \pi_{H,L}(X_s) ds \right],
$$

(9)

where $\tau_{FM}$ denotes the time at which the first mover enters the market; as before $\tau_F$ denotes the time of investment by the preassigned follower.

We follow similar steps as for the follower to solve for the optimal investment strategy and the corresponding firm value. The main difference is that the first mover takes into account when the follower decides to enter the market. We assume this happens the first time $X$ hits the low-quality firm’s optimal follower investment threshold, $X_{L,F}$. This subsequently reduces the leader’s profit flow scaling from 1 to $\delta_L$. We obtain that the value of the high-quality firm before investing is

$$
V_{H,FM}(X) = \left( \frac{w}{r - \tilde{\mu}_H} X_{H,FM}^{\gamma_H} - I \right) \left( \frac{X}{X_{H,FM}} \right)^{\nu} - (1 - \delta_L) \frac{w}{r - \tilde{\mu}_H} X_{L,F}^{\gamma_H} \left( \frac{X}{X_{L,F}} \right)^{\nu},
$$

(10)

and the optimal investment threshold for the first mover is

$$
X_{H,FM} = \left( \frac{\nu}{\nu - \gamma_H} \frac{I(r - \tilde{\mu}_H)}{w} \right)^{\frac{1}{\nu}}.
$$

(11)

The value of the high-quality firm as a leader, before the follower enters, is

$$
V_{H,M}(X) = \frac{w}{r - \tilde{\mu}_H} X^{\gamma_H} + \frac{(\delta_L - 1)w}{r - \tilde{\mu}_H} X_{L,F}^{\gamma_H} \left( \frac{X}{X_{L,F}} \right)^{\nu},
$$

(12)
and the follower enters the first time \( X = X_{L,F} \). The value of the high-quality firm as a leader, after the follower has entered, is \( V_{H,L}(X) = \frac{\delta_{L,X} X^{rH}}{r - \mu_H} \).

### 3.3 Preemption

We now turn to the problem of determining the optimal investment strategy and corresponding firm value when preemption is possible. To determine the optimal preemption strategy we follow, for example, Fudenberg and Tirole (1985) and Pawlina and Kort (2006). The firm must choose between becoming a first mover now or letting the other firm be the first mover. The cost of the former strategy is that it gives up the value of waiting, and the cost of the latter strategy is that it gives up profits. Thus, we first find the point at which it is at least as valuable for the firm to invest as the first mover as it is to postpone investment and invest as a follower. For the high-quality firm we denote this indifference point as \( X_{H,P} \), which satisfies:

\[
V_{H,F}(X_{H,P}) = V_{H,M}(X_{H,P}) - I. \tag{13}
\]

The indifference threshold, \( X_{H,P} \), has to satisfy a non-linear equation that does not have a closed-form solution. Therefore, we eventually have to this numerically.

As a second step, we need to characterize the preemptive equilibrium. To do so we use the indifference threshold for the high-quality firm, \( X_{H,P} \), and the low-quality firm, \( X_{L,P} \). The earliest point at which a firm is willing to invest is determined by the lower of the indifference thresholds and the optimal investment threshold when the firm is preassigned the role of first mover. That is, the preemption threshold of firm \( i \) is given by \( \hat{X}_{i,P} \stackrel{def}{=} \min(X_{i,P}, X_{i,FM}) \). The equilibrium is then determined by the firm with the lowest preemption threshold. This firm is willing to invest first and thereby secures the role of first mover. However, since the firm is investing
earlier than it would have done without the risk of preemption, it finds it optimal to postpone investment as long as possible. The firm can postpone investment just until its competitor would be willing to invest. For example, if the high-quality firm has $\bar{X}_{H,P} < \bar{X}_{L,P}$, then the high-quality firm becomes the first mover and it postpones investment until $X = \min\{\bar{X}_{L,P}, X_{H,FM}\} \overset{\text{def}}{=} X^*_H$. We collect our results in Proposition 1.

**Proposition 1.** Suppose the high-quality firm becomes the first mover. The ex ante value of the first mover for $X < X^*_H$ is given in (10) using $X^*_H$ instead of $X_{H,FM}$. The ex ante value of the low-quality firm investing as a follower is as in (6) using $\gamma_L$ instead of $\gamma_H$. The indifference threshold, $X_{H,P}$, solves the non-linear equation

$$
\left( \left( \frac{\delta_F w}{r - \hat{\mu}_H X_{H,F}^\gamma} - I \right) X_{H,F}^{-\nu} + \frac{(1 - \delta_L) w}{r - \hat{\mu}_H} X_{L,F}^{\gamma} X_{L,F}^{-\nu} \right) X_{H,P} = \frac{w}{r - \hat{\mu}_H} X_{H,P}^{\gamma} - I,
$$

where $X_{H,F}$ and $X_{L,F}$ are the follower thresholds of the high- and low-quality firms, respectively, following from (7).

