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Published in:
International Journal of Production Economics

DOI:
10.1016/j.ijpe.2019.03.003

Publication date:
2019

Document version:
Accepted manuscript

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Citation for published version (APA):
Accepted Manuscript

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PII: S0925-5273(19)30089-1
DOI: https://doi.org/10.1016/j.ijpe.2019.03.003
Reference: PROECO 7317


Received Date: 26 October 2017
Revised Date: 20 January 2019
Accepted Date: 5 March 2019


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An adjustable bi-level wholesale price contract for coordinating a three-level socially responsible supply chain under scenario-based stochastic demand

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Abstract
Recently, significant research attention has been paid to supply chain (SC) coordination. However, previous studies assume that the market demand is deterministic or has a certain probability distribution. Taking a new approach, this paper analytically addresses the coordination of an SC under a scenario-based demand and proposes an adjustable bi-level wholesale price contract. To analyse the coordination problem under a scenario-based demand, we focus on corporate social responsibility (CSR) investment as an important decision in the SC context and consider a three-tier SC (one manufacturer, one distributor, and one retailer) where the retailer faces a scenario-based stochastic CSR-sensitive demand. Considering a manufacturer-Stackelberg game model, the manufacturer as a leader determines the amount of investment in the CSR issues and the retailer as a follower decides on the order quantity by considering possible demand scenarios. First, we propose the decentralized and centralized decision-making models using a scenario-based approach. Afterward, to resolve the channel conflict in each possible demand scenario, an adjustable bi-level wholesale contract is proposed to entice all SC actors to take part in the coordination scheme. Moreover, a profit-sharing plan is developed based on the Nash-bargaining model to fairly share the extra profits among SC actors. Finally, in order to analyse the efficacy of proposed models, a numerical example and a set of sensitivity analyses are carried out. The results indicate that the proposed contract not only coordinates the three-level SC under a scenario-based stochastic demand, but also improves the CSR level in comparison with the decentralized structure.

Keywords: Channel coordination, Three-echelon supply chain, Scenario-based stochastic demand, Corporate social responsibility, Adjustable bi-level wholesale price contract.
1. Introduction

Recently, sustainability and social issues have attracted the attention of scholars, practitioners, governments, and so on. Corporate social responsibility (CSR) can be defined as a comprehensive set of voluntary activities, policies, and programs which are conducted by firms and incorporated into business operations with respect to the social subjects. Today, many international brand companies such as Walmart, Nike, and Adidas incorporate CSR activities into their supply chain (SC) networks (Panda et al., 2015). Cruz and Wakolbinger (2008) stated that many firms make the CSR efforts as a method for improving their corporate reputation, managing their risk, creating customer loyalty, and so on. In practice, investment in CSR issues will boost customer demand and consequently increase firm profitability. In the real-world cases, CSR is not limited to being within a company due to the interactions among SC actors. The influence of CSR issues extends to all the SC actors (Ashby et al., 2012). For instance, a manufacturer's CSR efforts increase the market share for the product, which in turn benefits the market share of all SC actors. Therefore, decisions related to the CSR should be coordinated in the SC context. Coordinating CSR issues will lead to achieving a sustainable competitive advantage and long-term success for the SC actors (Nematollahi et al., 2017a). In today's complex business environment, coordinating decisions throughout the SC can be an effective method of improving the performance of all SC actors. A decentralized setting, in which SC actors try to improve their individual profits, results in sub-optimal solutions and poor performance of the whole system. Using incentive mechanisms, coordination contracts aim to entice the SC actors to make optimal decisions from the perspective of the entire system (Chaharsooghi and Heydari, 2010).

In many real-world situations, it is difficult to obtain an exact estimation of major parameters such as market demand, lead time, production volume, and so on. The uncertainty of such parameters is an inevitable part of decision-making in the SC context. For instance, a retailer often has to decide on its order quantity before starting the selling season. In the case of fashion products, the retailer may only have some inaccurate knowledge about the future demand. Therefore, the retailer faces the difficult task of avoiding the risk of shortage or surplus inventory at the end of season. In practice, the uncertain demand of many products can be modeled using a scenario-based approach. For instance, the future demand of fuelwood, which is one of the main fuels for heating at a low price, can be forecasted using several distinct scenarios due to the nature of the fuelwood. Arabatzis et al. (2013) proposed a production-allocation model for the optimal management of fuelwood in Greece. They incorporated the uncertain demand using three scenarios: (1) the fuelwood demand will not increase, (2) the fuelwood demand will conservatively increase (i.e., by 50%), and (3) the fuelwood demand will extremely increase (i.e., by 100%) in the future while the fuelwood price is fixed. In addition to the fuelwood, a scenario-based demand can be observed in other industrial cases. Tsiakis et al. (2001) designed a multi-level SC network with multiple products in Europe. They considered three manufacturers producing 14 types of products with three possible demand scenarios and fixed retail prices. As noted above, channel coordination is a helpful method for
improving the entire SC’s performance and coordinating different decisions. However, most previous studies on channel coordination capture the uncertain nature of market demand with a unique probability and known distribution such as normal distribution. However, in real-world situations, it is often observed that the market demand can only be estimated by a set of discrete scenarios. It is a difficult task to determine the optimal order quantity when the market demand cannot be known with precision. Under such a case, the scenario-based approach should be applied to model this problem and optimize such decisions. However, to our knowledge, uncertainty has not yet been addressed using a scenario-based approach in the previous studies on SC coordination.

This study is motivated by a real-world industrial case study in the home appliances industry, where a leading manufacturer sells its products in the market through a distributor-retailer channel. The manufacturer aims to invest in CSR activities to positively influence the customers’ attitudes towards its products and consequently improve its market demand. When the manufacturer becomes involved in CSR, the distributor and retailer also benefit from the improvement of market demand. Therefore, the manufacturer plans to develop CSR in the SC in collaboration with the distributor and retailer. One of the main steps of a CSR collaboration in an SC is to determine how CSR investment would influence market demand (Nematollahi et al., 2017a). According to data from the retailer, who is in touch with the customers and determines the order quantity before starting the selling season, the market demand in the SC under study can be captured using a scenario-based approach. To be more precise, in the investigated case study, there are three scenarios for the market demand: (1) low-demand stochastic scenario, (2) medium-demand stochastic scenario, and (3) high-demand stochastic scenario. In the investigated case study, not only does the CSR investment of the manufacturer impact the other actors, but the order quantity of the retailer also influences the performance of other actors as it determines the amount of satisfied demand in the SC. The SC actors (i.e., the remanufacturer, distributor, and retailer) take part in a coordination model if and only if their individual profits under the coordination scheme are enhanced compared to those before coordination (Nematollahi et al., 2018). To coordinate the CSR decision under the scenario-based stochastic demand, in this paper, we propose a new contract, named adjustable bi-level wholesale price contract, not only to optimize the entire SC but also to ensure the participation of all SC actors.

Considering a socially three-level SC inspired by the case study discussed above, the current study tries to answer the following questions:

I. How can channel coordination be achieved when the stochastic future demand can only be predicted by a set of discrete scenarios?

II. How should contract parameters be set in a three-level SC in order to convince all SC actors to participate in the coordination plan in each demand scenario?

III. How can the manufacturer's CSR investment and the retailer's order quantity be coordinated when the SC faces a scenario-based stochastic CSR-dependent demand?
To answer the above research questions, in this paper, a decentralized three-level SC is considered where the manufacturer as a leader determines the CSR investment. The SC faces a scenario-based stochastic demand which is sensitive to the CSR investment made by the manufacturer. Using a scenario-based Newsvendor inventory system, the retailer determines the order quantity by considering all discrete possible demand scenarios. First, by considering a scenario-based approach, the optimal order quantity and CSR investment are determined under the decentralized structure, where SC actors try to optimize their individual profitability. Afterward, the investigated three-echelon SC is modeled as a single entity and the CSR investment and order quantity are optimized from point of view of the entire SC. The proposed scenario-based centralized model is designed in such a way that it improves the entire SC's profits compared to the decentralized model across all possible demand scenarios. To resolve the channel conflict and coordinate the SC, an adjustable bi-level wholesale price contract is proposed to incentivize the SC actors to participate in the coordination model. Moreover, a Nash-bargaining model is proposed to equitably share the surplus profit among the actors. The proposed contract not only maximizes the total SC profits but also ensures win-win profits for the SC actors across all demand scenarios.

This article is structured as follows. We provide related literature in Section 2. The problem definition and notations are presented in Section 3. Section 4 provides the decentralized and centralized models under single-scenario stochastic demand. Section 5 presents (1) decentralized, (2) centralized, and (3) coordination systems under the scenario-based stochastic demand. In Section 6, a numerical example and a set of sensitivity analyses are provided. The discussion and managerial insights are provided in Section 7. Section 8 presents conclusions and directions for future studies.

2. Literature review

In recent years, channel coordination has become one of the most interesting topics in the research field of supply chain (SC) management. Wholesale price, quantity discount, revenue sharing, quantity flexibility, buy-back and delay in payment are known as the most common contracts which are extensively used to coordinate SC systems. We refer the readers to the comprehensive reviews done by Cachon (2003) and Govindan et al. (2013) for more details.

Decision-making on replenishment policy and order quantity in the SC is of high importance both from an academic and practical point of view. There are numerous published papers which have used different incentive mechanisms to coordinate the order quantity decisions. Jaber and Osman (2006) developed a credit period scheme for order quantity coordination with constant demand. Li and Liu (2006) applied a quantity discount for coordinating order quantity decisions within a two-layer SC under stochastic demand. Chaharsoghi and Heydari (2010) used delay-in-payment as an incentive mechanism for the coordination of reorder points and order quantity decisions. Kamal Chaharsoghi et al. (2011) applied a quantity discount method for coordinating reorder points and order quantity within a serially connected two-level SC. Duan et al. (2012) proposed a quantity
discount scheme for order quantity coordination with short shelf life products. Gao et al. (2014) explored two-level SC coordination with stochastic demand by using a delay in payment scheme. Saha and Goyal (2015) proposed three different contracts in a two-stage SC where the demand was sensitive to both stock level and price. Ebrahimi et al. (2017) proposed a delay-in-payment contract to coordinate a two-level SC by considering a promotional effort-dependent demand. Jazianinejad et al. (2017) proposed a coordination model based on credit options to coordinate a two-level SC, where the market demand was stochastically dependent on the retail price. Recently, Hu and Feng (2017) developed a revenue-sharing scheme for order quantity coordination in a supplier-buyer chain with supply and demand uncertainty. Johari et al. (2018) proposed a bi-level coordination model based on the credit period in an SC under a periodic review inventory system. They assumed a price and credit-dependent demand and incorporated the impacts on time value of money into the model. Recently, Hosseini-Motlagh et al. (2018a) proposed a collaborative model to coordinate an SC consisting of a manufacturer and two competing retailers. They assumed a credit period and promotional effort-dependent demand and analyzed different behaviours of competing retailers.

Although coordination models have been extensively studied in two-echelon SCs, the literature on channel coordination in three-echelon SCs is scarce. As noted by Giri et al. (2016) and Panda et al. (2015), resolving channel conflict and coordinating decisions in a three-echelon SC is considerably more difficult than in two-echelon systems. Ding and Chen (2008) proposed a buy-back scheme for coordinating order quantity decisions in a three-level SC. Jaber et al. (2010) explored supplier-manufacturer-retailer SCs under continuous improvement with a learning-based approach. Van Der Rhee et al. (2010) investigated a multi-tier SC facing stochastic demand and used a special kind of revenue-sharing contract for order quantity coordination. By combining a quantity discount with a franchise fee contract, Modak et al. (2016) proposed an incentive plan to resolve the channel conflict in a manufacturer-distributor-duopolistic retailers chain under deterministic demand. In their study, the coordination model was discussed in terms of Cournot, Collusion, and Stackelberg games of the retailers. Hsueh (2015) developed a bi-level CSR collaboration problem for coordinating CSR performance level and order quantity decisions in a multiple suppliers-single manufacturer-multiple retailers chain.

