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**Highlights:**

Put forward the retailer’s overconfidence in a duopolistic supply chain

Categorize two cases on retailer’s overconfidence information and derive the retailer’s optimal pricing and ordering decision in Nash game

Extension to a Stackelberg-like model to investigate the impact of first-mover advantage on the outcome of overconfidence

Overconfidence does not necessarily damage the supply chain performance
Optimization on pricing and overconfidence problem in a duopolistic supply chain

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Abstract We analyze the impact of the retailer’s overconfidence on the supply chain performance. We consider a duopolistic market with uncertain demand where one overconfident retailer and one rational retailer compete in selling the same product. We identify two cases in terms of the rational retailer’s overconfidence awareness: (1) the rational retailer is aware of the other retailer’s overconfidence; (2) the rational retailer is not aware of the other retailer’s overconfidence. We first analyze the two retailers’ decision processes in a Nash game and derive their optimal decisions, and further discuss a Stackelberg-like game for an extension. We conduct a numerical analysis to examine the impact of overconfidence on the optimal pricing, ordering decisions and expected profits. We find that the higher the overconfident level is, the higher selling price that the overconfident retailer charges. Together, the results stress that the overconfidence does not necessarily damage the supply chain performance.

Keywords: Supply chain management, Overconfidence, Duopolistic market, Overconfidence awareness
1. Introduction

As one of the most commonly seen irrational behaviors in decision making, overconfidence has drawn much attention and been widely acknowledged having significant impacts in the economic and management areas. Nowadays, it is ubiquitous that retailing giants cut prices down to compete with their rivals in an aggressive way, even behave irrationally and fight like Kilkenny cats, due to their overconfidence on market outlooks. Although there is no general tendency toward overconfidence (Fowler and Johnson, 2011), previous market share or profit level, even their finance capital etc. may become anchors or reference points in their decision making, particularly their prominent successes in such business performances would bloat them to behave irrationally either trigger a price war in their markets or break into new markets. For example, with previous rapid expansion and ambition on booming market share, JD.com has successively launched two price wars in three years, with Dangdang.com in 2010\(^1\) and with Gome.com.cn and Suning.com in 2012\(^2\), yet benefitted from neither as wished. In contrast, the price war launched optimistically by Gree Electric Appliances in 2014 helped expand the market share and digest finished products inventory dramatically. It also happens in other markets, a well-known case is that in the crowdfunding market, MI Tech. Ltd. and PepsiCo Corp. almost simultaneously initiated their next generation smartphone in crowdfunding platform, the latter yet finally failed with an ambitious funding target\(^3\). In these cases above, the firms behave in overconfidence to some degree in their decision makings. However, the outcomes are not always the same.

Plous (1993) notes that “no problem in judgment and decision making is more prevalent and more potentially catastrophic than overconfidence”.

From a traditional perspective, overconfidence is perceived as a sabotaging factor in a wide range of areas: unprofitable merger and acquisition (Malmendier & Tate, 2005), mal-investment in secondary market (Heaton, 2002), even unwise channel entry (Camerer & Lovallo, 1999) and inventory imbalance (Ren & Croson, 2013) in operational level. Our study challenges the conventional wisdom that overconfidence makes decision maker (even supply chain) worse off by examining the impact of overconfidence in the context of duopolistic market. Specifically, our objective is to assess the impact of one retailer’s overconfidence in a duopolistic supply chain and how the rival’s overconfidence awareness adjusts this impact in different market power conditions. Our research question is motivated by the observation that the outcomes of overconfidence in duopolistic market have been confirmed to vary substantially in industries.

To address the research question, we employ a series of models with different conditions of the rival’s overconfidence awareness and compare the outcomes. The models consist of a supply chain with one supplier and two competing retailers, one retailer holds overconfident biased belief on market information, and the

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1 For details, refer to http://tech.ifeng.com/internet/special/360buypkdangdang/
2 Refer to http://tech.hexun.com/2012/jiagezhan2012/
3 PepsiCo Corp. initiated a project for Pepsi Phone in Nov 2015, and ambitiously set his funding target as 3 million RMB with an aggressively low price at 499 RMB. Ultimately this project failed with a poor fund-raising. Almost at the same time, MI Tech. Ltd. also initiated a project for its new generation smartphone with a lower funding target of 1 million and a relatively higher unit selling price of 1000 RMB, the project finally raised more than 35.995 times of the initial funding target. Refer to http://z.jd.com/project/details/30677.html and https://izhongchou.taobao.com/dreamdetail.htm?spm=a215p.128754.653087.26.4dfb51b6JYu Q2u&id=10050396
other retailer is rational. Further, the models also hinge on the market power to test the robustness of the results by assuming that the two retailers play Nash game and Stackelberg-like game respectively.

The paper proceeds as follows. We review the literature in §2. Following that, we describe the problem and present our models in §3. Section 4 discusses two retailers’ pricing and ordering decisions in a duopolistic market. Section 5 presents numerical studies to discuss the impact of overconfidence on pricing, ordering and profits. Finally, Section 6 concludes and provides direction for future research.

2. Literature review

There are two primary streams of research that relate to our analysis, which are studies conducted on pricing competition among retailers and studies on overconfidence, respectively. For the first stream, the competition among retailers is extensively addressed in academia and analyzed from different perspectives. Ingene and Parry (1995) analyze how two competing retailers make their pricing strategies when products are homogenous. Bernstein and Federgruen (2003) extend to address two competing retailers’ pricing strategies in Bertrand and Cournot game. Xiao and Qi (2008) consider a scenario of pricing competition between two retailers where the manufacturer’s production cost is disrupted. Parthasarathi et al. (2011) study how two competing retailers make pricing and ordering decisions when the manufacturer acts as a Stackelberg leader. More recently, scholars begin to analyze the dual channel problem. Mukhopadhyay et al. (2011) consider two firms sell complementary goods in a Stackelberg game under information asymmetry. Chen et al. (2013) consider the impact of competing for price on the supply chain in Nash game and Stackelberg game, respectively. Du et al. (2016) discuss the pricing competition between the incumbent and entrant firm on the innovative product. A literature review on the pricing competition and its extensions can be found in Agatz et al. (2008). Although many of these papers consider the pricing competition of the firms, none addresses the retailer’s decision preferences in a competing retailing environment. As a complement to the literature reviewed above, our paper explores the impact of overconfidence on pricing decision in a duopolistic supply chain context.