The two firms differ in their quality. Quality has a direct impact on a firm’s value in (3) through the market index and indirectly through the “quality-adjusted growth rate” in (4). This spills over to (14). A change in the growth rate, $\mu$, impacts the quality-adjusted growth rate, $\hat{\mu}$, more for the high-quality firm than for the low-quality firm. This means that a decrease in the growth rate decreases the present value of receiving the same cash flow more for the high-quality firm than for the low-quality firm.\footnote{More formally we have that $\frac{\partial^2}{\partial \gamma \partial \mu} \left( \frac{1}{r - \mu} \right) > 0$.} The high-quality firm is, therefore, less eager to invest. On the other hand, the high-quality firm gets a higher value effect of the scaling factor, $\delta$. The trade-off between these factors determines which firm gets most out of being the first mover.
4 Market profitability and first-mover decision

The above results provide the general framework for analyzing the trade-offs between a firm’s quality and market profitability. The interdependence of the firms’ strategies does not allow us to make further analytical discussions. Therefore, we turn to a numerical analysis to obtain further insights and relate to implications for empirical studies. Our objective is to analyze the implications of the trade-offs between a firm’s quality, the growth rate, and the first-mover advantage, and we focus on the central effects concerning these elements. In what follows we consider a set of benchmark parameters unless otherwise stated. The high-quality firm has $\gamma_H = 1.5$ and the low-quality firm has $\gamma_L = 1.25$. The investment cost is $I = 100$. We consider different levels of the growth rate, $\mu$, and keep the volatility at $\sigma = 0.20$. We let $\delta_F = 0.4$ and impose a persistent first-mover advantage of 30% ($\delta_L = 1.3 \delta_F$). Finally, we assume a risk-free rate of 4%. Below, we analyze variations of the parameters to generate a number of predictions.

We begin our analysis by considering the effects of quality, $\gamma$, and the growth rate, $\mu$, on the investment decision. The investment policies of the firms are shown in Figure 1. Each panel in the figure shows six thresholds, three for the low-quality firm (gray lines) and three for the high-quality firm (black lines). The dotted lines and the dashed lines represent the investment thresholds of a preassigned follower and a preassigned first mover, respectively. The solid lines represent the rent equalizing thresholds in the presence of preemption; see (14). For a given level of the growth rate the firms invest if the market profitability index becomes high enough.

Panel (a) and panel (b) show what happens as the quality of the low-quality

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6In the following we focus on effects through the growth rate $\mu$ rather than the volatility $\sigma$. We vary $\mu$ as it intuitively relates directly to the quality-adjusted growth rate $\hat{\mu}$. Clearly, a change in volatility also affects $\hat{\mu}$. We are grateful to a referee for pointing this out.
(a) Varying the low-quality firm’s quality, $\gamma_L$, for a low growth rate, $\mu = 0$. $\gamma_H$ is fixed at 1.5.

(b) Varying the low-quality firm’s quality, $\gamma_L$, for a high growth rate, $\mu = 1.6\%$. $\gamma_H$ is fixed at 1.5.

(c) Varying the growth rate, $\mu$, for $\gamma_L = 1.25$ and $\gamma_H = 1.5$. $\mu^*$ is indicated with the dash-dotted line.

Figure 1: **Investment thresholds.** The figure presents the investment thresholds of the high-quality (the black lines - $\gamma_H = 1.5$) and low-quality (the gray lines - $\gamma_L = 1.25$) firms. The dotted and dashed lines represent the investment threshold if the firm is preassigned the role of follower and first mover, respectively. The solid lines represent the indifference thresholds. When the black solid line is below the solid gray line, the high-quality firm is the first mover and vice versa.
firm, $\gamma_L$, is changed. Panel (a) considers the case of a low growth rate, $\mu = 0$, panel (b) considers the case of a high growth rate, $\mu = 1.6\%$. Both panels show the intuitive ordering that the follower’s investment threshold is above the preassigned first mover’s threshold. Due to preemption, a firm cannot optimally exploit the value of the option to wait, and hence the first mover invests earlier in the preemptive equilibrium than in the pre-assigned first mover case. A lower quality (decreasing $\gamma_L$) makes a firm invest earlier if it is a follower or a pre-assigned first mover. The reason for this is that a lower convexity of the profit implies that the value of waiting decreases. However, whether this effect carries over to the preemptive equilibrium depends on the growth rate. A higher growth rate increases a firm’s incentive to invest (the investment threshold decreases), and thus it becomes more important to win the preemption game. As discussed in relation to (14), this value effect of a higher $\mu$ is amplified the higher $\gamma$ is, because the two components complement each other in calculating the present value of cash flows. Thus, the net effect of a lower convexity depends on the growth rate. That is, when the growth rate is high, the value effect dominates implying that the high-quality firm preempts the low-quality firm. When the growth rate is low, the convexity effect dominates so that it is too expensive for the high-quality firm to preempt the low-quality firm.

We elaborate on the effect of the growth rate in panel (c), now holding $\gamma_L = 1.25$ and $\gamma_H = 1.5$ fixed. The ordering of the investment threshold of the preassigned first mover and preassigned follower is monotonic in the growth rate. However, when the growth rate decreases, the preemptive threshold increases implying that firms invest later. This is intuitive since a lower growth rate leads to a larger adjustment in the discount rate (see (4)) and therefore it reduces the value of the investment opportunity. As a result, with a high growth rate the high-quality firm preempts the low-quality firm, but as the growth rate gets low enough the low-quality firm
becomes the first mover in the preemptive equilibrium. Thus, there is a cut-off level of the growth rate such that the high-quality firm preempts the low-quality firm as long as the growth rate is larger than this cut-off level. For later use we denote the cut-off level as \( \mu^* \). In panel (c) we have \( \mu^* \) slightly higher than 1% as indicated with the dash-dotted line. The above results lead to the following prediction regarding how firm quality and the growth rate impact the preemption equilibrium:

**Prediction 1.** *In industries with a relatively low growth rate it is more likely that low-quality firms move first.*

Prediction 1 relates to Jiang et al. (2015) who provide evidence that a high growth rate leads to a positive relation between competition and investment and that industry leaders invest more.\(^7\) The prediction also fits well with R&D-intensive technology industries, in which less established firms are often the first ones to enter into new markets. This result is related to, for example, Phillips and Zhdanov (2013) and Huisman and Kort (2015), who find that small firms exercise investment options earlier and are more likely to invest in new technologies. What we show is that this effect can as well be caused by a sufficiently low growth rate on the new market.\(^8\)

5 Patents, profitability, and investment timing

We relate our analysis of the first-mover advantage to patents. The intention behind patents is to encourage investments in new technologies by deferring the entry of a

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\(^7\) In line with the literature on preemptive competition and investment (Boyer et al., 2004; Pawlina and Kort, 2006; Mason and Weeds, 2010) we also find that a lower growth rate postpones investments in the new market.