Due to the significant effects of CSR activities on the performance of the SCs, many studies on the SCs have been done by considering CSR. However, considering CSR issues in the SC coordination literature is still in its infancy. Ni et al. (2010) studied a two-stage SC with CSR-sensitive deterministic demand and proposed a wholesale discount scheme for coordinating both CSR and order quantity decisions. Ni and Li (2012) considered a two-stage SC including an upstream supplier and a downstream firm where the actors interact with each other regarding their CSR behaviour. They analyzed the effects of exogenous parameters on the interaction between actors and conducted a game-theoretical approach to achieve equilibrium solutions under simultaneous and sequential games. Our paper is significantly different from the work conducted by Ni and Li (2012), as our model focuses on coordinating CSR and order quantity under a stochastic scenario-
based demand while Ni and Li (2012) concentrate on strategic interactions of CSR conduct in SCs. Goering (2012) analyzed a two-stage game under a bilateral monopoly in a retailer-manufacturer chain under two scenarios: (1) the retailer exhibits CSR and (2) the manufacturer exhibits CSR. Panda (2014) coordinated a retailer-manufacturer SC by considering two cases: (1) the retailer is involved in CSR initiatives, (2) the manufacturer is involved in CSR initiatives. Our paper is significantly different from the work done by Panda (2014), as we focus on three-echelon channel coordination under a scenario-based stochastic demand while Panda (2014) coordinates a two-echelon SC under a deterministic price-sensitive demand. Moreover, Panda (2014) uses a traditional revenue sharing contract to coordinate the channel while in this paper we propose a new adjustable bi-level wholesale price contract. Last but not least, Panda (2014) accounts for a firm’s CSR in the form of consumer surplus while we use CSR performance level to quantify the CSR index.

Hsueh (2014) explored a two-level SC coordination by incorporating CSR into a revenue-sharing contract. He assumed that the normally distributed demand was affected by the manufacturer’s social efforts. Modak et al. (2014) examined profit distribution and coordination in a two-stage dual-channel SC where the upstream actor was in charge of improving CSR. They proposed a quantity discount and bargaining strategy to coordinate the channel, assuming the market demand was price-dependent. Modak et al. (2015) coordinated a dual channel SC through a hybrid contract based on quantity discounts and franchise fees, considering CSR issues. Panda et al. (2015) analyzed wholesale price discounts and bargaining in a three-stage SC in which the manufacturer exhibited CSR. Using strategic bargaining and sub-game perfect equilibrium, Panda and Modak (2016) explored profit distribution and coordination where the SC actors aimed to exhibit CSR. Recently, Nematollahi et al. (2017a) developed the study of Hsueh (2014) and proposed a novel collaboration model for order quantity and CSR coordination. In the work conducted by Nematollahi et al. (2017a), channel coordination is analyzed where the market demand is projected under a single scenario, whereas in this study the market demand is represented under a stochastic scenario-based demand. Different from the collaboration model proposed by Nematollahi et al. (2017a), the proposed adjustable bi-level wholesale price contract in this study aims to achieve channel coordination under a stochastic scenario-based demand. Moreover, the proposed coordination contract in this study is able to coordinate a three-stage SC while Nematollahi et al. (2017a) coordinates a two-echelon SC.

Panda et al. (2017) proposed a revenue-sharing and Nash-bargaining strategy to coordinate a two-stage closed-loop SC. They assumed that the manufacturer aimed to improve CSR by recycling end-of-life items. Ma et al. (2017) coordinated a manufacturer-retailer chain with information asymmetry, where the retailer and manufacturer invested in marketing efforts and CSR activities, respectively. They analyzed wholesale price and two-part tariff contracts under the asymmetric and symmetric situations. Recently, some papers have tried to incorporate CSR into the channel
coordination by considering new measures such as customer service level (Nematollahi et al., 2017b, 2018).

To form a CSR collaboration in a SC system, there are some important factors that should be analyzed precisely. First, the SC structure and the SC actors involved in the coordination scheme should be determined (i.e., SC structure). Second, it is essential to properly define and incorporate the CSR factor into the problem and to specify how SC actors aim to improve CSR in the SC (CSR type). Third, the impacts of CSR activities on actors’ financial performance should be identified. Many previous studies in this research field have considered a CSR-sensitive demand assuming the CSR investment impacts the demand function. Under such a case, the demand function of the downstream actor (e.g., retailer) in the SC should be captured (demand type). Fourth, an effective strategy (e.g., coordination contract) should be designed in order to create a win-win situation for all SC actors involved in the CSR collaboration.

To better highlight the main contributions of the current study, a summary of the papers reviewed above along with the focus of the current study is given in Table 1; further, each paper is compared based on the four factors discussed above. As Table 1 shows, all the previous studies on CSR collaboration have considered either a deterministic demand or a stochastic demand in a single scenario. Our paper fundamentally differs from the aforementioned studies, as we analyse CSR collaboration where the market demand can be captured using a scenario-based approach. In many real-world situations, the product demand can be forecast via a set of discrete scenarios with known probabilities (Gupta and Maranas, 2003; Jung et al., 2004). For instance, according to Eppen et al., 1989, the annual market demand of General Motors depends on various factors and its uncertainty can be captured using a scenario-based approach. In addition to the demand type, the current study completely differs from previous studies in terms of coordination strategy. All previous coordination contracts are applied only where the deterministic or stochastic market demand can be captured in a single scenario. However, in many real-world situations, the exact demand scenario cannot be predicted in advance and, therefore, a new coordination contract should be developed under a scenario-based demand. As can be seen in Table 1, we propose a new coordination contract, named adjustable bi-level wholesale price contract, to obtain channel coordination under a scenario-based stochastic demand. The proposed contract is capable of coordinating the SC under all demand scenarios. Unlike the traditional wholesale price contracts, in the proposed adjustable bi-level wholesale price contract in this study, two wholesale prices are considered contract parameters and more importantly, both wholesale prices are sensitive to the demand scenario.

Generally, the previous studies in the research field of SC coordination mainly consider either a deterministic or a stochastic demand with known probability distribution. To be more precise, in the previous studies, a single scenario for market demand is selected and the coordinated decisions and contract parameters are determined based on the selected scenario. Under such a case, the
coordination strategy is only optimal for the selected demand scenario (and not the other scenarios). Taking a novel approach in this paper, we propose a three-level coordination model where the uncertain demand can be captured using several scenarios with known probabilities. To analyse channel coordination under the scenario-based demand, in this study we focus on CSR decisions, assuming that the scenario-based market demand is sensitive to the CSR investment made by the manufacturer.

In summary, the main contributions of the current study to the research field of channel coordination are as follows: Unlike previous studies on SC coordination that assumed the exact form of demand distribution is known under a single scenario, we explore a coordination contract model with a scenario-based stochastic demand that is sensitive to CSR. To coordinate the investigated SC, a new adjustable bi-level wholesale discount is proposed to tackle channel conflict and to convince all SC actors to make globally optimal decisions in all possible scenarios. Last but not least, a scenario-based bargaining model among three actors is developed to divide surplus profit among them within all possible scenarios.

Table 1
Comparing current paper with previous studies

<table>
<thead>
<tr>
<th>Study</th>
<th>Supply chain structure</th>
<th>CSR issue</th>
<th>Demand type</th>
<th>Coordination contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni et al. (2010)</td>
<td>An upstream supplier-a downstream firm</td>
<td>CSR performance level</td>
<td>Deterministic CSR-sensitive demand</td>
<td>Wholesale price contract</td>
</tr>
<tr>
<td>Ni and Li (2012)</td>
<td>An upstream supplier-a downstream firm</td>
<td>CSR performance level</td>
<td>Deterministic price and CSR-sensitive demand</td>
<td>Mutual incentive</td>
</tr>
<tr>
<td>Panda (2014)</td>
<td>A manufacturer and a retailer</td>
<td>Consumer surplus</td>
<td>Deterministic price-sensitive demand</td>
<td>Revenue sharing contract</td>
</tr>
<tr>
<td>Modak et al. (2014)</td>
<td>A manufacturer and a retailer (dual channel)</td>
<td>Consumer surplus</td>
<td>Deterministic price-sensitive demand</td>
<td>Quantity discount contract</td>
</tr>
<tr>
<td>Hsueh (2014)</td>
<td>A manufacturer and a retailer</td>
<td>CSR investment</td>
<td>Stochastic CSR-sensitive demand</td>
<td>Revenue-sharing contract</td>
</tr>
<tr>
<td>Modak et al. (2015)</td>
<td>A manufacturer and two competitive retailers</td>
<td>Consumer surplus</td>
<td>Deterministic price-sensitive demand</td>
<td>Two-part tariff contract</td>
</tr>
<tr>
<td>Hsueh (2015)</td>
<td>Multiple suppliers and a manufacturer and</td>
<td>CSR performance level</td>
<td>Deterministic price and CSR-sensitive demand</td>
<td>Bi-level programming model</td>
</tr>
</tbody>
</table>
3. Problem description

Consider a three-level SC including one manufacturer, one distributor, and one retailer with a single item. The retailer sells a product with a scenario-based stochastic market demand, which is sensitive to the CSR investment of the manufacturer. Before starting the selling season, the retailer sets the order quantity. In this paper, it is assumed that the retailer is not a price setter; consequently, the retail price is exogenous and fixed. Such a situation can be observed in perfectly competitive markets where retailers are mere price-takers (Qin et al., 2011) and moreover in some industries (such as the health industry) in some countries, where the retail prices are under the control of the government (Hassett, 2004). The distributor provides the retailer's order quantity from the manufacturer. To be more precise, the distributor as an intermediary links the manufacturer to the retailer. According to Arya et al. (2015), many SCs usually include intermediaries, such as wholesalers, brokers, agents and even third-party logistics providers. A typical reason for the presence of intermediaries is that they play a transactional role by providing inventory and capacity, reducing search and matching costs, or aggregating supply/demand to
achieve economies of scale. In the problem under study, to improve the market share, the manufacturer determines the amount of CSR investment for each item, then sends all the finished products to the distributor in a single lot.

The demand can be affected by various factors like advertising, competitor activities, quality of products, etc. One of the issues that can potentially affect the market demand is CSR activity. In this study, a quantitative model is applied for capturing the impact of CSR investment on the market demand function. Paying more attention to CSR issues increases corporate goodwill, achieves competitive advantage and brand recognition, which in turn boosts the market demand of the product. In this section, two main subjects related to CSR activities have been investigated:

I. Defining the relationship between the value of CSR investment and CSR performance level

II. Evaluating how the CSR performance level influences the market demand

In this paper, it is supposed that the upstream manufacturer invests $\eta$ units in CSR activities for each item. The CSR performance level of the manufacturer depends on the amount of $\eta$ and is represented by a performance function $y(\eta)$, where $0 \leq y(\eta) \leq 1$ and $y(0) = 0$. The CSR performance function $y(\eta)$ is designed in such a way that more investment in CSR will yield to higher CSR performance level. In other words, the higher the investment in CSR, the higher the CSR performance level. As noted by Hsueh (2014), the CSR performance level function $y(\eta)$ is influenced by many socio-cultural factors and should consequently be calibrated using proper questionnaires and data from industrial cases in practice. In this paper, consistent with Hsueh (2014), we model the CSR performance function as $y(\eta) = \sqrt{\frac{1}{2}}$. Without loss of generality, this function lets us focus on the discussion of SC coordination.

Moreover, it should be evaluated how CSR performance level impacts the market demand. Obviously, an investment in CSR activities enhances the image of the brand among the end-consumers. Thus, CSR activities have a positive influence on demand. Consistent with Hsueh (2014) and Nematollahi et al. (2017a), we suppose that the market demand under each scenario follows a normal distribution $\mathcal{N}(\mu + \alpha y(\eta), \sigma^2)$, where $\mu$ denotes the demand mean and $\sigma$ shows the standard deviation of primary demand when no CSR investment is made. The term $\alpha y(\eta)$ measures how the CSR investment impacts the market demand, where $y(\eta)$ denotes the CSR performance level and parameter $\alpha$ represents the maximum amount of increase in the demand due to investing in CSR activities. As discussed above, the more the investment in CSR activities, the higher the CSR performance level, and the higher the market demand. When no CSR investment is made (i.e., $\eta = 0$), the CSR performance function (i.e., $y(\eta) = (1 - 1/(0.5\eta + 1))$) takes value zero and consequently the demand will not change. As the CSR investment increases, the CSR performance function takes a greater value, which in turn increases the market demand. Under a large amount of CSR investment, the CSR performance function takes a value close to one and consequently the market demand increases up to $\alpha$ units.
In this paper, we first consider the problem under single-scenario stochastic demand and propose the basic models (Section 4). Afterward, we extend the models to consider the scenario-based stochastic demand and analyse the problem under the decentralized, centralized, and coordinated models. In the decentralized model under scenario-based stochastic demand (Section 5.1), the retailer and the manufacturer individually decide on the order quantity and CSR investment by considering a scenario-based stochastic CSR-sensitive demand. Then, in the centralized model under scenario-based stochastic demand (Section 5.2), the optimal order quantity and CSR investment are obtained from the perspective of the entire SC. The optimal solutions under the centralized model might not be satisfactory to all three actors. Therefore, an adjustable bi-level wholesale price contract is proposed to convince three agents to enter into the coordination scheme. Under the proposed contract, the wholesale price of the distributor and manufacturer are treated as contract parameters and are determined in such a way that the proposed contract results in a win-win situation for all the SC actors.