Decision-makers’ behavior preference, the second stream, also gains great popularity in academia. Pertinent scholars have targeted their concerns on risk aversion (Yang et al., 2009), loss-averse (Liu et al., 2013), fairness concern (Cui et al., 2007; Ho et al., 2014), anchoring effects (Chandrashekaran and Grewal, 2006), the opportunism behavior (Hawkins et al., 2008), and spin off effect (Klepper and Sleeper, 2005), amongst many others. However, to the best of our knowledge, there has been limited research on the overconfidence in OM and MS area, whereas our paper considers the overconfidence in a supply chain with two retailers. The new trend is that scholars begin to pay close attention to overconfidence in its various forms to examine its impact and describe a variety of behavior in a supply chain environment (Croson et al., 2011; Ren and Croson, 2013).

Research on overconfidence is quite mature in the field of psychology and has been widely explored in the field of behavioral finance and behavior management. Particularly, Gervais et al. (2002) find that overconfident managers can make better decisions than rational managers in coordinating the interests of the
shareholders. Heaton (2002) thinks that managers’ overconfidence leads to the low efficiency of investment decision and then damages the interests of the enterprise. Nowadays, there is some literature starting to investigate the overconfidence in OM/MS area. Croson et al. (2011) summarize previous experimental study on human preference, and use salvage costs and price adjustments to revise the overconfident retailer’s ordering decisions. Ren and Croson (2013) show that the order bias could be caused by the overconfidence using a preference test. In a similar vein, Proeger and Meub (2014) show that overconfidence is a social rather than an individual bias through an experiment conducted in their study. Moreover, Wang et al. (2015) build a model of a retailer who is overconfident on mean and variance of the demand and discuss the effectiveness of supply contract in the supply chain coordination. Ren et al. (2017) conduct a theoretical research as an extension of their study (Ren and Croson, 2013). They model the newsvendor’s overprecision behavior as a biased belief that the distribution of demand has an underestimated variance. Nevertheless, these papers only consider a primary market containing one retailer, or a supply chain with one manufacturer and one retailer. Furthermore, experiment methods are always used in these studies to test and verify the overconfidence in the supply chain. Differing from most of the existing literature, under random and price-dependent demand, our paper analytically investigates different pricing games between one rational retailer and one overconfident retailer, and addresses the impact of overconfidence on supply chain performance under different overconfident information settings. More recently, Li et al. (2016) study the ordering game between two overconfident newsvendors. In comparison, our paper not only explores the impact of one retailer’s overconfidence on both retailers’ pricing and ordering decisions, but also examines how the rational retailer’s knowledge about his rival’s overconfidence adjusts this impact.

3. Problem descriptions

Consider a duopolistic supply chain wherein a rational decision-maker retailer 1 and retailer 2 with overconfident bias on the market demand compete in Nash game on selling prices under uncertain demand. According to retailer 1’s awareness about retailer 2’s overconfidence, we study two cases in the following: (1) retailer 1 is aware of retailer 2’s overconfidence; and (2) retailer 1 considers retailer 2 as a rational decision-maker as well. The manufacturer produces products with unit cost $c$, the two retailers place their orders $q_i$ ($i = 1, 2$) with a unit price $w$ and then sell products to consumers with a price $p_i$, respectively. The salvage value for the unsold products is $s$ per unit. The parameters used above have the following relationship: $c < s < w < p$.

In the duopolistic market, the demand is stochastic and price-dependent. Similar to the study of Mills (1959), we assume that the demand function is $D(p_1, p_2, \varepsilon) = y(p_1, p_2) + \varepsilon$. Therefore, the retailers’ actual demand functions are

$$D_1(p_1, p_2, \varepsilon_1) = m_1 + d_1p_2 - k_1p_1 + \varepsilon_1,$$

$$D_2(p_1, p_2, \varepsilon_2) = m_2 + d_2p_1 - k_2p_2 + \varepsilon_2.$$
where \( m_i \) is the market scale of retailer \( i \), \( k_i \) is the price sensitivity and \( d_i \) is the cross-price sensitivity satisfying \( d_i < k_i \). Moreover, \( \varepsilon_i \) is a continuous random variable on the range of \([A_i, B_i]\) with mean \( \mu_i \) and variance \( \sigma_i^2 \), and has a cumulative distribution function \( F(\varepsilon_i) \) and probability density function \( f(\varepsilon_i) \)(\( i = 1, 2 \)). For the tractability, it is assumed that \( \varepsilon_i \) follows a uniform distribution. Besides, to avoid trivial cases, it is assumed \( A_i > -m_i \) (Petruzzi and Dada, 1999).

Referring to the classification of overconfidence defined by Moore and Healy (2008), the retailer’s overconfident bias belongs to overprecision. According to Ren and Croson (2013), we adopt a mean-preserving but variance-reducing transformation of the actual market demand. Therefore, in the view of retailer 2, the demand functions he faces and retailer 1 faces are shown in (3) and (4), respectively,

\[
D_{22}(p_{21}, p_2, \varepsilon_2) = m_2 + d_2 p_{21} - k_2 p_2 + a\varepsilon_2 + (1 - a)\mu_2, \tag{3}
\]

\[
D_{21}(p_{21}, p_2, \varepsilon_{21}) = m_1 + d_1 p_2 - k_1 p_{21} + a\varepsilon_1 + (1 - a)\mu_1, \tag{4}
\]

where \( a \) indicates the overconfident level and satisfies \( 0 \leq a \leq 1 \). Here, the smaller \( a \) is, the higher the overconfident level is. Especially, when \( a = 1 \), retailer 2 has no overconfidence on demand variance (i.e., he is a rational decision-maker). Meanwhile, in the view of retailer 2, the mean of the market demand is \( ED_{22}(p_{21}, p_2, \varepsilon_2) = ED_2(p_1, p_2, \varepsilon_2) \) and the variance is \( VarD_{22}(p_{21}, p_2, \varepsilon_2) = a^2\sigma^2 = a^2 VarD_2(p_1, p_2, \varepsilon_2) \).