\(^8\) We also consider effects of other parameters. To save space we do not report detailed results. In brief, a higher volatility, \( \sigma \), (see also the appendix) or investment cost, \( I \), decreases \( \mu^* \), whereas a higher scaling, \( w \), increases \( \mu^* \). We return to implications of \( \delta_L \) later.
follower. That is, one role of patents is to ensure high enough future profits to the first mover thus facilitating firms’ willingness to incur potentially high investment costs. However, studies have shown that having patents in a competitive setting may lead firms to postpone investment (Weeds, 2002).

Patents can be captured by two channels. One channel relates to the effectiveness with which the patent curb the first mover’s profit reduction. This is the first-mover advantage measured by the difference between $\delta_L$ and $\delta_F$. Moreover, first-mover advantages are likely to increase with the lead time (Huff and Robinson, 1994), hence $\delta_L$ could be positively related to the length of the time period in which the firm has patent protection. The other channel is the time length of the patent. Thus, we augment the model to include the expected time of patent protection. To avoid making the framework unnecessarily complex we introduce the extension in a straightforward manner. Specifically, the first-mover advantage is lost by the occurrence of a Poisson driven event with intensity $\lambda$ once the follower has entered. In expected terms this implies that the first-mover advantage is lost after $1/\lambda$ years.

To have the high-quality firm preempting for more parameters ($\mu^*$ lower), we analyze the effects of $\delta_L$ and $\lambda$ with a slightly modified parametrization. We increase the investment cost, $I = 120$, decrease the level, $w = 10$, and let the follower’s profit scaling be low, $\delta_F = 0.15$. Figure 2 reports the results. To address the effect of the first-mover advantage we assume there is no risk of losing the first-mover advantage in panels (a) and (b); that is $\lambda = 0$. Panel (a) shows that the growth rate cut-off decreases in the first-mover advantage ($\delta_L$ increases). The intuition for this is that a higher first-mover advantage makes it more costly to become the follower in the preemption game. This creates a stronger incentive for a firm to preempt its

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9 We are grateful for a reviewer suggesting this extension to us. The details of the model are postponed to the appendix.
competitor, but the effect is not of equal importance for the high- and low-quality firm. Our analysis related to Figure 1 showed this in terms of the growth rate. Similarly, a higher first-mover advantage benefits the high-quality firm the most. Thus, the high-quality firm has a bigger incentive to preempt the low-quality firm implying that the high-quality firm is willing to invest early for a lower growth rate.

To elaborate on how patents affect incentives to invest we keep the growth rate constant at $\mu = 0.5\%$ in panel (b) which illustrates the preemption threshold as the first-mover advantage increases. The solid line corresponds to the high-quality firm, the dashed line corresponds to the low-quality firm. When $\delta_L = \delta_F$ there is no (persistent) first-mover advantage. When $\delta_L = 1$ the first mover has a very high advantage allowing the firm to obtain monopolist profits also after entry of a follower. When the entrance of a follower severely hampers the first mover’s profit ($\delta_L$ is close to $\delta_F$), firms optimally pre-empt later. This is due to the fact that the long-run difference in the value between being first mover or follower becomes small. Thus, a firm is better off exploiting the value of waiting to invest. This effect is strongest for the high-quality firm implying that the low-quality firm wins the preemption game. Similar to the discussion related to panel (a), we have that increasing the first-mover advantage benefits the high-quality firm relatively more. Hence, the high-quality firm eventually becomes the first mover (when $\delta_L = 0.25$).

The effect of the expected time length of patent protection is presented in panels (c) and (d). For a given $\delta_L$, panel (c) shows that $\mu^*$ increases when the risk of losing patent protection, $\lambda$, increases. That is, less patent protection tends to induce the low-quality firm to be the first mover. When $\delta_L$ increases, $\mu^*$ moves downwards as in panel (a). This holds also when the length of patent protection decreases, and thus there is a trade-off such that a high $\delta_L$ compensates for a shorter length of patent protection. Panel (d) shows the preemptive investment thresholds for the
Figure 2: **First-mover advantage and investment** For increasing first-mover advantage, $\delta_L$, panel (a) illustrates the growth rate cut-off value, $\mu^*$, with no termination of patent. Panel (b) presents the investment threshold in the preemption equilibrium fixing the growth rate at $\mu = 0.5\%$ with no termination of patent. Panel (c) shows the growth rate cut-off value, $\mu^*$, for increasing risk of termination of patent, $\lambda$, for three levels of $\delta_L$. Panel (d) shows the preemptive investment threshold for increasing risk of termination of patent, $\lambda$, for three levels of $\delta_L$. 

(a) Growth rate cut-off, $\mu^*$ (in percentages) for varying $\delta_L$. The high-quality (low-quality) firm is the first mover for $\mu > \mu^*$ ($\mu < \mu^*$).

(b) Investment thresholds fixing $\mu = 0.5\%$. The grey (black) line corresponds to the low-quality (high-quality) firm; the solid line is preemptive equilibrium.