The following decision variables and parameters are used in the proposed mathematical models.

**Parameters**

- $P$: Retail price
- $W_d$: Wholesale price of distributor
- $W_m$: Wholesale price of manufacturer
- $C$: Purchasing cost of manufacturer
- $S_i$: Index of demand scenario ($i = 1, 2, 3$)
- $\rho(S_i)$: Occurrence probability of scenario $S_i$
- $D_{S_i}$: Market demand under Scenario $S_i$
- $f(D_{S_i})$: Probability distribution function (p.d.f) of demand in scenario $S_i$
- $F(D_{S_i})$: Cumulative distribution function (c.d.f) of demand in scenario $S_i$
- $\mu_{S_i}$: Mean of market demand before investing in CSR in scenario $S_i$
- $\sigma_{S_i}$: Standard deviation of demand in scenario $S_i$
- $\alpha_{S_i}$: The highest increase in average demand by CSR investment in scenario $S_i$
- $\Pi^A_{B,S_i}$: Profit function of actor $B$ under decision-making structure $A$ in scenario $S_i$, where, the prefixes of $A$ can be $dc, c, co$, which depict decentralized, centralized, and coordinated structures, respectively. Furthermore, the suffixes $B$ can be $M, D, R, T$, which earmark manufacturer, distributor, retailer, and overall SC, respectively.
- $\Pi^A_B$: The expected profit function of $B$ under decision-making structure $A$: where, the prefixes of $A$ can be $dc, c, co$, which depict decentralized, centralized, and coordinated structures, respectively. Furthermore, the suffixes $B$ can be $M, D, R, T$, which earmark manufacturer, distributor, retailer, and overall SC,
respectively.

**Decision variables**

- \( Q \): Order quantity of retailer
- \( \eta \): Unit cost paid by the manufacturer on the CSR activities per unit product
- \( W_{s_i}^b \): Manufacturer’s wholesale price under the proposed contract in scenario \( s_i \)
- \( W_{d,s_i}^b \): Distributor’s wholesale price under the proposed contract in scenario \( s_i \)

### 4. Modelling the supply chain under single-scenario stochastic demand

In the investigated SC, the retailer and the manufacturer determine the order quantity and CSR investment, respectively. The retailer faces a CSR-sensitive stochastic demand with known normal distribution \( N(\mu + \alpha \eta, \sigma^2) \) and utilizes the Newsvendor inventory problem to set its order quantity. To better focus on the discussion of channel coordination, we assume that the retailer adopts the classic Newsvendor problem, in which neither shortage nor surplus imposes cost on the retailer (Simchi-Levi et al., 2004). The flow of payments among the SC actors under this condition is illustrated in Fig. 1. Before each selling season, the manufacturer supplies the raw materials at price \( C \) and invests \( \eta \) on CSR for each product, then sells them to the distributor at wholesale price \( W_{s_i} \) per unit. The distributor, who acts as a dealer between the manufacturer and retailer, sells each item at wholesale price \( W_{d} \) to the retailer. The retailer sells the items to customers at fixed price \( P \) for each item, determined by the market. In the investigated SC, the performances of the SC actors are interrelated to each other. To be more precise, the CSR investment made by the manufacturer impacts the retailer’s market demand and consequently the retailer’s order quantity is dependent on the manufacturer’s CSR investment. Moreover, the retailer’s order quantity determines the SC’s service level, which in turn influences the distributor and manufacturer’s market shares and profits. Therefore, the distributor’s profit is directly affected by the retailer’s order quantity and is indirectly affected by the manufacturer’s CSR investment.

![Fig. 1. Schematically representation of flow of payment in the basic model](image-url)
4.1. Decentralized basic model under single-scenario stochastic demand

Under the decentralized decision-making structure, the main aim of each SC actor is to maximize its own profit (Nouri et al., 2018). The decision process among the SC actors in the investigated SC is modeled as a Stackelberg game. To be more precise, the manufacturer first decides on the amount of investment in CSR. Afterward, the retailer sets the order quantity. Considering backward induction, in the following, the retailer’s and manufacturer’s problems will be solved.

The average sale and revenue of retailer per season are $E\left[\min\left(Q, D\right)\right]$ and $P \times E\left[\min\left(Q, D\right)\right]$, respectively. Moreover, the retailer pays $W_{D}Q$ to the distributor for its order quantity. Therefore, using a Newsvendor inventory model, the profit function of the retailer under the decentralized setting will be:

$$\Pi_{R}^{dc} = PE\left[\min\left(Q, D\right)\right] - W_{D}Q$$

(1)

According to Appendix A, the profit function of the retailer will be transformed to the following equation:

$$\Pi_{R}^{dc} = P \left[ \left(\mu_{0} + \alpha y(\eta) - Q\right) f_{z}\left(\frac{Q - \mu_{0} - \alpha y(\eta)}{\sigma}\right) - \sigma f'_{z}\left(\frac{Q - \mu_{0} - \alpha y(\eta)}{\sigma}\right) \right] + (P - W_{D})Q$$

(2)

where $f_{z}$ and $F_{z}$ are the p.d.f and c.d.f of standard normal distribution, respectively. $\Pi_{R}^{dc}$ is a function of two decision variables, (1) the order quantity, and (2) the CSR investment.

**Proposition 1:** For a specific value of $\eta$, $\Pi_{R}^{dc}$ is concave with respect to order quantity $Q$, and the optimal solution for $Q$ is:

$$Q^{*} = F_{z}^{-1}\left(\frac{P - W_{D}}{P}\right)\sigma + \mu_{0} + \alpha y(\eta)$$

(3)

**Proof:** See Appendix B

The Eq. (3) indicates that the optimal order quantity depends on the value of CSR level which should be determined by the manufacturer.

The distributor’s expected profit in the decentralized system can be stated as follows:

$$\Pi_{D}^{dc} = (W_{D} - W_{M})Q$$

(4)

According to Eq. (4), the profitability of the distributor is dependent on the retailer’s order quantity. Therefore, in the investigated SC, the maximum profit which can be achieved by the distributor is:

$$\Pi_{D}^{dc} = (W_{D} - W_{M})Q^{*}$$

(5)

Furthermore, the manufacturer’s profit function under the decentralized structure will be:
\[ \Pi_M^{dc} = (W_M - C - \eta)Q \]  

The manufacturer, as the leader of the investigated SC, aims to maximize his/her profitability by considering the retailer’s reaction. Hence, by substituting Eq. (3) for Eq. (6), we have:

\[ \Pi_M^{dc} = (W_M - C - \eta) \times \left\{ F_{z^{-1}} \left( \frac{P - W_{Dc}}{\sigma} \right) \alpha + \mu_0 + \alpha y(\eta) \right\} \]  

**Proposition 2:** Assuming \( y(\eta) = 1 - 1/(0.5\eta + 1) \), \( \Pi_M^{dc} \) is concave with respect to \( \eta \) and the optimal value of the unit CSR investment per product is:

\[ \eta^{dc}_{opt} = \frac{-(\alpha + \mu_o + J) + \sqrt{(\alpha + \mu_o + J)^2 - (\alpha + \mu_o + J)(-0.5\alpha(W_M - C) + \mu_o + J)}}{0.5(\alpha + \mu_o + J)} \]  

where

\[ J = F_{z^{-1}} \left( \frac{P - W_{Dc}}{\sigma} \right) \]  

**Proof:** See Appendix C

### 4.2. Centralized basic model under single-scenario stochastic demand

Under the centralized structure, all decision variables should be optimized from the perspective of the entire SC. In this problem, the decision variables (i.e., CSR investment and order quantity) will be optimized from the perspective of the entire SC. To this end, we first should calculate the profit function for the entire SC, which is the summation of the profit functions of all SC actors:

\[ \Pi^c_c(Q, \eta) = \Pi_R^{dc} + \Pi_D^{dc} + \Pi_M^{dc} \]

\[ = PE [\min(Q, D)] - W_DQ + (W_D - W_M)Q + (W_M - C - \eta)Q \]

\[ = \left\{ P \left[ (\mu_o + \alpha y(\eta) - Q)F_{z} \left( Q - \mu_o - \alpha y(\eta) \right) - \frac{\alpha}{\sigma} \left( Q - \mu_o - \alpha y(\eta) \right) \right] \right\} (10) \]

**Proposition 3:** The profit function of the entire SC under the centralized model, \( \Pi^c_c \) is concave with respect to \( Q \) and \( \eta \), and the optimal solutions for \( Q^{*c} \) and \( \eta^{*c} \) will be:

\[ Q^{*c} = F_{z^{-1}} \left( \frac{P - C - \eta^{*c}}{\sigma} \right) \]  

\[ (0.5\eta^{*c} + 1) \left[ \mu_o + \alpha + F_{z^{-1}} \left( \frac{P - C - \eta^{*c}}{\sigma} \right) \right] = 0.5\alpha (P - C) + \alpha \]  

\[ (12) \]
Proof: See Appendix D.

The optimal solution under the centralized model can be obtained by solving Eqs. (11) and (12).

5. Modelling the supply chain under scenario-based stochastic demand

In many real-world practical conditions, it is frequently observed that the behaviour of future market demand can only be predicted by a range of possible demand scenarios. In this section, we relax the assumption of single-scenario demand distribution and model the SC by considering scenario-based stochastic demand.

5.1 The decentralized model under scenario-based stochastic demand

In the following, the decentralized basic model will be extended by considering scenario-based stochastic demand. In many real-world cases, the market demand can be represented by a discrete random variable with only three possible scenarios as (1) high-demand scenario, (2) medium-demand scenario, and (3) low-demand scenario (Tsiakis et al. 2001; Arabatzis et al. 2013). Therefore, in this problem, three different possible scenarios are considered and are denoted by $S_1$, $S_2$, and $S_3$. To be more precise, the retailer’s future demand under each possible scenario $S_i$ ($i = 1, 2, 3$) is shown by $D_{si}$, in which $D_{si}$ follows a normal distribution $N(\mu_{si} + \sigma_{si} \eta, \sigma_{si}^2)$. In addition, the occurrence probability of each scenario is denoted by $\rho(S_i)$. It is noteworthy that the number of demand scenarios can be easily extended by making small adjustments in the proposed models.

Under the decentralized structure, we model the problem as a Manufacturer-Stackelberg game, where the manufacturer optimizes its decision by considering the retailer’s best response. The optimal solutions of a Stackelberg game can be obtained via the backward induction method (Yu et al., 2009). In this method, we first optimize the retailer’s problem and obtain the retailer’s best response for any CSR investment made by the manufacturer. Under scenario-based stochastic demand, the retailer should decide on the order quantity to optimize its profit function across future demand scenarios. Moreover, the determined order quantity should guarantee that the retailer will not incur losses under each demand scenario. Afterward, we optimize the manufacturer’s problem by incorporating the retailer’s best response into the manufacturer’s profit function and obtain the optimal CSR investment. Finally, the retailer’s optimal order quantity is achieved based on the manufacturer’s CSR investment.
Fig. 2 depicts the flow of payments in the investigated SC under scenario-based demand.