Based on the demand functions above, we aim to analyze two competing retailers’ pricing and ordering decisions in two cases. To clarify our exposition, (i) we use expected profit to represent the actual expected profit of manufacturer and retailers, and anticipated profit to represent the fictitious expected profit of manufacturer and retailers, respectively; (ii) we use the subscripts 1 and 2 to represent retailer 1 and 2, and use superscripts 1, 2, 3, and 4 to represent the benchmark case, Case I, Case II, Case III, and Case IV, respectively throughout the paper. To be specific, we first consider a Nash game, in which retailer 1 realizes retailer 2’s overconfidence in Case I while retailer 1 is not aware of retailer 2’s overconfidence in Case II. Then, as an extension, a Stackelberg game (retailer 2 is the leader) is studied in Case III and IV. Similar to Case I and Case II, retailer 1 has a foresight on retailer 2’s overconfident behavior in Case III, while he considers retailer 2 as a rational decision maker in Case IV. In order to provide a consistent notation throughout this paper, we briefly describe the parameters and variables appeared in the derivation of the theorem and corollaries, as shown in Table 1.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$c$</td>
<td>production cost per unit</td>
</tr>
<tr>
<td>$s$</td>
<td>salvage value for unsold product per unit</td>
</tr>
<tr>
<td>$m_i$</td>
<td>market scale of retailer $i$</td>
</tr>
<tr>
<td>$d_i$</td>
<td>cross-price sensitivity</td>
</tr>
<tr>
<td>$\varepsilon_i$</td>
<td>price sensitivity</td>
</tr>
<tr>
<td>$F(\varepsilon_i)$</td>
<td>cumulative distribution function of $\varepsilon_i$</td>
</tr>
<tr>
<td>$f(\varepsilon_i)$</td>
<td>probability density function of $\varepsilon_i$</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>mean of $\varepsilon_i$</td>
</tr>
<tr>
<td>$\sigma_i^2$</td>
<td>variance of $\varepsilon_i$</td>
</tr>
<tr>
<td>$\pi_i^N$</td>
<td>retailer $i$’s profit when the two retailers are rational</td>
</tr>
<tr>
<td>$\pi^K$</td>
<td>the profit of the entire supply chain in case $K$</td>
</tr>
<tr>
<td>$E(\pi^K_i)$</td>
<td>retailer $i$’s expected profit in case $K$</td>
</tr>
</tbody>
</table>

$p_i^{K*}$ represents the optimal pricing of retailer $i$ in case $K$ in retailer $i$’s view.

$q_i^{K*}$ represents the optimal ordering quantity of retailer $j$ in case $K$ in retailer $i$’s view.

$\pi_j^K$ represents the profit of retailer $j$ in case $K$ in retailer $i$’s view.

$D_{ij}(p_{ij}, p_i, \varepsilon_j)$ represents the demand of retailer $j$ in retailer $i$’s view which is related with $p_{ij}, p_i, \varepsilon_j$.

### 4. The analysis of two cases

It is known that the retailers’ objectives are to maximize their own profits by choosing prices and ordering quantities. As a benchmark where retailer 1 and 2 are both rational, retailer $i$’s expected profit is $\pi_i^N (i = 1, 2)$.

#### 4.1 Case I

In this case, both retailers make their pricing and ordering decisions simultaneously. Retailer 1 knows that retailer 2 is overconfident on his demand variance and the overconfidence level; however, in retailer 2’s view, they both are rational, thus their market demand functions are equations (3) and (4). That is, retailer 2 does not recognize his own overconfident behavior. Accordingly, we build both retailers’ profit functions in retailer 2’s view as follows.

Retailer 1’s profit function is

\[
\pi_{21}^I(q_{21}, p_{21}, p_{21}^I) = \begin{cases} 
  p_{21}^I D_{21}^I(p_{21}^I, p_{21}, \varepsilon_{21}^I) - w q_{21}^I + s[q_{21} - D_{21}^I(p_{21}^I, p_{21}, \varepsilon_{21}^I)], & \text{if } D_{21}^I(p_{21}^I, p_{21}, \varepsilon_{21}^I) \leq q_{21} \\
  p_{21}^I q_{21}^I - w q_{21}^I, & \text{if } D_{21}^I(p_{21}^I, p_{21}, \varepsilon_{21}^I) > q_{21} 
\end{cases}
\]

Retailer 2’s profit function is

\[
\pi_{22}^I(q_{22}, p_{21}^I, p_{21}^I) = \begin{cases} 
  p_{21}^I D_{22}^I(p_{21}^I, p_{21}^I, \varepsilon_{21}^I) - w q_{21}^I + s[q_{22} - D_{22}^I(p_{21}^I, p_{21}^I, \varepsilon_{21}^I)], & \text{if } D_{22}^I(p_{21}^I, p_{21}^I, \varepsilon_{21}^I) \leq q_{22} \\
  p_{21}^I q_{22}^I - w q_{22}^I, & \text{if } D_{22}^I(p_{21}^I, p_{21}^I, \varepsilon_{21}^I) > q_{22} 
\end{cases}
\]

Since each retailer seeks to maximize his own profit, in the view of retailer 2, the optimal prices are shown in Theorem 1.

**Theorem 1.** In the view of retailer 2, the optimal selling prices are
\[ p_{21}^* = \frac{2a(B_1-A_1)(m_1+k_1w+d_1p_{12}^*+\mu_1)-(z_{21}^*-aB_1)^2}{4ak_1(B_1-A_1)}, \]
\[ p_{12}^* = \frac{2a(B_2-A_2)(m_2+k_2w+d_2p_{12}^*+\mu_2)-(z_{12}^*-aB_2)^2}{4ak_2(B_2-A_2)}, \]
where \( z_{21}^* = \frac{aB_1(p_{21}^*-w)+aA_1(w-s)}{p_{21}^*-s} \) and \( z_{12}^* = \frac{aB_2(p_{12}^*-w)+aA_2(w-s)}{p_{12}^*-s}. \)

Given the values of \( p_{21}^*, z_{21}^*, p_{12}^*, \) and \( z_{12}^* \), the optimal ordering, from retailer 2’s perspective, are \( q_{21}^* = m_1 + d_1p_{21}^* - k_1p_{12}^* + (1-\alpha)\mu_1 + z_{21}^* \) and \( q_{12}^* = m_2 + d_2p_{12}^* - k_2p_{12}^* + (1-\alpha)\mu_2 + z_{12}^* \), respectively.

Retailer 2 believes he can predict retailer 1’s optimal pricing and ordering decisions accurately, and maximize his own profit accordingly. However, he is overconfident on the market demand variance and makes a distorted pricing decision, which would lead to market disruption, thereby causing a decrease in both his profit and profits of retailer 1.

Compared with retailer 2, retailer 1 can predict the market demand accurately. Meanwhile, he knows retailer 2’s overconfident level and decision process clearly. Thus, retailer 1 can predict retailer 2’s optimal pricing and ordering decisions accurately, which embodies the value of information (Yu et al., 2001).