(c) Growth rate cut-off, $\mu^*$ (in percentages) for varying $\lambda$. The dotted (dashed; solid) line corresponds to $\delta_L = 0.3$ (0.5; 0.8).

(d) Preemptive investment thresholds fixing $\mu = 0.5\%$ and varying $\lambda$. The dotted (dashed; solid) line corresponds to $\delta_L = 0.3$ (0.5; 0.8).

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same combination of $\lambda$ and $\delta_F$ as in panel (c), keeping $\mu = 0.5\%$ as in panel (b).\textsuperscript{10} When the length of patent protection decreases, the low-quality firm eventually becomes the first mover. This effect is counterbalanced by a higher $\delta_L$. We also note that the investment time increases in $\lambda$. Hence, stronger protection either in terms of a higher $\delta_L$ or a lower $\lambda$ decreases the investment threshold, and thus patent protection expedites investment by the high-quality firm. This is in line with the evidence in Kanwar and Evenson (2003), who show that intellectual property rights provide incentives for spurring innovation.

Our analysis shows that if a firm cannot exclude future entrants, it is likely that the low-quality firm becomes the first mover and the initial investment in a new market is deferred. Hence, allowing for patents reduces the underinvestment problem and increases R&D. This result contrasts the findings in Weeds (2002), who shows that the option to get a patent is likely to induce a waiting game and increase underinvestment. Our analysis thus complements the one in Weeds (2002) and it highlights that patents also have positive effects. Furthermore, patents make it more likely that entry in the new market is carried out by the high-quality firm. This leads to the following predictions regarding the effects of patents:

**Prediction 2.a.** A higher level of patent protection expedites investment in new markets.

**Prediction 2.b.** An increase in patent protection increases the probability that new technology is introduced by high-quality firms.

\textsuperscript{10}The investment thresholds are similar to the solid black-gray line in panel (b). That is, we do not show the investment thresholds for the firm which does not become the first mover.
6 Observing the first-mover advantage

The notion that early investment is associated with a first-mover advantage is widely debated in the literature (Lieberman and Montgomery, 1988, 1998; Suarez and Lan- zolla, 2005). Several papers try to quantify the first-mover advantage using revenue or market-share based measures (Robinson et al., 1994; Boulding and Christen, 2003; Coeurderoy and Durand, 2004). However, the literature has reached mixed conclusions (Golder and Tellis, 1993; Suarez and Lanzolla, 2007). We argue that empirical studies risk over- or underestimating the first-mover advantage if they do not take the underlying reasons for a firm to become the first mover into account.

One method for quantifying the first-mover advantage is to consider differences in firm values once both firms have invested. Thus, we calculate a firm’s value as

$$W(X; \gamma, \delta) = \frac{\delta w}{r - \mu} X^\gamma.$$  \hspace{1cm} (15)

To understand the problem of estimating the first-mover advantage we consider the difference between the value of the two firms. We denote this as $\Delta W_H$ if the high-quality firm is the leader and we split it into two components:

$$\Delta W_H = [W(X; \gamma_H, \delta_L) - W(X; \gamma_H, \delta_F)] + [W(X; \gamma_H, \delta_F) - W(X; \gamma_L, \delta_F)].$$ \hspace{1cm} (16)

Similarly, $\Delta W_L$ is the difference if the low-quality firm is the leader. The first term represents the net value stemming from the difference in market position; that is, the difference between leader and follower profits given the same level of quality. The second term is the net value due to the intrinsic difference in quality. Proposition 2 shows how the two components depend on the profit scaling and the convexity effect.
(a) High growth rate ensuring that the high-quality firm is the first mover, \( \mu = 1.2\% \).

(b) Low growth rate ensuring that the low-quality firm is the first mover, \( \mu = 0.8\% \).

Figure 3: First-mover advantage. The figure illustrates the value of the high- and low-quality firm, respectively, as a function of the market index, \( X \), for two levels of growth rate. The solid (dashed) line represents the value of the first mover (follower). The dotted line represents the hypothetical value of the first-mover firm, given it had invested as a follower. The gray area between the dashed and dotted lines represents the difference in value due to quality differences; the hatched area between the solid and dotted lines represents the true first-mover advantage.

Proposition 2. The true first-mover advantage increases in \( \delta_L - \delta_F \). The quality advantage, when the high-quality (low-quality) firm is the first mover, is positive (negative), if \( X > \left( \frac{r-\hat{\mu}_H}{r-\hat{\mu}_L} \right)^{\frac{1}{\gamma_H - \gamma_L}} \). The absolute value of the quality advantage increases in \( \delta_F \) and, if \( \mu > -\frac{\sigma^2}{2}(2\gamma_L - 1) \) and \( X > 1 \), also in \( \gamma_H - \gamma_L \).

Figure 3 illustrates how the quality advantage complicates an empirical study of the first mover advantage, cf. the sign effect stated in Proposition 2. The figure shows the value of being on the new market as the market index changes (recall that \( \delta_L \) is 30\% higher than \( \delta_F \)). The solid line represents the value of the first mover, and the dashed line represents the value of the follower. The dotted line represents the value of the first mover, had it instead entered the market as a follower. In panel (a) the high-quality firm is the first mover (\( \mu = 1.2\% > \mu^* \)), in panel (b) the low-quality firm is the first mover (\( \mu = 0.8\% < \mu^* \)). The hatched area between the solid and
dotted lines represents the true first-mover advantage, whereas the quality advantage is measured by the gray area between the dashed and dotted lines. Panel (a) shows that both the true first-mover advantage component and the quality component in (16) are positive. Since it is only possible to observe the aggregate difference in market value in data, the true first-mover advantage is likely to be overestimated in empirical studies. For a low growth rate, panel (b) shows that the quality advantage component is negative (the dotted line is below the dashed line). This implies that the true first-mover advantage is underestimated. As stated in Proposition 2, a larger asymmetry between the firms implies a more negative quality component. This eventually dominates the true first-mover advantage component and, thus, one is falsely led to conclude that there is a first-mover disadvantage. Hence, the estimation bias is larger the more asymmetric firms are. Depending on the sign and size of the components, an empirical analysis thus over- or underestimates the true first-mover advantage. This gives us the following predictions:

**Prediction 3.** *The estimation bias is increasing in firms’ asymmetry; the sign of the bias depends on the determinants of a firm being the first mover.*

Hence, in line with Robinson et al. (1992) and VanderWerf and Mahon (1997) our analysis demonstrates that an empirical analysis of first-mover advantages must control for firms’ competitive strengths. Measures such as revenue, market share, or market value may lead to false conclusions if one does not adequately control for firms’ asymmetry and market growth. With a large pool of firms it very likely that one includes a broader mixture of the cases considered here. Our analysis shows that such an analysis is likely to give a wrong estimate of the first-mover advantage. This provides a possible explanation for why empirical studies covering a broad range of
industries cannot establish whether there is a first-mover advantage or not (Golder and Tellis, 1993; Suarez and Lanzolla, 2007). In contrast, studies focusing on specific industries are able to find positive results (Huff and Robinson, 1994; Shankar et al., 1998; Makadok, 1998; Coeurderoy and Durand, 2004).

6.1 Implications of the strength of competitors

The interplay between firms’ quality and market growth has interesting implications for investment timing and the firm’s value before entry in the market. We denote this value as the ex-ante value. Figure 4 depicts the value as a function of the competitor’s quality. In each panel, the solid (dashed) line depicts the case of the high-quality (low-quality) firm. To emphasize the forces at play, we consider a high and low growth rate as in Figure 1.

Consider panel (a) with $\mu = 1.6\%$. If the firms have the same quality ($\gamma_H = \gamma_L = 1.5$) it is random which firm becomes the first mover. When we decrease $\gamma_L$ we know from Figure 1 that the high-quality firm is the first mover ($\mu > \mu^*$) and the low-quality firm’s willingness to preempt decreases ($X_{L,P}$ increases). This allows the high-quality firm to execute the investment decision later and hence better exploit the value of waiting to invest. Therefore, the ex ante value of the high-quality firm increases, and since the low-quality firm gets a lower profit once it invests as a follower, its ex ante value intuitively decreases as $\gamma_L$ decreases. Panel (b) uses $\mu = 0 < \mu^*$, thus the low-quality firm is the first mover. Symmetric firms ($\gamma_L = \gamma_H = 1.25$) have the same ex ante value. An increase in the quality of the high-quality firm intuitively increases the high-quality firm’s value. Interestingly, the low-quality firm’s value also increases. When the growth rate is low, a high-quality firm considers a follower status as being less bad implying that the preemption
threshold of the high-quality firm increases. This allows the low-quality firm to postpone its preemptive investment and thus it better exploits the value of the option to wait. Our Nokia–Apple example points in a similar direction. Had Nokia initially been in a weaker position, it may have entered the smart phone market much earlier. This would have had a negative impact on the profits of Apple. This result relates to Carlson et al. (2014), who show that competition partly reduces a firm’s incentives to undertake new investments when a positive shock occurs. What we find is that the preemption risk by a higher productivity, $\gamma_H$, of the high-quality firm is partly hedged by its incentive to exploit the value of waiting; this has positive spill-over effects to the low-quality firm.

More generally, the ex ante firm value can be non-monotonic in its rival’s competitive strength, $\gamma$. The reason for the non-monotonic relationship is that the higher growth rate makes preemption more valuable for the high-quality firm, and this effect eventually comes into play when the quality level is high enough. Our
results lead to the following predictions regarding the effect of competitor quality on how a firm’s value and investment timing depend on the growth rate:

**Prediction 4.a.** *Depending on which firm becomes the first mover, a change in the quality of a competitor has a decreasing, non-monotonic, or increasing impact on firm value.*

**Prediction 4.b.** *Depending on which firm becomes the first mover, a change in the quality of a competitor can either expedite or postpone investment.*

A change in a rival’s competitive strength does not have straightforward implications. For example, if a firm invests to improve its $\gamma$, it is insufficient to measure the success of that strategy by looking at changes in a competitor’s value. In addition to Carlson et al. (2014), our first prediction relates to Pawlina and Kort (2006), who also find that improved conditions of one firm can benefit its rival. Specifically, a lower investment cost of one firm can increase its competitor’s value. Related to our second prediction, Bulan et al. (2009) find that competition erodes option value, thus impacting investment timing. Considering an IPO as entry into a new market, Çolak and Günay (2011) show that high-quality firms do not always enter first. In an M&A cross-border wave context, Xu (2017) find that deals undertaken later in waves outperform those earlier in waves. Akdoğan and MacKay (2008) find empirical support for a trade-off between the value of waiting and strategic investing. Indeed, they find a non-monotonic relationship between industry concentration and investment speed with the investment speed being highest in oligopolistic industries.
7 Conclusion

In a dynamic model with two asymmetric firms we analyze a firm’s first-mover decision. The key contribution of the paper is that the asymmetry of profit convexity results in important effects on the order of entry and a firm’s valuation. Interestingly, the low-quality firm can become the first mover. The interplay between the determinants in the model highlights the challenges for empirical studies to avoid biasing the results when estimating first-mover advantages. The interplay also affects a firms ex ante value which can be non-monotonic in its competitor’s quality. Finally, we provide two insights on the effect of patents. First, patents induce firms to preempt earlier, thereby speeding up investment. Second, patents increase the probability of high-quality firms entering the market as first movers. Altogether, we demonstrate that the interplay among asymmetric profit convexity, market profitability, and first-mover advantage has wide implications.
Appendix A  Values and investment strategies

A.1  The value of the new market.