According to the general framework introduced by Birge and Louveaux (2011) for the scenario-based environment, we propose a scenario-based stochastic model for the retailer’s profit function under the decentralized decision-making structure as follows:

\[
\Pi^d_R = \rho(s_1) \times \{\Pi^d_{R,s_1}(Q)\} + \rho(s_2) \times \{\Pi^d_{R,s_2}(Q)\} + \rho(s_3) \times \{\Pi^d_{R,s_3}(Q)\}
\]

Subject to:

\[
\begin{align*}
\Pi^d_{R,s_1}(Q) & \geq 0; \\
\Pi^d_{R,s_2}(Q) & \geq 0; \\
\Pi^d_{R,s_3}(Q) & \geq 0;
\end{align*}
\]

The objective function \(\Pi^d_R\) is the mean of the expected profit for the retailer across all possible scenarios, which is the summation of the expected profit of the retailer in each scenario \(\Pi^d_{R,s_i}(Q)\), multiplied by the probability of that scenario. In the proposed model in Eq. (13), \(\Pi^d_{R,s_1}(Q)\), \(\Pi^d_{R,s_2}(Q)\), \(\Pi^d_{R,s_3}(Q)\) have the same equation with different coefficients. By calculating \(\Pi^d_{R,s_1}(Q), \Pi^d_{R,s_2}(Q), \Pi^d_{R,s_3}(Q)\) from Eq. (2) and replacing them into Eq. (13), the retailer’s scenario-based optimization model can be expressed as:
\[ \bar{\Pi}_{D}^{\text{dc}} = \rho(s_i) \left\{ P \left[ \frac{Q - \mu_{s_i} - \alpha_{s_i} y(\eta)}{\sigma_{s_i}} \right] - \sigma_{s_i} f_{\xi} \left( \frac{Q - \mu_{s_i} - \alpha_{s_i} y(\eta)}{\sigma_{s_i}} \right) + (P - W_D)Q \right\} \\
+ \rho(s_j) \left\{ P \left[ \frac{Q - \mu_{s_j} - \alpha_{s_j} y(\eta)}{\sigma_{s_j}} \right] - \sigma_{s_j} f_{\xi} \left( \frac{Q - \mu_{s_j} - \alpha_{s_j} y(\eta)}{\sigma_{s_j}} \right) + (P - W_D)Q \right\} \\
+ \rho(s_k) \left\{ P \left[ \frac{Q - \mu_{s_k} - \alpha_{s_k} y(\eta)}{\sigma_{s_k}} \right] - \sigma_{s_k} f_{\xi} \left( \frac{Q - \mu_{s_k} - \alpha_{s_k} y(\eta)}{\sigma_{s_k}} \right) + (P - W_D)Q \right\} \right\} \\
\text{subject to :} \\
P \left[ \frac{Q - \mu_{s_i} - \alpha_{s_i} y(\eta)}{\sigma_{s_i}} \right] - \sigma_{s_i} f_{\xi} \left( \frac{Q - \mu_{s_i} - \alpha_{s_i} y(\eta)}{\sigma_{s_i}} \right) + (P - W_D)Q \geq 0; \\
P \left[ \frac{Q - \mu_{s_j} - \alpha_{s_j} y(\eta)}{\sigma_{s_j}} \right] - \sigma_{s_j} f_{\xi} \left( \frac{Q - \mu_{s_j} - \alpha_{s_j} y(\eta)}{\sigma_{s_j}} \right) + (P - W_D)Q \geq 0; \\
P \left[ \frac{Q - \mu_{s_k} - \alpha_{s_k} y(\eta)}{\sigma_{s_k}} \right] - \sigma_{s_k} f_{\xi} \left( \frac{Q - \mu_{s_k} - \alpha_{s_k} y(\eta)}{\sigma_{s_k}} \right) + (P - W_D)Q \geq 0; \]

Since the order quantity is set before the occurrence of each demand scenario, the profit of the distributor and the manufacturer are independent of the possible scenarios. Therefore, the average profit of the distributor and manufacturer in all scenarios is equal to Eq. (4) and Eq. (6), respectively. Considering these explanations, the distributor and manufacturer’s profit functions under the scenario-based model will be:

\[ \bar{\Pi}_{D}^{\text{dc}} = \sum_{s_i} \rho(s_i) \times \left\{ \bar{\Pi}_{D,S_i}^{\text{dc}} \right\} \]

\[ \rightarrow \bar{\Pi}_{D}^{\text{dc}} = \sum_{s_i} \rho(s_i) \times (W_D - W_M)Q \]

\[ \bar{\Pi}_{M}^{\text{dc}} = \sum_{s_i} \rho(s_i) \times \left\{ \bar{\Pi}_{M,S_i}^{\text{dc}} \right\} \]

\[ \rightarrow \bar{\Pi}_{M}^{\text{dc}} = \sum_{s_i} \rho(s_i) \times (W_M - C - \eta)Q \]

In the proposed scenario-based decentralized decision-making model, the manufacturer first optimizes the unit CSR investment \( \eta \). Then the retailer will choose the optimal order quantity. Since the optimal value of the order quantity and unit CSR investment cannot be directly calculated from Eq. (14) and Eq. (16), in the following, a solution procedure is established to obtain the optimal amount of order quantity and CSR investment in the decentralized scenario-based model.
Since Eqs. (14) and (16) are interrelated, the iterative search procedure which is proposed below can be applied to find the optimal amounts of decision variables.

**Decentralized iterative solution procedure (DC-ISP)**

**Step 1:** (Initial)
Let \( K_1 = 1, \eta_{K_1} = 0 \).

**Step 2:** (Optimal order quantity)
Set \( K_2 = 1, Q_{K_2} = 0, \eta_{K_2} = 0 \).

**Step 2.1:** Set \( K_2 = K_2 + 1, Q_{K_2} = Q_{K_2} + \varepsilon_2 \) (where \( \varepsilon_2 \) is a small value) and calculate \( \Pi^{dc} (Q_{K_2}, \eta_{K_2}) \) from Eq. (14).

**Step 2.2:** If \( (Q_{K_2}, \eta_{K_2}) \) satisfies all the constraints in Eq. (14), afterward, go to step 2.3; otherwise, go to step 2.4.

**Step 2.3:** If \( (Q_{K_2}^{*}, \eta_{K_2}^{*}) > (Q_{K_2}, \eta_{K_2}) \), then set \( Q_{K_2} = Q_{K_2}^{*} \) and go to step 2.4.

**Step 2.4:** If \( Q_{K_2} \leq \mu_3 + 3\sigma_3 \), afterward, go to step 2.1. Else, go to step 3.

**Step 3:** (Optimal CSR investment)
Calculate \( \Pi^{dc} (Q_{K_2}^{*}, \eta_{K_2}^{*}) \) from Eq. (16). Go to step 3.1.

**Step 3.1:** If \( \eta_{K_1} + \varepsilon_1 \leq \eta^{\text{max}} \), then Set \( K_1 = K_1 + 1, \eta_{K_1} = \eta_{K_1} + \varepsilon_1 \) (where \( \varepsilon_1 \) is a small value) go to step 2 to calculate the related optimal order quantity; otherwise, go to step 4.

**Step 4:** (Optimal values)
Find the maximum amount for \( \Pi^{dc} (Q_{K_1}^{*}, \eta_{K_1}^{*}) \), recall its parameter values \( (Q_{K_1}^{*}, \eta_{K_1}^{*}) \) and set \( Q^{*dc} = Q_{K_1}, \eta^{*dc} = \eta_{K_1} \). Return the optimal values for \( (Q^{*dc}, \eta^{*dc}) \) and stop the search procedure.

**5.2 The centralized model under scenario-based stochastic demand**
Under the centralized structure, SC actors act as a vertically integrated SC with a unique decision-maker. The unique decision-maker of the SC which faces a scenario-based stochastic demand should set \( Q \) and \( \eta \) before starting the selling season to obtain the maximum expected profit of the entire SC. The entire SC’s profit function under the scenario-based stochastic demand \( \Pi_{c, T, S} \) can be calculated as follows:
Similar to the methodology applied in the decentralized model, the scenario-based centralized model can be expressed as follows:

\[
\prod_{T,S_i}^c = \prod_{M,S_i}^{dc} + \prod_{D,S_i}^{dc} + \prod_{R,S_i}^{dc} = (P) \left[ (\mu_{S_i} + \alpha_{S_i}, y(\eta) - Q) F_z \left( \frac{Q - \mu_{S_i} - \alpha_{S_i}, y(\eta)}{\sigma_{S_i}} \right) \right. \\
\left. - \sigma_{S_i} f_z \left( \frac{Q - \mu_{S_i} - \alpha_{S_i}, y(\eta)}{\sigma_{S_i}} \right) \right] + (P - C - \eta)Q \quad \forall S_i
\]

(17)

\[
\text{Max} \quad \prod_{T}^c = \sum_{S_i} \rho(s_i) \times \left\{ \prod_{T,S_i}^c (Q, \eta) \right\}
\]

Subject to:

\[
\prod_{T,S_i}^c (Q, \eta) \geq \prod_{T,S_i}^{dc} ; \quad (18)
\]

In the proposed scenario-based stochastic model for the centralized system, the mathematical model aims to maximize the expected profits of the entire SC by considering various demand scenarios. In the proposed model, \( \prod_{T}^c \) is the sum of the expected profit of the overall SC in scenario \( S_i \) (i.e., \( \prod_{T,S_i}^c \)) multiplied by its probability. The constraints of the above model guarantee that the profit of the entire SC in the centralized setting will be greater than that of the decentralized setting under all of the scenarios. The optimal solutions will not be desirable if the expected profit for the entire SC under the centralized mode in one scenario (e.g., \( S_i \)) is less than that under the decentralized mode, \( \prod_{T,S_i}^{dc} \) in the same scenario \( S_i \). In other words, the equations \( \left[ \prod_{T,S_i}^c (Q, \eta) \geq \prod_{T,S_i}^{dc} \forall S_i \right] \) should be satisfied. In the above model, \( \prod_{T,S_i}^{dc} \) can be obtained as follows:

\[
\prod_{T,S_i}^{dc} = \prod_{M,S_i}^{dc} (Q^{dc}, \eta^{dc}) + \prod_{D,S_i}^{dc} (Q^{dc}) + \prod_{R,S_i}^{dc} (Q^{dc}, \eta^{dc}) \quad \forall S_i
\]

(19)

By replacing Eq. (17) with Eq. (18), the proposed scenario-based optimization model under the centralized model can be rewritten as follows:
\(\bar{\Pi}_T^c = \rho(s_1) \left\{ P \left[ (\mu_s + \alpha_s y(\eta) - Q) F_z \left( \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right) - \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right] + (P - C - \eta)Q \right\} \)

\[+ \rho(s_1) \left\{ P \left[ (\mu_s + \alpha_s y(\eta) - Q) F_z \left( \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right) - \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right] + (P - C - \eta)Q \right\} \]

\[+ \rho(s_1) \left\{ P \left[ (\mu_s + \alpha_s y(\eta) - Q) F_z \left( \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right) - \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right] + (P - C - \eta)Q \right\} \]

Subject to:

\[P \left[ (\mu_s + \alpha_s y(\eta) - Q) F_z \left( \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right) - \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right] + (P - C - \eta)Q \geq 0; \]

\[P \left[ (\mu_s + \alpha_s y(\eta) - Q) F_z \left( \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right) - \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right] + (P - C - \eta)Q \geq 0; \]

\[P \left[ (\mu_s + \alpha_s y(\eta) - Q) F_z \left( \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right) - \frac{Q - \mu_s - \alpha_s y(\eta)}{\sigma_s} \right] + (P - C - \eta)Q \geq 0; \]

The optimal values for two decision variables (i.e., the order quantity and unit CSR investment) cannot be calculated in a straightforward manner from Eq. (20). In the following, an iterative approach is developed for calculating the optimal values for \((Q^c, \eta^c)\) in the centralized scenario-based setting:

**Centralized iterative solution procedure (C-ISP)**

**Step 1:** (Initial)

Set \(K_1 = 1, \eta_{K_1} = 0\).

**Step 2:**

If \(\eta_{K_1} \leq \eta_{\max}\), then set \(K_2 = 1, Q_{K_2} = 0\) and go to step 3. Otherwise, go to step 5.

**Step 3:** If \(Q_{K_1} \leq \mu_s + 3\sigma_s\), then go to step 3.1. Otherwise, go to step 4.

**Step 3.1:** Calculate \(\bar{\Pi}_T^c(Q_{K_1}, \eta_{K_1})\) from Eq. (20).

**Step 3.2:** If \((Q_{K_1}, \eta_{K_1})\) cannot satisfy at least one of the constraints in Eq. (20), set \(\bar{\Pi}_T^c(Q_{K_1}, \eta_{K_1}) = -\infty\). Go to step 3.