Retailer 1’s profit function is given as follows,
\[
\pi_{11}(q_1^*, p_1^*, p_{12}^*) = \begin{cases} 
 p_1^*D_{11}(p_1^*, p_{12}^*, \epsilon_1^*) - wq_1^* + s[q_1^* - D_{11}(p_1^*, p_{12}^*, \epsilon_1^*)], & D_{11}(p_1^*, p_{12}^*, \epsilon_1^*) \leq q_1^* \\
 p_1^*D_{12}(p_1^*, p_{12}^*, \epsilon_1^*) & D_{12}(p_1^*, p_{12}^*, \epsilon_1^*) \leq q_1^* 
\end{cases} \tag{5}
\]

Based on the analysis above, the best response of retailer 1 is shown in Theorem 2.

**Theorem 2.** Retailer 1’s optimal selling price is
\[ p_1^* = \frac{2a(B_1-A_1)(m_1+k_1w+d_1p_{12}^*+\mu_1)-(z_{12}^*-B_1)^2}{4k_1(B_1-A_1)}, \]
where \( z_{12}^* = \frac{B_1(p_{12}^*-w)+A_1(w-s)}{p_{12}^*-s} \) and \( p_{12}^* \) is retailer 2’s optimal price predicted by retailer 1.

Besides, retailer 1’s optimal ordering quantity is \( q_1^* = m_1 + d_1p_{12}^* - k_1p_{12}^* + z_{12}^* \), where \( p_{12}^* \) is retailer 2’s optimal pricing predicted by retailer 1. Here, note that \( p_{12}^* = p_{22}^* \) in case I, which means retailer 1 can predict retailer 2’s pricing decision accurately.

4.2 Case II

Different from case I, in case II, retailer 1 is not aware of retailer 2’s overconfidence. In other words, retailer 1 thinks that retailer 2 has the same knowledge of demand as he does.

4.2.1 The rational retailer problem

As illustrated at the beginning of this section, retailer 1 thinks that both of them are rational and their demand functions have been given in (1) and (2). We utilize Theorem 3 to demonstrate the optimal pricing strategy in retailer 1’s point of view.
Theorem 3. In the view of retailer 1, the optimal pricings are

\[
p_{11}^{llr} = \frac{2(B_1 - A_1)(m_1 + k_1 w + d_1 p_{12}^{llr} + \mu_1) - (B_1 - z_1^{llr})^2}{4k_1(B_1 - A_1)},
\]
\[
p_{12}^{llr} = \frac{2(B_2 - A_2)(m_2 + k_2 w + d_2 p_{12}^{llr} + \mu_2) - (B_2 - z_1^{llr})^2}{4k_2(B_2 - A_2)}.
\]

where \(z_1^{llr} = \frac{p_{11}^{llr} - w}{p_{11}^{llr} - s}\) and \(z_1^{llr} = \frac{p_{12}^{llr} - w}{p_{12}^{llr} - s}\).

In the view of retailer 1, the optimal ordering quantities of both retailers are \(q_{11}^{llr} = m_1 + d_1 p_{11}^{llr} - k_1 p_{11}^{llr} + z_1^{llr}\), \(q_{12}^{llr} = m_2 + d_2 p_{12}^{llr} - k_2 p_{12}^{llr} + z_1^{llr}\), respectively, where the values of \(p_{11}^{llr}, z_1^{llr}, p_{12}^{llr}\) and \(z_1^{llr}\) are shown in Theorem 3.

4.2.2 The overconfident retailer problem

As mentioned above, retailer 2 thinks that both of them are rational and the market demands are given in equations (3) and (4), respectively. He predicts his own optimal pricing and ordering decisions. Meanwhile, he believes that his rival has the same prediction and also considers the market demand functions to follow equations (3) and (4). Similar to the results obtained in section 4.2.1, retailer 2 makes the same decisions.

Corollary 1. The optimal pricing \(p_2^{*}\) is decreasing with respect to \(a\).

The result of Corollary 1 implies that the higher retailer 2’s overconfident level is, in retailer 2’s view, the smaller the demand fluctuation is and the more precision his prediction is.

Corollary 2. \(p_{12}^{*}\) is strictly convex with respect to \(p_{11}^{*}\).

This corollary reveals the relationship between the prices of retailers 1 and 2 in case I, it tells us that in retailer 1’s view, he could adjust his rival’s selling price by designing his selling price, and because of the convexity, the results implies that retailer 1 has two pricing options for the same price of retailer 2. However, he can choose the lower one to capture the competitive pricing advantage.

Corollary 3. Retailer 2’s expected profit has the following properties,

1. \(E(\pi_{22}) > E(\pi_2)\):
2. \(\Delta = E(\pi_{22}) - E(\pi_2)\) increases with respect to the overconfident level.

Corollary 3 shows the negative effect of overconfidence which can lead to order bias and a loss of profit.

After that, we aim to analyze the relationship among \(E(\pi_{11}^N), E(\pi_1^I), \) and \(E(\pi_{11}^I)\), which is given in Corollary 4.

Corollary 4. \(E(\pi_1^I) > E(\pi_1^I) > E(\pi_{11}^N)\).

Corollary 4 implies that retailer 1 can make more profit when competing with an overconfident retailer than with a rational retailer. When retailer 1 realizes retailer 2’s overconfidence, he can benefit from the information on retailer 2’s overconfidence.

4.3 Extension

4.3.1 A Stackelberg game problem
Our analysis has assumed that two competing retailers sell products in Nash game in a duopolistic market and decide selling prices simultaneously. However, in reality, different retailers may have different market power and the game leader always has the first-mover advantage. Given this knowledge, in this section, we consider the case that the retailer who has greater market power is more prone to behave in overconfidence in the duopolistic market environment. Hence both retailers interact in a Stackelberg-like game, where the overconfident retailer (retailer 2) is the Stackelberg leader and the rational retailer (retailer 1) is the follower. The sequence of events is defined as follows. Firstly, retailer 2 announces $p_2$ and $q_2$ to maximize his profit (i.e., $\pi_2$). Then, in response to retailer 2’s pricing and ordering strategy, retailer 1 chooses $p_1$ and $q_1$ to achieve his profit (i.e., $\pi_1$) maximization.