Using the profit flow in (2) an application of Itô’s Lemma implies that \( \pi \) follows a geometric Brownian motion with growth rate \( \hat{\mu} \) and volatility \( \gamma \sigma \), where \( \hat{\mu} = \gamma \mu + \frac{1}{2} \sigma^2 \gamma (\gamma 1) \). This proves (4). With the dynamics of \( \pi \) it follows that

\[
E_t[\pi(X_s)] = e^{\hat{\mu}(s-t)} \pi(X_t),
\]

and thus

\[
V(X_t) = \int_t^\infty e^{-r(s-t)} e^{\hat{\mu}(s-t)} w X_\gamma^\gamma ds = \frac{w}{r - \hat{\mu}} X_\gamma^\gamma.
\]

This proves (3).

A.1.1  Example: A relation between convexity and internal flexibility

We illustrate how the convexity parameter \( \gamma \) can be related to internal flexibility. The basics of the setting is as in Section 2.1, but as an example we let the firm continuously adjust its use of labor, \( L \). The firm’s profit stems from the proceeds of selling the output less the cost of labor used for production. Therefore, if we keep the scaling with \( \delta \) as in the main text, the profit is

\[
\pi = X \delta L^\alpha - cL,
\]

where \( c > 0 \) is the unit cost of labor and \( \alpha \in (0, 1) \) represents the productivity of labor (e.g., Dixit and Pindyck, 1994). We interpret a higher productivity as a higher quality of the produced good. By producing a good of higher quality the firm is able to service a larger fraction of the market at the same price. Let \( \gamma = 1/(1 - \alpha) > 1 \) and use the first order condition, \( \frac{d\pi}{dL} = 0 \), to get the optimal use of labor:

\[
L^*(X) = \left( \frac{\alpha \delta}{c} X \right)^{\gamma}.
\]
Inserting (20) in (19) and rewriting yields that the profit from the market is

$$\pi(X) = \delta^{\gamma} \hat{K} X^{\gamma},$$  \hspace{1cm} (21)

where \( \hat{K} = \frac{(1-\alpha)c}{\alpha} \left( \frac{\alpha}{c} \right)^{\gamma} \). Thus, profit can be convex in the market profitability, \( X \).

### A.2 The follower

To derive the optimal investment threshold corresponding to the stopping time \( \tau_F \) we follow the usual steps in analyzing real option problems (e.g., Dixit and Pindyck, 1994; Hackbarth et al., 2007). Thus, we have that the value of the follower before investment satisfies an ordinary differential equation (ODE) in the continuation region and we set up the value matching and smooth pasting condition corresponding to the follower’s decision to invest. We henceforth assume the firm is a high-quality firm so that \( \gamma = \gamma_H \) etc. After investing the follower’s value is simply the value in (3) (or (18)) scaled with \( \delta_F \). That is,

$$V_F^+(X) = \frac{w \delta_F X^{\gamma}}{r - \hat{\mu}},$$  \hspace{1cm} (22)

which is (8). We now determine the value of the follower before investment, \( V_F(X) \).

Since the follower does not receive (nor pays) any stream of payments until the exercise of the investment option, the ODE in the continuation region is:

$$0 = \frac{1}{2} \sigma^2 X^{2\gamma} \frac{\partial^2 V_F}{\partial X^2} + \mu X \frac{\partial V_F}{\partial X} - r V_F.$$  \hspace{1cm} (23)

We conjecture that the value function before investment can be written as

$$V_F(x) = k_{\nu_+} X^{\nu_+} + k_{\nu_-} X^{\nu_-}.$$  \hspace{1cm} (24)

Inserting this into (23) and rearranging terms shows that \( \nu \) (short hand for \( \nu_+ \) and \( \nu_- \)) needs to satisfy the quadratic equation

$$\frac{1}{2} \sigma^2 (\nu - 1) \nu + \mu \nu - r = 0.$$  \hspace{1cm} (25)
Hence, \( \nu \) can be written as

\[

\nu = \frac{1}{\sigma^2} \left( -\left( \mu - \frac{\sigma^2}{2} \right) \pm \sqrt{\left( \mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right),
\]

(26)

where \( \nu_+ \) is the positive and \( \nu_- \) the negative solution, respectively. Furthermore, setting \( \nu = 1 \) in (25) shows that \( \nu_+ > 1 \). To derive the constants in (24) we consider what happens as \( X \) decreases and increases, respectively. As \( X \) tends to 0, the value of the follower must tend to 0 as well. This implies that \( k_{\nu_-} = 0 \) and hence (after renaming \( \nu_+ \) to \( \nu \)):

\[

V_F(X) = \mathbb{A}X^\nu.
\]

(27)

On the other hand, as \( X \) tends to the investment threshold, \( X_F \), we have the value-matching condition (using (22)):

\[

V_F(X_F) = \mathbb{A}X^\nu \Delta = \frac{w_\delta_F X_F^\gamma}{r - \hat{\mu}} - I,
\]