**Step 3.3:** Set \(K_2 = K_2 + 1, Q_{K_2} = Q_{K_1} + \varepsilon_2\) (where \(\varepsilon_2\) is a small value) and go to step 3.

**Step 4:**

Set \(K_1 = K_1 + 1, \eta_{K_1} = \eta_{K_1} + \varepsilon_1\) (where \(\varepsilon_1\) is a small value). Go to step 2.

**Step 5:**
Find the maximum amount for \( \Pi_f(Q_{k_1}, \eta_{k_1}) \), recall its parameter values \((Q_{k_1}, \eta_{k_1})\) and set \( Q^{*c} = Q_{k_1}, \eta^{*c} = \eta_{k_1} \). Return the optimal values for \((Q^{*c}, \eta^{*c})\) and stop the search procedure.

Although the centralized system increases the total SC profit compared to the decentralized one, the centralized solution will not necessarily raise the profitability of all individual SC actors. Since the independent SC actors aim to maximize their own profits, the SC actors who incur losses will not be interested in choosing the centralized decisions. In the next section, a new coordination contract based on the wholesale price scheme is proposed to resolve the channel conflict and achieve coordination system under the scenario-based stochastic demand.

5.3 The coordinated model under scenario-based stochastic demand

The optimal decision variables obtained under the centralized model might reduce the retailer or manufacturer’s profit compared to the decentralized model; however, the centralized solution is of great benefit to the whole SC system (Hosseini-Motlagh et al. 2018b). In the SC management context, coordination contracts have been used to persuade decentralized SC actors to make centralized decisions. Various coordination schemes such as wholesale price, delay-in-payment, buy-back, and quantity flexibility contracts have been developed to coordinate SC networks. The interested readers are referred to the papers by Cachon (2003) and Govindan et al. (2013) for more discussions and literature review on the SC coordination issue. The previous coordination contracts are only applied where the future market demand can be predicted by a single scenario. However, in many situations, the uncertain demand can be captured via a set of discrete scenarios with their respective probabilities. To the best of our knowledge, no previous study has ever proposed a contract for coordinating SC systems under a scenario-based demand. In this section, a new coordination contract named adjustable bi-level wholesale price contract is proposed to achieve channel coordination among three SC actors under the scenario-based stochastic demand. The proposed contract is designed in such a way that it benefits all SC actors regardless of which demand scenario occurs in the future. This can be considered the unique feature of the proposed contract. The previous coordination contracts that existed in the literature can only be applied when the future market demand can be predicted under a single scenario. In the problem under study, the profits of the actors directly depend on the demand scenario that occurs in the future and consequently a new coordination contract is needed to provide a win-win situation regardless of which scenario occurs. To resolve this issue, in the proposed contract in this paper, the amounts of the contract parameters are designed by considering the possible demand scenarios in the future.

In the proposed contract, once the actual demand in scenario \( s_i \) is revealed, the manufacturer offers wholesale fee \( W_{M,S_i}^b \) to the distributor. Then, the distributor specifies wholesale fee \( W_{D,S_i}^b \) to the
Therefore, the parameters of the proposed contract for each scenario can be expressed as \((W^b_{M,S_i}, W^b_{D,S_i})\). These two parameters should be determined in such a way that the profits of all actors increase after their participation in the coordination model. It is noteworthy that the amounts of contract parameters \((W^b_{M,S_i}, W^b_{D,S_i})\) in each scenario \(S_i\) are independent of the other scenarios.

The proposed coordination contract should be acceptable to all three actors including the manufacturer, distributor, and retailer. Otherwise, they refuse to participate in the coordination model. Therefore, in the proposed coordination contract, the minimum requirements of three actors for participation should be considered. The term "bi-level" in the name of the contract refers to the fact that the proposed contract not only sets the wholesale price of the manufacturer, but also simultaneously determines the wholesale price of the distributor in such a way that all three actors benefit from the coordination model under various demand scenarios. These two contract parameters should be simultaneously set so that all three actors participate in the coordination model.

In the previously developed coordination mechanisms, the SC actors determine the exact amounts of the contract parameters before starting the selling season. The proposed adjustable bi-level wholesale price contract differs from the other contracts as its parameters are dependent on the demand scenario and therefore a set of contract parameters should be determined prior to the demand realization. When the selling season starts, the SC actors choose the contract parameters based on the realized demand scenario. The operation of the proposed contract is depicted schematically in Fig.3.

The main steps of the proposed coordination model can be summarized as follows:

**Step 1.** Before starting the selling season (i.e., before demand realization), the retailer places an order equal to the centralized order quantity.

**Step 2.** The manufacturer produces the order quantity of the retailer while investing in CSR for each product consistent with the centralized CSR investment.

**Step 3.** The distributor transfers the products from the manufacturer to the retailer.

**Step 4.** After starting the selling season (i.e., once the demand scenario is realized), the contract parameters \((W^b_{M,S_i}, W^b_{D,S_i})\) will be chosen based on the demand scenario that occurred.
Step 5. The retailer pays the distributor $W^b_{D,S1}Q^{c^*}$ and the distributor pays the manufacturer $W^b_{M,S1}Q^{c^*}$.

Under the proposed coordination model, the SC agents (i.e., the manufacturer, the distributor and the retailer) cooperatively make an agreement on the order quantity, CSR investment, and the contract parameters prior to the realization of the demand scenario. Therefore, following the logic of Hu et al. (2016), the distributor’s order quantity and the manufacturer’s production quantity should be equal to the optimal retailer’s order quantity under the centralized model.

After applying the proposed contract, the expected profit of the SC actors under each scenario can be calculated as follows:

$$\Pi_{R,S_i}^{co} = P \left[ \left( \mu_{S_i} + \alpha_{S_i} y (\eta^{c^*}) - Q^{c^*} \right) F_z \left( \frac{Q^{c^*} - \mu_{S_i} - \alpha_{S_i} y (\eta^{c^*})}{\sigma_{S_i}} \right) - \sigma_{S_i} f_z \left( \frac{Q^{c^*} - \mu_{S_i} - \alpha_{S_i} y (\eta^{c^*})}{\sigma_{S_i}} \right) \right] + (P - W^b_{D,S_i})Q^{c^*}$$  \hspace{1cm} (21)

$$\Pi_{D,S_i}^{co} = \left( W^b_{D,S_i} - W^b_{M,S_i} \right) Q^{c^*}$$  \hspace{1cm} (22)

$$\Pi_{M,S_i}^{co} = \left( W^b_{M,S_i} - C - \eta^{c^*} \right) Q^{c^*}$$  \hspace{1cm} (23)

As can be seen, under the coordinated model, the retailer and manufacturer will choose the centralized solutions of the order quantity and CSR investment, respectively. Moreover, the manufacturer and distributor will reconsider their decisions on their wholesale price. The contract parameters should be set in such a manner that by choosing the centralized solution under each scenario, the profits of all SC actors increase compared to the decentralized model. Otherwise, they
will refuse to cooperate in the coordinated decision-making structure. Therefore, contract parameters \((W_{M,S_i}^b, W_{D,S_i}^b)\) should satisfy the following constraints:

\[
\begin{align*}
\Pi_{R,S_i}^c & \geq \Pi_{R,S_i}^{dc} \\
\Pi_{D,S_i}^c & \geq \Pi_{D,S_i}^{dc} \\
\Pi_{M,S_i}^c & \geq \Pi_{M,S_i}^{dc} \quad \forall S_i
\end{align*}
\]  

(24)

By substituting Eqs. (21), (22) and (23) for Eq. (24), we have:

\[
\begin{align*}
P & \geq \left( \frac{Q^c - \mu_{S_i} - \alpha_{S_i} y (\eta_i)}{\sigma_{S_i}} \right) + (P - W_{D,S_i}^b) Q^c \geq \Pi_{R,S_i}^{dc} \\
\left( W_{D,S_i}^b - W_{M,S_i}^b \right) Q^c & \geq \Pi_{D,S_i}^{dc} \\
\left( W_{M,S_i}^b - C - \eta^c \right) Q^c & \geq \Pi_{M,S_i}^{dc} \quad \forall S_i
\end{align*}
\]  

(25)

Through some simplifications and expanding Eq. (25), we can obtain an acceptable range for contract parameters \((W_{M,S_i}^b, W_{D,S_i}^b)\) as follows:

\[
\begin{align*}
W_{D,S_i}^b & \leq \frac{\left( \mu_{S_i} + \alpha_{S_i} y (\eta_i) - Q^c \right) F_z \left( \frac{Q^c - \mu_{S_i} - \alpha_{S_i} y (\eta_i)}{\sigma_{S_i}} \right) - \sigma_{S_i} f_z \left( \frac{Q^c - \mu_{S_i} - \alpha_{S_i} y (\eta_i)}{\sigma_{S_i}} \right)}{Q^c} + PQ^c - \Pi_{R,S_i}^{dc} \\
\left( W_{D,S_i}^b - W_{M,S_i}^b \right) & \geq \frac{\Pi_{D,S_i}^{dc}}{Q^c} \\
W_{M,S_i}^b & \geq \frac{\Pi_{M,S_i}^{dc}}{Q^c} + (C + \eta^c) \quad \forall S_i
\end{align*}
\]  

(26)

Each pair of contract parameters \((W_{M,S_i}^b, W_{D,S_i}^b)\) which satisfy Eq. (26) will increase the profits of all actors under the coordination model compared to the decentralized model. Therefore, the new wholesale prices (contract parameters) obtained under the proposed contract are satisfactory for all the SC actors as they simultaneously benefit from the coordination model. However, the surplus profit in each scenario (i.e., \(\Pi_{R,S_i}^c (Q^c, \eta^c) - \Pi_{R,S_i}^{dc} (Q^{dc}, \eta^{dc})\)) will be shared between the SC actors.
according to the amounts of the chosen contract parameters. In the following, a profit-sharing scheme based on the Nash-bargaining model is used in order to equitably share the surplus profit among SC actors. The proposed bargaining model suggests how the wholesale prices are to be set to resolve the channel conflict and provide a win-win situation. In the proposed game model, the objective function is the multiplication of the actors’ profit functions after participating in the coordination plan (Modak et al., 2016). The proposed game model can be formulated as:

$$\text{Max} \left( \Pi_{R,S_i}^{\text{co}} - \Pi_{R,S_i}^{\text{dc}} \right) \left( \Pi_{D,S_i}^{\text{co}} - \Pi_{D,S_i}^{\text{dc}} \right) \left( \Pi_{M,S_i}^{\text{co}} - \Pi_{M,S_i}^{\text{dc}} \right)$$

$$\Pi_{R,S_i}^{\text{co}} \geq \Pi_{R,S_i}^{\text{dc}}$$

$$\Pi_{D,S_i}^{\text{co}} \geq \Pi_{D,S_i}^{\text{dc}}$$

$$\Pi_{M,S_i}^{\text{co}} \geq \Pi_{M,S_i}^{\text{dc}}$$ \hspace{1cm} \forall S_i$$

(27)

By substituting the profit functions and simplifying Eq. (27), we have:

$$\text{Max}$$

$$\left[ \left( \mu_{S_i} + \alpha_{S_i} y (\eta^*) - Q^* \right) F_z \left( \frac{Q^* - \mu_{S_i} - \alpha_{S_i} y (\eta^*)}{\sigma_{S_i}} \right) \right]$$

$$\left[ -\sigma_{S_i} f_z \left( \frac{Q^* - \mu_{S_i} - \alpha_{S_i} y (\eta^*)}{\sigma_{S_i}} \right) \right]$$

$$\left\{ (W_{D,S_i}^b - W_{M,S_i}^b) Q^* - \Pi_{D,S_i}^{\text{dc}} \right\} \times \left\{ (W_{M,S_i}^b - C - \eta^*) Q^* - \Pi_{M,S_i}^{\text{dc}} \right\}$$

Subject to:

$$\left[ \left( \mu_{S_i} + \alpha_{S_i} y (\eta^*) - Q^* \right) F_z \left( \frac{Q^* - \mu_{S_i} - \alpha_{S_i} y (\eta^*)}{\sigma_{S_i}} \right) \right]$$

$$\left[ -\sigma_{S_i} f_z \left( \frac{Q^* - \mu_{S_i} - \alpha_{S_i} y (\eta^*)}{\sigma_{S_i}} \right) \right]$$

$$+ P Q^* - \Pi_{R,S_i}^{\text{dc}}$$

$$W_{D,S_i}^b \leq \frac{Q^*}{Q^*}$$

$$\left( W_{D,S_i}^b - W_{M,S_i}^b \right) \geq \frac{\Pi_{D,S_i}^{\text{dc}}}{Q^*}$$

$$W_{M,S_i}^b \geq \frac{\Pi_{M,S_i}^{\text{dc}}}{Q^*} + (C + \eta^*)$$ \hspace{1cm} \forall S_i$$

(28)

By solving the above bargaining model, the exact amounts of the contract parameters which are acceptable to all three actors will be determined. The proposed Nash-bargaining model will result
in a win-win situation for all three actors after achieving channel coordination across all possible demand scenarios.