Firstly, we discuss case III, in which retailer 1 knows that retailer 2 is overconfident. In the view of retailer 2, retailer 1’s expected profit function is

$$E[\pi_{1}^{III}(p_{12}^{III}, p_{2}^{III}, z_{21}^{III})] = \int A_{1}^{a} f(u_{1}) du_{1} + \int A_{1}^{a} s\left(z_{21}^{III} - a u_{1}\right) f(u_{1}) du_{1} - w\left[m_{2} + d_{2} p_{2}^{III} - k_{2} p_{2}^{III} + (1 - a) \mu_{1} + z_{21}^{III}\right] + \int A_{2}^{b} f(u_{2}) du_{2}$$

Solving the first-order condition of retailer 1’s anticipated profit function with respect to $p_{2}^{III}$ and $z_{21}^{III}$, respectively, we can obtain

$$p_{21}^{III} = \frac{2a(B_{1}-A_{1})(m_{2} + k_{2} w + a_{2} p_{2}^{III} + \mu_{1}) - \left(ab_{1} z_{21}^{III}\right)^{2}}{4a k_{1}(B_{1}-A_{1})},$$

(6)

$$z_{21}^{III} = \frac{a B_{1}(p_{21}^{III} - w) + a A_{1}(w - s)}{p_{21}^{III} - s}.$$  

(7)

Due to the complexity of mathematical calculation, closed-form expressions for the optimal prices are difficult to obtain and $p_1$ cannot be represented explicitly by $p_2^{III}$. Therefore, by defining $p_{21}^{III} = g(p_{2}^{III})$ and substituting equations (5) and (6) into retailer 2’s anticipated profit function, we can obtain

$$E[\pi_{2}^{III}(p_{12}^{III}, p_{2}^{III}, z_{22}^{III})] = \int A_{1}^{a} f(u_{1}) du_{1} + \int A_{2}^{a} s(z_{22}^{III} - a u_{2}) f(u_{2}) du_{2} - w\left[m_{2} + d_{2} p_{21}^{III} - k_{2} p_{2}^{III} + (1 - a) \mu_{2} + a u_{2}\right] + \int A_{2}^{b} f(u_{2}) du_{2}$$

$$= \int A_{1}^{a} f(u_{1}) du_{1} + \int A_{2}^{b} f(u_{2}) du_{2} - w\left[m_{2} + d_{2} g(p_{2}^{III}) - k_{2} p_{2}^{III} + (1 - a) \mu_{2} + a u_{2}\right] + s(z_{22}^{III} - a u_{2}) f(u_{2}) du_{2}$$

$$- w\left[m_{2} + d_{2} g(p_{2}^{III}) - k_{2} p_{2}^{III} + (1 - a) \mu_{2} + \frac{z_{22}^{III}}{a}\right]$$

(8)
Taking the first-order condition of retailer 2’s anticipated profit function with respect to \( p_2 \) and \( z_2 \), respectively, we get \( p_2^{III} \) and \( z_2^{III} \). Therefore, retailer 2’s optimal ordering quantity can be represented as \( q_2^{III} = m_2 + d_2 g(p_2^{III}) - k_2 p_2^{III} + (1 - a) \mu_2 + z_2^{III} \).

As mentioned above, retailer 1 has predicted retailer 2’s decisions correctly before retailer 2 makes decisions. Retailer 1’s optimal response (i.e., \( p_1^{III}(p_2^{III}, z_2^{III}) \)) is obtained by setting \( \frac{\partial \pi_1}{\partial p_1} = 0 \) to find \( p_1^{III} \) and \( q_1^{III} \). Finally, we can obtain that \( p_1^{III} = \frac{2(B_1-A_1)(m_1+k_1 w+d_1 p_1^{IV}+\mu_1)-(z_1^{IV}-B_1)^2}{4k_1(B_1-A_1)} \) and \( z_1^{III} = \frac{B_1(p_1^{III}-w)+A_1(w-s)}{p_1^{IV}-s} \), and retailer 1’s optimal ordering quantity is \( q_1^{III} = m_1 + d_1 p_1^{III} - k_1 p_1^{III} + z_1^{III} \).

Then we discuss case IV: retailer 1 thinks retailer 2 is a rational decision-maker. Here, retailer 2’s decision process is the same with case III, that is, \( p_2^{IV} = p_2^{III} \), \( q_2^{IV} = q_2^{III} \), and \( z_2^{IV} = z_2^{III} \). In the view of retailer 1, retailer 2’s predictions on his decision variables (\( p_1^{IV} \) and \( z_1^{IV} \)) are
\[
p_1^{IV} = \frac{2(B_1-A_1)(m_1+k_1 w+d_1 p_1^{IV}+\mu_1)-(z_1^{IV}-B_1)^2}{4k_1(B_1-A_1)},
\]
\[
z_1^{IV} = \frac{B_1(p_1^{IV}-w)+A_1(w-s)}{p_1^{IV}-s}.
\]

When \( \alpha = 1 \), the equations (10) and (11) are equal to the equations (13) and (14), respectively. Meanwhile, retailer 2’s profit-maximizing function is a special case of equation (12). In a similar vein, let \( p_i = h(p_i) \) and substitute equations (13) and (14) into retailer 2’s anticipated profit function \( E[\pi_{12}^{IV}(z_1^*, p_1, p_2)] \), we can obtain
\[
E[\pi_{12}^{IV}(p_1^{IV}, p_1^{IV}, z_1^{IV})] = \int_{A_2}^{B_2} \left[p_1^{IV} (m_2 + d_2 p_1^{IV} - k_2 p_1^{IV} + u_2) + s(z_1^{IV} - u_2) \right] f(u_2)du_2
\]
\[+ \int_{z_1^{IV}}^{B_2} \left[p_1^{IV} (m_2 + d_2 p_1^{IV} - k_2 p_1^{IV} + z_1^{IV}) \right] f(u_2)du_2 - w[m_2 + d_2 p_1^{IV} - k_2 p_1^{IV} + z_1^{IV}].
\]

Solving the first-order condition of retailer 2’s expected function with respect to \( p_2 \) and \( z_2 \), respectively, we can obtain \( p_2^{IV} \) and \( z_2^{IV} \). Similarly, by setting \( \frac{\partial \pi_1}{\partial p_1} = 0 \), \( p_1^{IV} \) and \( q_1^{IV} \) is derived, thereby retailer 1’s maximum-profit response function \( q_1^{IV}(p_1^{IV}, z_1^{IV}) \) can be obtained. In the next step, retailer 1’s optimal pricing and ordering quantity are calculated as
\[
p_1^{IV*} = \frac{2(B_1-A_1)(m_1+k_1 w+d_1 p_1^{IV}+\mu_1)-(z_1^{IV}-B_1)^2}{4k_1(B_1-A_1)},
\]
\[
z_1^{IV*} = \frac{B_1(p_1^{IV}-w)+A_1(w-s)}{p_1^{IV}-s},
\]
\[
q_1^{IV*} = m_1 + d_1 p_1^{IV*} - k_1 p_1^{IV*} + z_1^{IV*},
\]
respectively. From the above results, retailer 2’s decisions keep the same in all four cases.