(28)

implying that

\[

\mathbb{A} = \left( \frac{w_\delta_F X_F^\gamma}{r - \hat{\mu}} - I \right) X_F^{-\nu},
\]

and hence

\[

V_F(X) = \left( \frac{w_\delta_F X_F^\gamma}{r - \hat{\mu}} - I \right) \left( \frac{X}{X_F} \right)^\nu,
\]

(29)

as stated in (6). The smooth-pasting condition related to (28) yields

\[

\nu \left( \frac{w_\delta_F X_F^\gamma}{r - \hat{\mu}} - I \right) \left( \frac{X}{X_F} \right)^{-\nu - 1} \left| X_F \right. = \gamma \frac{w_\delta_F X_F^\gamma}{r - \hat{\mu}} \left| X = X_F \right.,
\]

(30)

thus

\[

X_F = \left( \frac{\nu}{\nu - \gamma} \left( \frac{r - \hat{\mu}}{w_\delta F} \right) \right)^{\frac{1}{\gamma}},
\]

(31)

for \( \gamma < \nu \). This proves (7).

### A.3 The first mover

Using derivations similar to (22) the leader’s value, after the follower has entered, is

\[

V_L(X) = \frac{w_\delta_L X^\gamma}{r - \hat{\mu}},
\]

(32)
Before the follower enters, the firm is a leader and gets a profit of \( wX^\gamma \). The first mover’s value satisfies

\[
0 = \mu X \frac{\partial V_{H,M}}{\partial X} + \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 V_{H,M}}{\partial X^2} - rV_{H,M}(X) + wX^\gamma \mu.
\]  

(33)

We conjecture a value function

\[
V_M(X) = D_0 + D_1 X^\gamma + D_2 X^\nu.
\]

(34)

Inserting this into (33) gives us \( D_0 = 0 \), \( D_1 = \frac{w}{r - \mu} \) and

\[
V_M(X) = \frac{w}{r - \mu} X^\gamma + D_2 X^\nu,
\]

(35)

where \( \nu \) is given in (26). The constant \( D_2 \) is determined using the optimal entry level of the follower. After the follower invests, the leader has a value determined by (32). Hence, \( D_2 \) must ensure that the value matching condition holds in \( X_F \):

\[
\frac{wX_F^\gamma}{r - \mu} X_F^\gamma + D_2 X_F^\nu = \frac{w\delta L X_F^\gamma}{r - \mu},
\]

(36)

which gives \( D_2 \) as

\[
D_2 = -\frac{(1 - \delta L)w}{r - \mu} X_F^{-\nu}.
\]

(37)

Notice that \( D_2 < 0 \). This makes perfect sense because the first mover is effectively short in the follower’s option to enter. Collecting (35) and (37) yields (12).

Before investment the ODE is equivalent to that of the follower prior to investing; see (23). Hence, we can write the first-mover’s value function as

\[
V_{FM}(X) = D_2 X^\nu + D_3 X^\nu.
\]

(38)

The value matching condition in \( X_{FM} \) gives us

\[
D_3 = \left( \frac{w}{r - \mu} X_{FM}^\gamma - I \right) X_{FM}^{-\nu}.
\]

(39)

Using smooth pasting in \( X_{FM} \) and (39) we obtain the optimal investment trigger of
the first mover as
\[
X_{FM} = \left( \frac{\nu}{\nu - \gamma} \frac{r - \hat{\mu}}{w} \right)^{\frac{1}{\gamma}}.
\]
(40)

This shows (11). Using equations (37)–(39) we can write the value of the first-mover firm prior to investment. To emphasize that the first-mover investment threshold and the follower threshold correspond to two different firms, we state the value for firm \(i \in \{L, H\}\) as being the first mover and firm \(j \neq i\) as being the follower; i.e. (10):
\[
V_{i,FM}(X) = \left( \frac{w}{r - \hat{\mu}_i} X_{i,FM}^{\gamma_i} - I \right) \left( \frac{X}{X_{i,FM}} \right)^{\nu} - \frac{(1 - \delta_L)w}{r - \hat{\mu}_i} X_{j,F}^{\gamma_j} \left( \frac{X}{X_{j,F}} \right)^{\nu}.
\]
(41)

### A.4 Preemption

The ex-ante value of the high-quality firm (before investment) follows from (10) (or (41)) using that the high-quality firm invests as the first mover the first time \(X = X_{H,P}^\ast\). The ex-ante value of the low-quality firm investing as a follower can be derived from (6) using that the high-quality firm invests the first time \(X = X_{H,P}^\ast\).

To derive the indifference threshold \(X_{H,P}\) we need to solve condition (13). The left-hand side follows from (6) using that the other firm has already invested due to preemption. The right-hand side follows from (12). Both expressions are evaluated in \(X = X_{H,P}\), which gives us
\[
\left( \frac{w \delta_F X_{H,F}^{\gamma_H}}{r - \hat{\mu}_H} - I \right) \left( \frac{X_{H,P}}{X_{H,F}} \right)^{\nu} = \frac{w}{r - \hat{\mu}_H} X_{H,P}^{\gamma_H} - \frac{(1 - \delta_L)w}{r - \hat{\mu}_H} X_{L,F}^{\gamma_L} \left( \frac{X_{H,P}}{X_{L,F}} \right)^{\nu} - I,
\]
(42)
from which it follows that
\[
\left( \frac{w \delta_F}{r - \hat{\mu}_H} X_{H,F}^{\gamma_H} - I \right) X_{H,F}^{-\nu} + \frac{(1 - \delta_L)w}{r - \hat{\mu}_H} X_{L,F}^{\gamma_L - \nu} \right) X_{H,P}^{\nu} = \frac{w}{r - \hat{\mu}_H} X_{H,P}^{\gamma_H} - I,
\]
(43)
which is condition (14) in the proposition.
A.4.1 Effect of volatility