6. Numerical example and sensitivity analysis

In the following, a numerical example is performed so as to demonstrate the efficiency of the proposed coordination model. The parameters are provided in Table 2.

<table>
<thead>
<tr>
<th>Scenario-1</th>
<th>Scenario-2</th>
<th>Scenario-3</th>
<th>P</th>
<th>W_M</th>
<th>W_D</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(μ₁₁, σ₁₁)</td>
<td>α₁₁</td>
<td>ρ₁₁</td>
<td>(μ₂₂, σ₂₂)</td>
<td>α₂₂</td>
<td>ρ₂₂</td>
<td>(μ₃₃, σ₃₃)</td>
</tr>
<tr>
<td>(100,20)</td>
<td>30</td>
<td>0.3</td>
<td>(120,30)</td>
<td>40</td>
<td>0.4</td>
<td>(150,35)</td>
</tr>
</tbody>
</table>

Table 3 indicates the results of solving the test problem under different decision-making structures. The optimal solutions of decision variables, which include CSR investment and order quantity, along with the expected profits of the investigated actors as well as the entire SC, are shown in Table 3. The profits of individual SC actors in each scenario under different decision-making structures are also provided. Although the profitability of the entire SC will be raised by adopting the centralized decision-making system in comparison with the decentralized profit, the retailer will incur losses. To be more precise, under the decentralized model, the retailer's order quantity is equal to 117.34 and the retailer will obtain positive profit across all demand scenarios. Under the centralized model, the retailer will lose 121.94 if scenario 1 occurs. Moreover, the retailer's expected profit under the decentralized model (943.98) will decrease under the centralized model (499.78). Therefore, the retailer will refuse to take part in the centralized model. However, the centralized model will increase the profits of both the distributor and manufacturer compared to the decentralized structure. As can be seen, the proposed contract will resolve the channel conflict of the investigated SC in each scenario. Under the coordinated model, the profits of all three actors will increase compared to the decentralized model and consequently the proposed contract is applicable in practice. The proposed coordination model results in the centralized profit and moreover it will improve all three actors’ profits across all demand scenarios.

From a social responsibility point of view, the proposed coordination model is of great benefit as well. Under the decentralized model, the manufacturer will invest 0.36 on CSR issues per unit product. However, under the coordinated model, the manufacturer will invest 1.06 on CSR for each unit product. Therefore, the proposed coordination contract not only improves the economic performance of the whole channel and individual SC actors but also significantly improves the CSR performance level.
Table 3
Results of the test problem

Optimal solution for variables

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Decentralized case</th>
<th>Centralized case</th>
<th>Coordinated case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q^{*}_{dc}$</td>
<td>$Q^{*}_c$</td>
<td>$Q^{*}_{co}$</td>
</tr>
<tr>
<td></td>
<td>117.34</td>
<td>192.11</td>
<td>192.11</td>
</tr>
<tr>
<td></td>
<td>$\eta^{*}_{dc}$</td>
<td>$\eta^{*}_c$</td>
<td>$\eta^{*}_{co}$</td>
</tr>
<tr>
<td></td>
<td>0.36</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>$y(\eta^{*}_{dc})$</td>
<td>$y(\eta^{*}_c)$</td>
<td>$y(\eta^{*}_{co})$</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Optimal solution for bargaining variables

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Scenario-1</th>
<th>Scenario-2</th>
<th>Scenario-3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W^b_D$</td>
<td>$W^b_D$</td>
<td>$W^b_D$</td>
</tr>
<tr>
<td></td>
<td>14.01</td>
<td>13.03</td>
<td>12.81</td>
</tr>
<tr>
<td></td>
<td>$W^b_M$</td>
<td>$W^b_M$</td>
<td>$W^b_M$</td>
</tr>
<tr>
<td></td>
<td>9.10</td>
<td>8.09</td>
<td>7.45</td>
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</table>

The mean of the expected profit under three decision-making cases

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Decentralized case</th>
<th>Centralized case</th>
<th>Coordinated case</th>
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<tbody>
<tr>
<td></td>
<td>$\Pi^{*}_{dc}$</td>
<td>$\Pi^{*}_c$</td>
<td>$\Pi^{*}_{co}$</td>
</tr>
<tr>
<td></td>
<td>943.98</td>
<td>499.78</td>
<td>1090.73</td>
</tr>
<tr>
<td></td>
<td>$\Pi^{*}_d$</td>
<td>$\Pi^{*}_D$</td>
<td>$\Pi^{*}_{co}$</td>
</tr>
<tr>
<td></td>
<td>586.70</td>
<td>960.55</td>
<td>733.25</td>
</tr>
<tr>
<td></td>
<td>$\Pi^{*}_M$</td>
<td>$\Pi^{*}_M$</td>
<td>$\Pi^{*}_{co}$</td>
</tr>
<tr>
<td></td>
<td>1013.76</td>
<td>1525.35</td>
<td>1161.69</td>
</tr>
<tr>
<td></td>
<td>$\Pi^{*}_T$</td>
<td>$\Pi^{*}_T$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2544.50</td>
<td>2985.68</td>
<td></td>
</tr>
</tbody>
</table>

Profitability of SC actors in scenario 1 under three decision-making cases

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Decentralized case</th>
<th>Centralized case</th>
<th>Coordinated case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Pi^{*}_{dc}$</td>
<td>$\Pi^{*}_c$</td>
<td>$\Pi^{*}_{co}$</td>
</tr>
<tr>
<td>$R,S_1$</td>
<td>732.71</td>
<td>-121.94</td>
<td>742.56</td>
</tr>
<tr>
<td>$D,S_1$</td>
<td>586.70</td>
<td>960.55</td>
<td>597.46</td>
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<tr>
<td>$M,S_1$</td>
<td>1013.76</td>
<td>1525.35</td>
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<tr>
<td>$T,S_1$</td>
<td>2332.16</td>
<td>2363.97</td>
<td>2363.97</td>
</tr>
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Profitability of SC actors in scenario 2 under three decision-making cases

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Decentralized case</th>
<th>Centralized case</th>
<th>Coordinated case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Pi^{*}_{dc}$</td>
<td>$\Pi^{*}_c$</td>
<td>$\Pi^{*}_{co}$</td>
</tr>
<tr>
<td>$R,S_2$</td>
<td>971.04</td>
<td>457.32</td>
<td>1095.12</td>
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<tr>
<td>$D,S_2$</td>
<td>586.70</td>
<td>960.55</td>
<td>710.81</td>
</tr>
<tr>
<td>$M,S_2$</td>
<td>1013.76</td>
<td>1525.35</td>
<td>1137.27</td>
</tr>
<tr>
<td>$T,S_2$</td>
<td>2571.58</td>
<td>2943.22</td>
<td>2943.22</td>
</tr>
</tbody>
</table>

Profitability of SC actors in scenario 3 under three decision-making cases

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Decentralized case</th>
<th>Centralized case</th>
<th>Coordinated case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Pi^{*}_{dc}$</td>
<td>$\Pi^{*}_c$</td>
<td>$\Pi^{*}_{co}$</td>
</tr>
<tr>
<td>$R,S_3$</td>
<td>1119.16</td>
<td>1178.08</td>
<td>1433.58</td>
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<tr>
<td>$D,S_3$</td>
<td>586.70</td>
<td>960.55</td>
<td>901.00</td>
</tr>
<tr>
<td>$M,S_3$</td>
<td>1013.76</td>
<td>1525.35</td>
<td>1329.40</td>
</tr>
<tr>
<td>$T,S_3$</td>
<td>2719.68</td>
<td>3663.98</td>
<td>3663.98</td>
</tr>
</tbody>
</table>
In the following, a set of sensitivity analyses is carried out with respect to the primary parameters. In the proposed model, the impact of the CSR on market demand is measured by \( \alpha_{S_i} \). The greater the value of \( \alpha_{S_i} \), the greater the effect of CSR investment on demand. Fig. 4 and Fig. 5 respectively show the variation of the CSR value \( \eta \) and the CSR performance level \( y(\eta) \) against changes in \( \alpha_{S_i} \) under the decentralized and coordinated decision-making structures. As can be seen, the coordinated model considerably improves the CSR index of the chain compared to the decentralized model under various values of the CSR impact coefficient (see arrow in Fig. 4). This finding indicates that the proposed coordination model, which is obtained using the proposed adjustable bi-level wholesale price contract, improves the CSR index in the SC regardless of the CSR impact coefficient.

Moreover, by increasing the CSR impact coefficient \( \alpha_{S_i} \), the CSR value and the CSR performance level improve in both the decentralized and coordinated systems (see the trends in Fig. 4 and Fig. 5). Therefore, it can be concluded that the SC actors have a greater tendency to invest in CSR subjects when such efforts have a higher impact on the market demand. In practice, the consumers’ awareness about the CSR and social issues determine their purchasing behaviour towards environmentally and socially-responsible products. Therefore, the governments should promote awareness of the importance of CSR and encourage customers to purchase products from socially responsible SCs. Under such a case, the CSR efforts made by the SC actors will significantly boost their market demand and consequently they will invest more in CSR issues. Moreover, as shown in Fig. 4 and Fig. 5, the difference between the decentralized and coordinated CSR performance level gets larger by increasing \( \alpha_{S_i} \) (see the gap between lines in Fig. 4 and Fig. 5). Therefore, the efficiency of the proposed contract improves as the CSR impact coefficient increases.

![Fig. 4. Impact of \( \alpha_{S_i} \) on the CSR value of the SC in the decentralized and coordinated/centralized models.](image-url)
As shown above, the coordination of the CSR investment and order quantity using the proposed adjustable bi-level wholesale price contract significantly improves the CSR index compared to the decentralized state. In the following, the performance of the proposed contract is analyzed in terms of economic performance. Fig. 6 indicates changes in the profit of the entire SC by increasing the CSR coefficient impact $\alpha_S$ in the decentralized and coordinated models. As can be observed, the proposed coordination model significantly increases the profit of the entire SC in comparison with the decentralized system under different values of $\alpha_S$ (see the arrow in Fig. 6). The results show the efficiency of the proposed adjustable bi-level wholesale price contract in coordinating the channel under scenario-based stochastic demand.

**Fig. 5.** Influences of $\alpha_S$ on the level of CSR performance of the SC under decentralized and coordinated/centralized models

Increase of the CSR performance level in transition from the decentralized to coordinated/centralized model

As shown above, the coordination of the CSR investment and order quantity using the proposed adjustable bi-level wholesale price contract significantly improves the CSR index compared to the decentralized state. In the following, the performance of the proposed contract is analyzed in terms of economic performance. Fig. 6 indicates changes in the profit of the entire SC by increasing the CSR coefficient impact $\alpha_S$ in the decentralized and coordinated models. As can be observed, the proposed coordination model significantly increases the profit of the entire SC in comparison with the decentralized system under different values of $\alpha_S$ (see the arrow in Fig. 6). The results show the efficiency of the proposed adjustable bi-level wholesale price contract in coordinating the channel under scenario-based stochastic demand.
In this paper, we coordinate the CSR investment and order quantity when the SC faces a scenario-based stochastic CSR-sensitive demand. In the following, we analyse how the uncertainty in market demand affects the performance of the proposed adjustable bi-level wholesale price contract. To this end, a sensitivity analysis is conducted with respect to the standard deviation of the market demand $\sigma_{S_i}$. Fig. 7 and Fig. 8, respectively, show profitability and CSR performance level of the entire SC under different values of the standard deviation of demand. As can be seen, the proposed coordination model not only provides more profit for the entire SC in comparison with the decentralized state (see the arrow in Fig. 7), but also improves the CSR performance level under various values of demand uncertainty (see the arrow in Fig. 8). These results indicate the proposed adjustable wholesale price contract performs quite well under different levels of demand uncertainty.