Previous literature suggests that the Stackelberg leader has the first-mover advantage, which is either on the assumption of information symmetry or rational decision-makers. However, in this study, our results show that retailer 1 has a late-mover advantage in case III. According to our analysis, when the leader is an irrational decision-maker and the follower has the information that the leader is irrational, the leader never has the first-mover advantage again, since the follower can make use of this information and benefit from it.
4.3.2 Overconfidence on other market variables

So far we assume that the retailer is only overconfident in his estimation of the demand variance. In reality, the retailer may be overconfident in the market scale and demand sensitivity. For example, the retailer may overvalue the potential demand scale he faces. This kind of overconfidence is considered as overestimation, which is different from overprecision (see the Introduction section). From this perspective, if retailer 2 overestimates the market scale, in his view, the demand function he faces is

\[ D_{22}(p_{21}, p_2, \varepsilon_2) = (1 + a)m_2 + d_2 p_{21} - k_2 p_2 + \varepsilon_2 \]

where \( a \) (0 ≤ \( a \) ≤ 1) indicates the overconfident level, and the larger \( a \) is, the higher the overconfident level is. Especially, when \( a = 0 \), retailer 2 has no overconfidence on the market scale.

The overconfident retailer may not only overestimate the market demand scale, but may also overrate the demand variance in the overprecision type. That is, he has two kinds of overconfidence – overestimation and overprecision. Under this circumstance, the market demand he faces can be expressed as

\[ D_{22}(p_{21}, p_2, \varepsilon_2) = (1 + a_1)m_2 + d_2 p_{21} - k_2 p_2 + a_2 \varepsilon_2. \]

where \( a_1 \) and \( a_2 \) denote the level of the overestimation and overprecision, respectively. In particular, a larger \( a_1 \) and a smaller \( a_2 \) indicate a higher overconfident level. Consequently, the mean of the market demand in this case would be higher than that considered in the previous cases (i.e., \( ED_{22}(p_{21}, p_2, \varepsilon_2) = ED_2(p_1, p_2, \varepsilon_2) + a_1 m_2 \geq ED_2(p_1, p_2, \varepsilon_2) \)), and the variance becomes smaller (i.e., \( Var[ED_{22}(p_{21}, p_2, \varepsilon_2)] = a_2 Var[ED_2(p_1, p_2, \varepsilon_2)] \)). Since retailer 2’s demand expectation will increase, the ordering quantity also increases, leading to an inventory problem. Meanwhile, as the two variables (\( a_1, a_2 \)) change simultaneously, both retailers’ decisions will be more complex.

Furthermore, the overconfident retailer overestimates his prediction on the price sensitivity. Therefore, in the view of retailer 2, the demand function he faces is

\[ D_{22}(p_{21}, p_2, \varepsilon_2) = m_2 + d_2 p_{21} - a k_2 p_2 + \varepsilon_2. \]

where \( a \) (0 ≤ \( a \) ≤ 1) indicates the overconfident level, and the smaller \( a \) is, the higher the overconfident level is. Especially, when \( a = 1 \), retailer 2 has no overconfidence on the price sensitivity. In the view of retailer 2, the demand increases (decreases) when he reduces (raises) the selling price.

The overconfident retailer’s optimal price is\( p_2^* = \frac{2(B_2 - A_2)(m_2 + ak_2 w + d_2 p_{21}^2 + \mu_2 - (\varepsilon_2^2 - B_2^2))}{4ak_2(B_2 - A_2)} \), where \( Z_2^* = \frac{B_2 (p_2^* - w) k_2 w (w - e)}{p_2^* - e} \).

Last but not least, the overconfident retailer may overestimate the demand scale and demand sensitivity, and predict the demand variance in overprecision preference at the same time. Therefore, in the view of retailer 2, the demand function he faces is

\[ D_{22}(p_1, p_2, \varepsilon_2) = (1 + a_2)m_2 + d_2 p_{21} - a_1 k_2 p_2 + a_3 \varepsilon_2. \]
where $a_1$, $a_2$ and $a_3$ indicate the overconfident level in terms of the demand scale, demand sensitivity and demand variance, respectively. The smaller $a_1$ and $a_2$ is and the larger $a_3$ is, the higher the overconfident level is. This overconfident model integrates all types of the retailer’s overconfidence over the demand completely. For retailer 2, the overconfidence may promote him to make decision with a higher price and lower order quantity, thereby leading to a smaller profit. Moreover, the cases of retailer 1’s knowledge on retailer 2’s overconfidence become more complex.

Since the profit functions contain a large number of unknown parameters, it is difficult to compare the optimal pricing, the optimal order quantity and maximal profit under different cases. Therefore, numerical methods are utilized to conduct further analysis.

5. Numerical analysis

In this section, our study releases the assumption made in Section 3 to examine the robustness of the obtained results and the impact of overconfident level on the supply chain performance. Here, detailed parameter settings are referred to the existing studies (see Choi, 1991; Lau and Lau, 2003).

To capture the effects of the overconfidence and avoid the complex impacts of cost parameters, we select the “actual” market demand function as

$$D_1(p_1, p_2, e_1) = 300 + 6p_2 - 14p_1 + e_1,$$
$$D_2(p_1, p_2, e_2) = 400 + 6p_1 - 14p_2 + e_2.$$

where $e_i \sim U(-30,30), i = 1,2$. Other parameters are set as $c = 7, w = 15, s = 2$, respectively.

Considering the overconfidence, the demand functions in the view of retailer 2 turn to be

$$D_{21}(p_{21}, p_2, e_{21}) = 300 + 6p_2 - 14p_{21} + a e_1, \quad D_{22}(p_{21}, p_2, e_2) = 400 + 6p_{21} - 14p_2 + a e_2.$$

Given all these settings, the optimal price, ordering quantity and the maximal profit are solved via Maple. When both retailers are rational, as defined in the benchmark model, the optimal prices are $p_1^N = 23.5060$ and $p_2^N = 26.5216$, the optimal ordering quantities are $q_1^N = 123.7766$ and $q_2^N = 167.9249$, the maximum profits are $\pi_1^N = 951.9160$ and $\pi_2^N = 1772.359$, and the manufacturer’s maximum profit is $\pi_M^N = 2333.6120$. In case II, $p_1'' = 23.5060$ and $q_1'' = 123.7766$.