A change in volatility also affects $\tilde{\mu}$ and the cut-off level for investment (we thank a referee for suggesting this to us). Figure 5(a) illustrates the cut-off $\mu^*$ as a function of $\sigma$; clearly, the cut-off decreases in the volatility. This implies that a more volatile environment makes it more likely that the high-quality firm is the first mover. Given the monotonicity in the figure we could have analyzed a cut-off in the volatility instead of a cut-off in the growth rate. Thus, we include in Figure 5(b) the preemption thresholds for the $\gamma_H$-firm (black) and $\gamma_L$-firm (gray); the figure is similar to Figure 1(c), but now we vary the volatility. As seen in the figure, we could define a cut-off $\sigma^*$ so that the high-quality firm is the first mover for $\sigma > \sigma^* \approx 0.23$. Hence, the volatility matters, but in a similar fashion as the growth rate and our predictions can be reinterpreted accordingly.

A.5 Patents

In Section 5 we introduce the possibility that the leader’s first-mover advantage is not permanent. This can be done in various ways. To avoid a time-dependent model
we let the loss occur according to a Poisson driven event with intensity (arrival rate) $\lambda$. However, this can also be done in several different ways. For example, an event can partially reduce the advantage, its impact can depend on the stage of the model, and the reduction can be ex ante unknown (for example following a beta distribution or depending on the level of the stat-variable $x$). To have a model which, on the one hand, is tractable, but, on the other hand, shows the implications of a diminished first-mover advantage we introduce a Poisson process when the follower enters and which terminates the first-mover advantage. Thus, the leader’s $\delta_L$ eventually becomes equal to the follower’s $\delta_F$.

The extension of the model implies the following main changes. First, the leader’s value after entry of the follower decreases from the value in (32) to

$$V_L(X) = \frac{\delta_L(\lambda)}{r - \bar{\mu}} w X^\gamma,$$  

(44)

where $\delta_L(\lambda) = \delta_L + \lambda \frac{\delta_F}{r - \bar{\mu}}$. After the Poisson jump the value decreases further to

$$V_L(X) = \frac{\delta_F}{r - \bar{\mu}} w X^\gamma.$$  

(45)

This change in the leader’s value spills over to the previous stages of the model. For example, the value of firm $H$ as the first mover changes from (41) to:

$$V_{H,FM}(X) = \left( \frac{\delta_M}{r - \bar{\mu}_H} w X_{H,FM}^{\gamma_M} - I \right) \left( \frac{X}{X_{H,FM}} \right)^\nu$$

$$- \left( \frac{\delta_M}{r - \bar{\mu}_H} - \frac{\delta_L(\lambda)}{r - \bar{\mu}_H + \lambda} \right) w X_{L,F}^{\gamma_M} \left( \frac{X}{X_{L,F}} \right)^\nu.$$  

(46)

Consequently, the condition for the indifference threshold, (14), changes to

$$\left( \left( \frac{\delta_F w}{r - \bar{\mu}_H} X_{H,F}^{\gamma_H} - I \right) X_{H,F}^{\nu - \nu} + \left( \frac{\delta_M}{r - \bar{\mu}_H} - \frac{\delta_L(\lambda)}{r - \bar{\mu}_H + \lambda} \right) w X_{L,F}^{\gamma_M - \nu} \right) X_{H,F}^{\nu}$$

$$= \frac{\delta_M w}{r - \bar{\mu}_H} X_{H,F}^{\gamma_H} - I.$$  

(47)
A.6 Observing the first-mover advantage

This section proves Proposition 2. We first rewrite the difference in the value of the two firms from (16). Inserting (15) yields:

$$\Delta W_H = (\delta_L - \delta_F)w \left( \frac{X^{\gamma_H}}{r - \bar{\mu}_H} - \frac{X^{\gamma_L}}{r - \bar{\mu}_L} \right).$$

(48)

Similarly

$$\Delta W_L = (\delta_L - \delta_F)w \left( \frac{X^{\gamma_L}}{r - \bar{\mu}_L} - \frac{X^{\gamma_H}}{r - \bar{\mu}_H} \right).$$

(49)

It follows that the true first-mover advantage (the first term) is increasing in the difference $\delta_L - \delta_F$, because the restriction on admissible $\gamma$s implies that $\bar{\mu} < r$.

Second, the quality advantage (the second term), when the high-quality firm is the first mover, is positive iff.

$$\frac{X^{\gamma_H}}{r - \bar{\mu}_H} - \frac{X^{\gamma_L}}{r - \bar{\mu}_L} > 0,$$

(50)

iff.

$$X > \left( \frac{r - \bar{\mu}_H}{r - \bar{\mu}_L} \right)^{\frac{1}{\gamma_H - \gamma_L}}.$$

(51)

Since the quality advantage has the opposite sign, when the low-quality firm is the first mover, this proves the second claim. Third, a higher $\delta_F$ clearly increases the absolute value of the second term in (49). Finally, from (4) it follows that $\frac{d\bar{\mu}}{d\gamma} > 0$ when $\mu > -\frac{\sigma^2}{2}(2\gamma - 1)$. By assumption only $X > 1$ is considered, so the numerators in the quality advantage are increasing in $\gamma$. Thus, the claim follows if

$$\frac{1}{r - \bar{\mu}_H} - \frac{1}{r - \bar{\mu}_L}$$

(52)

is increasing in $\gamma_H$. This in turn follows because $\frac{d\bar{\mu}}{d\gamma} > 0$. 

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References