**Fig. 6.** Impact of $\sigma_{S_i}$ on the profitability of entire SC under the decentralized and coordinated/centralized models

Increase of the total SC profit in transition from the decentralized to coordinated/centralized model
Fig. 7. Impact of $\sigma_{S_i}$ on profitability of the whole SC under the decentralized and coordinated/centralized models.

Increase of the profitability of the total SC in transition from the decentralized to coordinated/centralized system.

Fig. 8. Impact of $\sigma_{S_i}$ on the CSR performance value under the decentralized and coordinated/centralized models.

Enhancement of the profitability of the whole SC in transition from the decentralized to coordinated/centralized system.
The rational SC actors take part in a coordination model if and only if the coordination contract improves their individual profits compared to the decentralized model. Otherwise, they will not enter into the coordination contract. In the following, we explore how the proposed adjustable bi-level wholesale price contract impacts the economic performance of the SC actors. Fig. 9, Fig. 10, and Fig. 11, respectively, demonstrate the profits of the retailer, the distributor, and the manufacturer under the three decision-making structures by increasing the standard deviation of the demand. Although the centralized model greatly improves the profits of both the distributor and manufacturer compared to the decentralized model (see Fig. 10 and Fig. 11), the profit of the retailer under the centralized model is lower than under the decentralized one (see Fig. 9). Therefore, the retailer will refuse to cooperate under the centralized system. However, the proposed coordination model simultaneously improves the individual profits of all SC actors (i.e., the retailer, distributor, and manufacturer) compared to the decentralized state (see and compare the decentralized and coordinated profits in Fig. 9, Fig. 10, and Fig. 11). Therefore, the proposed coordination contract is mutually beneficial and provides a win-win outcome for the SC actors. The results show the efficacy of the proposed adjustable bi-level wholesale price contract in resolving the channel conflicts and motivating SC actors to enter the coordination scheme.

![Graph](image.png)

**Fig. 9.** Impact of $\sigma_S$ on retailer’s profitability under the decentralized, centralized, and coordinated models
Fig. 10. Impact of $\sigma_{x_i}$ on distributor’s profitability under the decentralized, centralized, and coordinated models.

Fig. 11. Impact of $\sigma_{x_i}$ on manufacturer’s profitability under the coordinated, centralized, and decentralized systems.
In this paper, an adjustable bi-level wholesale price contract is proposed to coordinate the investigated three-stage SC under the scenario-based demand. The proposed contract has two primary parameters \( W_{M,S_i} \) and \( W_{D,S_i} \). Once the demand scenario is revealed, these contract parameters should be simultaneously set in such a way that the proposed contract benefits all SC actors. The feasible ranges of these contract parameters are mathematically obtained in Eq. (28). In the following, a sensitivity analysis is conducted to better explore the efficacy of the proposed contract and show how it works. The feasible area of the contract parameters under scenario (1), scenario (2), and scenario (3) are shown in Fig. 12-a, 12-b, and 12-c, respectively. These feasible spaces are formed by intersection among the inequalities obtained in Eq. (28). For instance, the feasible space I is formed by intersection among the following constraints: \( W_{M,S_1} \geq 7.34, W_{D,S_1} \leq 10.56, \) and \( W_{D,S_1} - W_{M,S_1} \geq 3.05 \). Under scenario 1, each pair of contract parameters within the feasible area (I) will be acceptable to all SC actors. However, not all pairs will share the surplus benefits among SC actors in a similar way. For instance, using the pair of \( W_{M,S_1} = 7.34 \) and \( W_{D,S_1} = 10.56 \) (i.e., point A), the retailer and manufacturer’s profits will be equal to those under the decentralized model (i.e., 732.71 for the retailer and 1013 for the manufacturer) and the distributors profit will be improved compared to the decentralized one (i.e., from 586.70 to 618.51). Therefore, at point A, all benefits obtained by the coordination will be gained by the distributor. Similarly, at points B and C all the benefits obtained under the proposed coordination contract will be acquired by the manufacturer and retailer, respectively.

In this paper, the channel coordination is investigated under a stochastic scenario-based demand. Each demand scenario has its associated probability level and represents the expected occurrence of a particular outcome. In the following, a sensitivity analysis is conducted to explore the performance of the proposed models under different sets of probabilities. Fig. 13 demonstrates the profitability of the SC under the decentralized and coordinated models by changing the scenario probabilities. As can be seen, the SC profit improves by increasing the probability of the high-demand scenario. Moreover, the SC profit improvement under the coordinated model increases by increasing the probability of the high-demand scenario (see the arrow in Fig. 13). This finding indicates the appropriateness of the proposed coordination model, especially where the product demand is strongly predicted to be high.

To better analyse the impact of the CSR on the proposed models, we compare the profitability of the SC under the coordinated model with and without the CSR investment. Fig. 14 shows the profit of the SC with and without CSR efforts under various scenario probabilities. As can be seen, the CSR investment made by the manufacturer improves the SC profitability compared to the situation where the manufacturer refuses to invest in CSR. Moreover, Fig. 14 shows that the CSR investment is highly beneficial when the probability of the high-demand scenario is high. The results indicate
that the social activities done by the manufacturer not only improve the CSR index of the entire SC, but also economically benefit the chain.
$$W_{b_{M,31}}^b \geq 7.34$$
$$W_{b_{D,31}}^b - W_{b_{A,31}}^b \geq 3.05$$

Feasible area for the contract parameters in the first scenario

Point A: 7.34, 10.56, 1013.76, 618.51, 732.71
Point B: 9.27, 12.33, 1385.40, 586.70, 971.04
Point C: 7.34, 10.40, 1013.76, 586.70, 1342.68

Feasible area for the contract parameters in the second scenario

Point A: 7.34, 15.31, 1013.76, 1531.00, 1119.16
Point B: 12.25, 15.31, 1958.06, 586.70, 1119.16
Point C: 7.34, 10.40, 1013.76, 586.70, 2063.46

Feasible area for the contract parameters in the third scenario

Point A: 7.34, 15.31, 1013.76, 1531.00, 1119.16
Point B: 12.25, 15.31, 1958.06, 586.70, 1119.16
Point C: 7.34, 10.40, 1013.76, 586.70, 2063.46

Fig. 12. Feasible area for the contract parameters in different scenarios
Fig. 13. SC profitability under different sets of demand scenario probabilities in the centralized/coordination and decentralized systems.

Fig. 14. SC profitability under different sets of demand scenario probabilities with and without the CSR investment.
7. Discussion and managerial insights

In this paper, the channel coordination issue is addressed under a scenario-based stochastic demand. In the investigated manufacturer-distributor-retailer chain, the manufacturer invests in CSR initiatives to stimulate the market demand of the chain. On the other side, the retailer, who faces a scenario-based stochastic CSR-dependent demand, decides on the order quantity and directly impacts the amount of satisfied demand. We first analyse the problem under the decentralized and centralized models. The initial results of this study indicate that jointly made decisions under the centralized model can improve the SC's profitability compared to the decentralized one. This finding broadly supports the results of earlier studies on SC coordination (Ni and Li, 2012; Panda et al. 2015; Hosseini-Motlagh, 2018a). Moreover, consistent with the results of the research done by Nematollahi et al. (2017a), the current study finds that the integrated decisions on CSR and order quantity can improve the CSR performance level of the chain. However, all previous studies on CSR coordination assumed a single scenario for the deterministic (Panda, 2014; Ni and Li, 2012) or stochastic (Hsueh, 2014; Nematollahi, 2017a) demand and did not analyse the channel coordination and SC actors' performance under various demand scenarios.

In many real markets, the stochastic market demand can be captured by a set of discrete scenarios, each of which has its own probability distribution parameters (Arabatzis et al., 2013). In this study, inspired by a real case study, we analyse the channel coordination under a scenario-based stochastic demand and explore how the jointly-made decisions in the SC context impact the SC actors' profits under various demand scenarios. Interestingly, the current study finds that the centralized decisions under a scenario-based stochastic demand can negatively impact the profits of some actors. As shown in the numerical experiment, the retailer’s profit in the centralized model might even be negative under some scenarios and consequently they would refuse to accept the centralized plan.

To persuade all actors to transfer from their individually optimized solution (i.e., decentralized model) to a system-wide optimal solution (i.e., centralized model), a coordination contract is needed to carefully orchestrate the problem under study. However, the common contracts existing in the channel coordination literature cannot be applied in the investigated SC due to the scenario-based nature of the stochastic market demand. All previous coordination contracts are applied where the market demand can be approximated by a single scenario. Designing a coordination contract will be more complicated when the future stochastic demand can be captured by a set of discrete scenarios, mainly because the coordination contract must benefit all SC actors regardless of which scenario occurs in the future. To resolve this issue and fill the research gap, in this paper, we propose an adjustable bi-level wholesale price contract to coordinate the investigated SC.

The proposed contract in this study, which can be applied under a scenario-based stochastic demand, is a tailored version of the wholesale price contract. According to Cachon (2003), the initial version of the wholesale price generally cannot coordinate SC systems. However, in the
literature, a number of authors have analyzed behavioural interactions of CSR conduct under wholesale price contracts. Ni et al. (2010) apply a game-theoretical concept to obtain the optimal allocation of CSR between a firm and a supplier under a wholesale price contract. The results of the study conducted by Ni et al. (2010) indicate that the optimal CSR allocation plan is to designate the supplier to perform CSR initiatives. Furthermore, they show that the system-wide profit cannot be obtained under the wholesale price contract due to the double-marginalization. In contrast to this finding, the current study shows that the proposed adjustable bi-level wholesale price contract can result in the system-wide profit even in a more complicated system, i.e., under a three-echelon SC with a scenario-based stochastic demand. Another important finding obtained by Ni et al. (2010) is that the profit of the whole SC improves by investing in CSR compared to the case without considering CSR. This finding is consistent with that of our study. Recently, Nouri et al. (2018) propose an improved version of the wholesale price contract, named compensation-based wholesale price contract, to coordinate a manufacture's innovation with a retailer's replenishment and promotion efforts. The proposed contract by Nouri et al. (2018) is designed to coordinate a two-level SC with a single scenario for demand, while the proposed adjustable bi-level wholesale price contract in this study is able to coordinate a three-stage SC with a scenario-based stochastic demand.

In a recent study, Nematollahi et al. (2017a) find that coordinating order quantity and CSR investment can improve the economic profits and CSR performance in a two-level SC. They propose a collaboration model to coordinate the investigated SC. However, the proposed collaboration model does not consider the possibility of different occurring demand scenarios. Moreover, the model proposed by Nematollahi et al. (2017a) cannot be applied in a three-stage channel where the supplier sells its products to the retailer through a distributor. Last but not least, the collaboration model proposed by Nematollahi et al. (2017a) distributes the surplus benefits between SC actors in a haphazard way. In the current study, an adjustable bi-level wholesale price contract is proposed to coordinate the CSR investment and order quantity in a three-stage channel when the future stochastic demand can only be predicted by a set of discrete scenarios. Using the proposed adjustable bi-level wholesale price contract, our results suggest that the amounts of contract parameters can be determined in such a way that all actors simultaneously enjoy the extra profits generated regardless of which scenario occurs. Moreover, a Nash-bargaining model is proposed to fairly share the surplus benefits among the actors.

Some important managerial insights obtained from the investigated problem are summarized in the following:

- The current study provides a practical method for SC actors to coordinate their decisions when the SC faces a scenario-based stochastic demand. The unpredictability of this type of demand makes the coordination process rather difficult technically. Under such a market demand, a feasible coordination contract should be able to benefit all actors regardless of
which scenario occurs in the future. In the current study, an adjustable bi-level wholesale price contract is proposed to resolve the channel conflict under a stochastic scenario-based demand and encourage all SC actors to take part in the coordination system. The proposed contract provides a win-win status for all three SC actors when they enter the coordination model regardless of which scenario occurs in the future (see Figs. 9, 10 and 11).