5.1 The impact of overconfident level on optimal decisions

Fig.1 shows the relationships between the overconfident level and the optimal selling prices. With the increase of retailer 2’s overconfident level, retailer 1 raises his selling price in case I. However, since retailer 1 considers retailer 2 as a rational decision-maker, his pricing strategy is not influenced by retailer 2’s overconfidence in case II. In contrast, retailer 2 raises his selling price both in cases I and II. Results obtained from the numerical analysis can also be verified in Corollary 1. Moreover, it is noticed that retailer 1 always sets a lower selling price compared with retailer 2 (i.e., $p_1' < p_2'$) to attract more customers in case I. From Fig.
2, it is found that retailer 2’s optimal ordering quantity increases as $\alpha$ increases in both cases I and II. On the contrary, retailer 1’s optimal ordering quantity decreases with $\alpha$ in case I and remains the same in case II.

Figs. 1 and 2 summarize the impact of the overconfidence on the profits of the supply chain. According to the figures, retailer 1’s expected profit increases with the overconfident level. When it comes to retailer 2, as he mistakenly predicts his rival’s decision, his expected profit decreases compared with the benchmark model (i.e., $\pi_2^N > \pi_2^I, \pi_2^N > \pi_2^H$). Furthermore, a higher overconfident level results in a higher selling price and a lower ordering quantity for the overconfident retailer. Retailer 2’s anticipated profit is greater than the actual expected profit, which is also verified in Corollary 2.

5.2 The value of information on overconfidence

For retailer 1, it is evident that his pricing and ordering strategies are different when he has or doesn’t have the knowledge of retailer 2’s overconfidence. In order to reflect the value of this knowledge, we seek to analyze both retailers’ profits in cases I and II, as shown in Tab. 2 and Fig. 5.

<table>
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<th>$\alpha$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{1I}$</td>
<td>972.6176</td>
<td>970.6021</td>
<td>968.5783</td>
<td>966.5360</td>
<td>964.4856</td>
<td>962.4218</td>
<td>960.3430</td>
<td>958.2599</td>
<td>956.1566</td>
<td>954.0454</td>
<td>951.9160</td>
</tr>
<tr>
<td>$\pi_{1II}$</td>
<td>972.5091</td>
<td>970.5136</td>
<td>968.5078</td>
<td>966.4817</td>
<td>964.4454</td>
<td>962.3937</td>
<td>960.3319</td>
<td>958.2496</td>
<td>956.1521</td>
<td>954.0442</td>
<td>951.9160</td>
</tr>
</tbody>
</table>

Fig. 1. The effect of overconfidence on optimal prices

Fig. 2. The effect of overconfidence on optimal ordering quantities

Fig. 3. The effect of overconfidence on profits in case I

Fig. 4. The effect of overconfidence on profits in case II
Fig. 5. The effect of overconfidence on retailer 2’s profit

Tab. 2 shows that compared with case II, retailer 1 can obtain more profit in case I (i.e., $\pi_1^I > \pi_1^II$). The reason is that retailer 1 has the private information about retailer 2’s overconfidence in case I, thus he can make better decisions compared with case II. Compared case I with case II, we have $\pi_1^I > \pi_1^{II} > \pi_1^N$. The reason may be that retailer 2’s selling price is too high, which increases retailer 1’s product demand, thereby improving retailer 1’s profit. Last but not least, as is shown in Fig. 5, retailer 2’s expected profit decreases as the overconfident level increases.

It is important to note that $\pi_1^I > \pi_2^{II}$. It means that although the two retailers are competitors, they can share some market information, which can make them recognize more information on market environment and then make more rational decisions and does not necessarily result in profit loss.

In both cases I and II, retailer 2 makes a higher selling price and a lower ordering quantity because of his overconfidence. Besides, from Fig. 6, we know that $\pi_M > \pi_M^N > \pi_M^{II}$ apparently, therefore, the manufacturer can benefit from retailer 1’s knowledge of retailer 2’s overconfident level and lose considerable profit when retailer 1 does not have the knowledge. Specifically, when retailer 1 is not aware of retailer 2’s overconfident level, the manufacturer’s expected profit decreases sharply with the overconfident level.

Fig. 6. The effect of the overconfidence on the manufacturer’s profit

5.3 The impact of overconfidence on the supply chain

In the area of finance, overconfident decision-makers tend to overestimate their understanding of the market environment and underestimate the uncertainty of the market, which leads to some consequences such as unreasonable mergers and excessive investment. In a similar vein, the overconfident retailer often overestimates the market demand, and determines a higher selling price and a lower sales volume as a
consequence. The overconfidence can increase retailer 2’s price, however, whether it reduces the profit of the whole supply chain needs to be further studied.

We calculate the profit of the entire supply chain in cases I and II, and make comparisons with the benchmark model. From Fig. 7, we find that $\pi^T > \pi^H > \pi^N$ when $\alpha < 1$. Meanwhile, the lower retailer 2’s overconfident level is, the smaller the profit deviates from $\pi^N$. Obviously, the overconfidence is not necessarily harmful to the whole supply chain. A possible theoretical interpretation is as follows: the decisions in the benchmark model (decentralized scenario) deviate from those in a centralized setting, thus bring about lower profit than that in the centralized scenario, while the overconfidence may make both retailers’ pricing and ordering decisions closer to the optimal decisions in the centralized scenario. Our conclusion is different from the conclusion in Croson et al. (2011), they consider a two-echelon supply chain composed of one rational manufacturer and one overconfident retailer. Besides, the demand is independent of the selling price. They conclude that the higher the overconfident level is, the more the loss of the whole supply chain will suffer. However, in our paper, we study a two-echelon supply chain composed of one rational manufacturer, one rational retailer and one overconfident retailer in a duopolistic market. Besides, the demand is price-dependent.