• To form a CSR collaboration plan within the SC, it is needed to evaluate the impact of CSR investment on the market demand (Hsueh, 2014). In practice, implementing CSR coordination under a scenario-based stochastic demand will be more difficult than a typical demand that can be captured by a single deterministic or probability distribution function. Our results assist the managers in CSR collaboration within the SC systems when the future demand can be represented by a set of discrete scenarios with given probability of occurrence.

• Under a scenario-based stochastic demand, our findings indicate that joint decision-making on the order quantity and CSR investment considerably increases the whole SC profitability and CSR performance level in comparison to the decentralized system. Nevertheless, the centralized decision will not necessarily provide more profit for all SC actors than the decentralized one under all demand scenarios (see Figs. 9, 10, and 11). Surprisingly, the results indicate that some scenarios occurring might even lead to negative profit for some SC actors. Therefore, implementing the joint decision-making structure (centralized model) is practically impossible in practice when the SC faces a scenario-based stochastic demand.

• The proposed adjustable bi-level wholesale price contract not only increases the economic profits of the SC actors and the whole SC under various values of demand uncertainty (see Fig. 7), but also improves the CSR performance level compared to the decentralized model (see Fig. 8). Moreover, the proposed contract improves the CSR index of the chain under various values of demand sensitivity to the CSR activities (see Figs. 4 and 5).

• The proposed adjustable bi-level wholesale price contract gives the SC actors the opportunity to share the surplus profits among themselves in different ways (see Fig. 12). All proposed contract parameters create a win-win-win situation for all three agents and consequently they willingly enter into the coordination contract. Moreover, to equitably share the surplus profits, the contract parameters can be set using the proposed Nash-bargaining model.

• The proposed coordination model performs well under various settings of demand scenario probabilities. Moreover, the results indicate that the proposed contract considerably improves the SC profitability compared to the decentralized model, especially when the high-demand season has a greater probability than the low-demand season (see Fig. 13).

8. Conclusions and future research directions

40
In this paper, for the first time, a new contract was proposed to achieve channel coordination in a supply chain (SC) facing scenario-based stochastic demand. For this purpose, a decentralized manufacturer-distributor-retailer chain was considered and corporate social responsibility (CSR) was treated as the coordinated decision in the chain. To be more precise, in the investigated SC, the manufacturer aimed to invest in CSR activities and stimulate the scenario-based stochastic demand, which was dependent on the amount of CSR investment made by the manufacturer. Under such a case, the manufacturer's CSR investment would improve the market share of both the distributor and retailer, and consequently a CSR collaboration plan should be designed to share the CSR costs. On the other side, to cope with demand uncertainty, the retailer adopted a classic Newsvendor inventory system based on the scenario-based approach and determined the order quantity, which directly influenced the market share of both the distributor and manufacturer. Firstly, we proposed the mathematical models for scenario-based decentralized and centralized systems. The results indicated that the centralized decision-making on the CSR investment and order quantity would considerably increase the total SC profit and CSR performance level in comparison with the decentralized system. Nevertheless, as widely discussed in the literature, the centralized decisions might not be satisfactory to all SC actors and therefore a coordination plan should be designed. To resolve the channel conflict in the investigated SC, we developed an adjustable bi-level wholesale price contract to coordinate the investigated three-echelon SC under all possible demand scenarios. In the proposed contract, the wholesale price of the distributor and manufacturer were simultaneously adjusted in such a way that the contract resulted in a Pareto improving win-win-win situation after realization of the market demand. Moreover, a profit-sharing scheme based on the Nash-bargaining model was proposed to share the surplus profit among the SC actors. Finally, besides a test problem, a set of sensitivity analyses was conducted to evaluate the performance of our proposed models. The results indicated that the proposed incentive mechanism could coordinate the SC under various values of parameters. To be more precise, the results showed that economic profitability of each SC actor and the whole SC increased under the coordinated structure in comparison with the decentralized one in all possible scenarios. The results suggested that the manufacturer would not be willing to invest highly in CSR activities under the decentralized model. However, the proposed contract provided enough incentives for the manufacturer to invest in CSR activities even under the situation where the CSR investment had negligible effects on the market demand. Moreover, the results showed that the proposed adjustable bi-level wholesale price contract had the capability of coordinating the three-echelon SC under high demand uncertainty. In the real-world business environments, it can be extensively observed that the market demand is dependent on the retail price. Considering a price and CSR-sensitive stochastic scenario-based demand could be a future study. Moreover, one could apply other coordination contracts in the investigated problem to extend the current study.
References


& Chemical Engineering. 27, 1219–1227. https://doi.org/10.1016/S0098-1354(03)00048-6


Appendix A:

In the decentralized framework, the retailer's expected profit can be defined as:

$$
\Pi_{R}^{de} = PE \left[ \min (Q, D) \right] - W_{D}Q
$$

The expected sales $E[\min(Q, D)]$ can be calculated as:

$$
E[\min(Q, D)] = \int_{-\infty}^{\infty} Df(D)dD + \int_{0}^{\infty} Qf(D)dD = \int_{-\infty}^{\infty} D \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(D-\mu)^2}{2\sigma^2}} dD + Q(1 - F(Q))
$$

By replacing $\frac{(D - \mu)}{\sigma}$ with $x$. Then $dD = \sigma dx$

$$
E[\min(Q, D)] = \int_{-\infty}^{0} (\mu + \sigma x) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x)^2} dx + Q(1 - F_{\frac{x}{\sigma}}(\frac{Q - \mu}{\sigma}))
$$
\[ = \int_{-\infty}^{0} \frac{\mu}{\sigma} e^{-\frac{1}{2} x^2} \, dx + \int_{0}^{\infty} \frac{\sigma x}{2\pi} e^{-\frac{1}{2} x^2} \, dx + Q \left( 1 - F(z) \frac{Q - \mu}{\sigma} \right) \]

\[ = (\mu - Q) F(z) \frac{Q - \mu}{\sigma} + \sigma \int_{-\infty}^{0} \frac{x}{2\pi} e^{-\frac{1}{2} x^2} \, dx + Q \]

By replacing \( x^2 / 2 \) with \( w \), then \( x \, dx = dw \)

\[ E[\min(Q, D)] = (\mu - Q) F(z) \frac{Q - \mu}{\sigma} + \sigma \int_{-\infty}^{0} \frac{1}{2\pi} e^{-w} \, dw + Q \]

\[ = (\mu - Q) F(z) \frac{Q - \mu}{\sigma} + \sigma f(z) \frac{Q - \mu}{\sigma} + Q \]

By substituting \( E[\min(Q, D)] \) with the associated expanded equations we obtain:

\[ \Pi^k_R = (P) \left( (\mu - Q) F(z) \frac{Q - \mu}{\sigma} + \sigma f(z) \frac{Q - \mu}{\sigma} \right) + (P - W_D) Q \]

By replacing \( \mu = \mu_0 + \alpha \eta(\eta) \), we have:

\[ \Pi^k_R = P \left[ (\mu_0 + \alpha \eta(\eta) - Q) F(z) \frac{Q - \mu_0 - \alpha \eta(\eta)}{\sigma} - \sigma f(z) \frac{Q - \mu_0 - \alpha \eta(\eta)}{\sigma} \right] - (P - W_D) Q \]

The proof is completed.

**Appendix B:**

In order to obtain the optimal solution of \( Q \) with a fixed \( \eta \), the first and second order derivatives are calculated with respect to \( Q \) as:

\[ \frac{\partial \Pi^k_R(Q)}{\partial Q} \to -pF(z) \frac{Q - \mu_0 - \alpha \eta(\eta)}{\sigma} + (P - W_D) \]
\[ \frac{\partial^2 \Pi_{R}^{dc}(Q)}{\partial Q^2} \rightarrow -(P) \left( \frac{Q - \mu_0 - \alpha y(\eta)}{\sigma} \right) < 0 \]

Since the second order derivative of \( \Pi_{R}^{dc} \) is negative, \( \Pi_{R}^{dc} \) is concave with respect to \( Q \) with a given \( \eta \). Set \( \frac{\partial \Pi_{R}^{dc}(Q)}{\partial Q} = 0 \) and we have:

\[ F_z \left( \frac{Q - \mu_0 - \alpha y(\eta)}{\sigma} \right) = \frac{(P - W_D)}{P} \]

\[ Q_{^{dc}} = F_z^{-1} \left( \frac{P - W_D}{P} \right) \sigma + \mu_0 + \alpha y(\eta) \]

The proof is completed.

**Appendix C:**

Calculating the first and second derivatives of \( \Pi_{M}^{dc} \) with respect to \( \eta \):

\[ \Pi_{M}^{dc} = (W_M - C - \eta) \times \left\{ F_z^{-1} \left( \frac{P - W_D}{P} \right) \sigma + \mu_0 + \alpha y(\eta) \right\} \]

\[ \frac{\partial \Pi_{M}^{dc}(\eta)}{\partial \eta} = -(J + C_0 + \alpha y(\eta)) + (W_M - \eta - C)(J + \alpha y(\eta)) \]

\[ \frac{\partial^2 \Pi_{M}^{dc}(\eta)}{\partial \eta^2} = -2(\alpha y'(\eta)) + (W_D - \eta - C)(\alpha y''(\eta)) \]

where

\[ y(\eta) = 1 - 1/(0.5\eta + 1) \]

\[ y'(\eta) = 0.5/(0.5\eta + 1)^2 \]

\[ y''(\eta) = -0.5/(0.5\eta + 1)^3 \]
\[ J = F_z^{-1}(P - W_P)\sigma \]

It is clear that for any profitable firm, the equation \( W_P - \eta - C \) should be positive. Hence, \( \Pi_{M}^k \) is a concave function with respect to \( \eta \). Set \( \frac{\partial \Pi_{M}^k(\eta)}{\partial \eta} = 0 \) we have:

\[
\eta_{dc}^* \equiv - (\alpha + \mu_0 + J) + \sqrt{(\alpha + \mu_0 + J)^2 - (\alpha + \mu_0 + J)(-0.5\alpha(W_m - C) + \mu_0 + J)} \]

\[ 0.5(\alpha + \mu_0 + J) \]

The proof is completed.

**Appendix D:**

In order to prove the concavity of the function \( \Pi_c^T \) with respect to \( Q \) and \( \eta \), the Hessian matrix of \( \Pi_c^T \) with respect to these variables should be calculated.

\[
H_{\Pi_c^T} = \begin{bmatrix}
\frac{\partial^2 \Pi_c^T}{\partial Q^2} & \frac{\partial^2 \Pi_c^T}{\partial Q \partial \eta} \\
\frac{\partial^2 \Pi_c^T}{\partial \eta \partial Q} & \frac{\partial^2 \Pi_c^T}{\partial \eta^2}
\end{bmatrix}
\]

Where,

\[
\frac{\partial^2 \Pi_c^T}{\partial Q^2} = \frac{(P\sigma f_z'(Q - \mu_0 - \alpha y(\eta)))}{\sigma}
\]

\[
\frac{\partial^2 \Pi_c^T}{\partial Q \partial \varphi} = \frac{\partial^2 \Pi_c^T}{\partial \varphi \partial Q} = -1 + \frac{\alpha y(\eta)(P\sigma f_z(Q - \mu_0 - \alpha y(\eta)))}{\sigma}
\]

\[
\frac{\partial^2 \Pi_c^T}{\partial \varphi^2} = (P\alpha y)(\eta) - \left\{ \alpha(P\sigma f_z(Q - \mu_0 - \alpha y(\eta))) + \frac{\alpha y(\eta)^2}{\sigma} \right\}
\]
Obviously, the first principal minor of the calculated Hessian matrix is always negative. Moreover, under the following condition, the second principal minor of that is positive:

\[
\frac{\alpha(P)}{\sigma} \left[ \frac{Q - \mu_0 - \alpha y \eta}{\sigma} \right] \left[ 2 y' \eta - (P) y'' \eta \right] + (P) y'' \eta (1 - F \left( \frac{Q - \mu_0 - \alpha y \eta}{\sigma} \right)) > 1
\]

This condition is tested in our investigated data set and it is observed that the condition holds under a wide range of parameters. Consequently, the SC profit function is concave with respect to Q and \( \eta \).