![Fig. 7. The effect of overconfidence on profit of the whole supply chain](image)

### 6. Conclusions

In this paper, we examine a duopolistic supply chain with two retailers compete in selling the same product, one retailer is rational decision-maker, and the other retailer is overconfident. In such context, we aim to investigate the impact of overconfidence on supply chain members’ optimal decisions and their profits under uncertain demand, particularly in two cases: (1) the rational retailer realizes the other retailer’s overconfidence; (2) the rational retailer is not aware of the other retailer’s overconfidence. After investigating the effect of overconfidence and its information value by using modeling and numerical analysis, we put forth three main findings: 1) **Findings on pricing policies.** In both cases, the overconfident retailer’s optimal prices are higher than that in the benchmark model, and increase with the overconfident level. In case I, for the rational retailer, similar to the overconfident retailer’s pricing decision, he raises his price, but the amount of the price increasing is smaller than that of the overconfident retailer. In contrast, in case II, his pricing
decision is the same with that in the benchmark model since he is not aware of his rival’s overconfidence. 2) **Findings on expected profits.** Compared with the benchmark model, the overconfident retailer’s expected profit decreases in both cases, the profit loss in case II is higher than that in case I. However, the rational retailer’s expected profits increase with the overconfident level. Moreover, for the same overconfident level, his expected profit in case I is higher than that in case II. This is because the rational retailer has the information about his rival’s overconfident level, which not only leads to an increase in his expected profit but also an increase in his rival’s expected profit (compared with case II). Besides, the manufacturer’s expected profit increases in case I while decreases in case II. This result implies that sharing overconfident information is beneficial to each member in the supply chain. 3) **Findings on supply chain performance.** The retailer’s overconfident behavior is not necessarily detrimental to the supply chain profit (compared to the benchmark model). Previous research finds that the overconfident behavior has a negative effect on the supply chain (Croson et al., 2011). In this paper, we complement their results by analyzing a duopolistic market, and find that the profit of the supply chain increases in cases I and II (compared to the benchmark model).

Our analysis can be extended in several directions. First, we assume that the demand is uniformly distributed, in reality the demand follows a variety of distributions. Thus our model can be extended to cover other demand distribution cases, such as normal distribution. Besides, our paper studies the impact of overconfidence in the supply chain members’ optimal decisions and profits. In fact, in a duopolistic supply chain with overconfidence, how the manufacturer designs incentive to motivate the retailers and coordinate the supply chain is also an interesting topic.
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Appendix

Proof of Theorem 1. Let’s solve retailer 2’s optimal pricing first. Define \( z_2^* = q_2^* - [m_2 + d_2p_{21}^* - k_2p_{21}^* + (1 - \alpha)\mu_2] \), and then retailer 2’s anticipated profit can be rephrased as

\[
E[\pi_{22}(p_{21}^*, p_2^*, z_2^*)] = \int_{A_2}^{z_2^*} \left( p_2^*[m_2 + d_2p_{21}^* - k_2p_{21}^* + (1 - \alpha)\mu_2 + au_2] + s(z_2^* - au_2) \right) f(u_2)du_2 + \\
\int_{z_2^*}^{B_2}(p_2^*[m_2 + d_2p_{21}^* - k_2p_{21}^* + (1 - \alpha)\mu_2 + z_2^*]) f(u_2)du_2 - w[m_2 + d_2p_{21}^* - k_2p_{21}^* + (1 - \alpha)\mu_2 + z_2^*].
\]

Solving the first-order condition of retailer 2’s profit function with respect to \( p_2^* \) and \( z_2^* \), we can obtain that

\[
p_2^* = \frac{2a(B_2 - A_2)(m_2 + k_2w + d_2p_{21}^* + \mu_2) - (z_2^* - aB_2)^2}{4ak_2(B_2 - A_2)},
\]

(11)

\[
z_2^* = \frac{aB_2(p_2^* - w) + aA_2(w - s)}{p_2^* - s}.
\]

(12)

With the same logic above, we analyze retailer 1’s optimal pricing in the view of retailer 2. Solving the first-order condition of retailer 1’s profit function with respect to \( p_{21}^* \) and \( z_{21}^* \), respectively, we can obtain

\[
p_{21}^* = \frac{2a(B_1 - A_1)(m_1 + k_1w + d_1p_{12}^* + \mu_1) - (z_{21}^* - aB_1)^2}{4ak_1(B_1 - A_1)},
\]

(13)

\[
z_{21}^* = \frac{aB_1(p_{21}^* - w) + aA_1(w - s)}{p_{21}^* - s}.
\]

(14)

Jointly solving the equations (10)-(13), optimal values of \( p_{21}^*, z_{21}^*, p_{12}^* \), and \( z_{21}^* \) could be derived. Since the complexity of the optimal solutions, we only reserve the succinct form shown in Theorem 1. Q.E.D

Proof of Theorem 2. Similar to Theorem 1, we define \( z_1^* = q_1^* - (m_1 + d_1p_{12}^* - k_1p_{12}^*) \). Therefore, retailer 1’s expected profit can be rewritten as

\[
E[\pi_{11}(p_1^*, p_{12}^*, z_1^*)] = \int_{A_1}^{z_1^*} \left( p_1^*[m_1 + d_1p_{12}^* - k_1p_{12}^* + \mu_1] + s(z_1^* - u_1) \right) f(u_1)du_1 + \\
\int_{z_1^*}^{B_1}(p_1^*[m_1 + d_1p_{12}^* - k_1p_{12}^* + z_1^*]) f(u_1)du_1 - w[m_1 + d_1p_{12}^* - k_1p_{12}^* + z_1^*].
\]

Solving the first-order condition of retailer 1’s profit function with respect to \( p_1^* \) and \( z_1^* \), respectively, we can obtain

\[
p_{12}^* = \frac{2(B_1 - A_1)(m_1 + k_1w + d_1p_{12}^* + \mu_1) - (z_{12}^* - B_1)^2}{4k_1(B_1 - A_1)},
\]

(15)

\[
z_{12}^* = \frac{B_1(p_{12}^* - w) + A_1(w - s)}{p_{12}^* - s}.
\]

(16)

where \( p_{12}^* \) is retailer 2’s optimal price predicted by retailer 1. Q.E.D

Proof of Theorem 3. Proofing of the Theorem 3 is similar to the Theorem 1 and 2 in case I. We define \( z_{11}^* = q_{11}^* - (m_1 + d_1p_{11}^* - k_1p_{11}^*) \), then retailer 1’s anticipated profit function is

\[
E[\pi_{11}(p_1^*, p_{12}^*, z_{11}^*)] = \int_{A_1}^{z_{11}^*} \left( p_1^*[m_1 + d_1p_{12}^* - k_1p_{12}^* + \mu_1] + s(z_{11}^* - u_1) \right) f(u_1)du_1 + \\
\int_{z_{11}^*}^{B_1}(p_1^*[m_1 + d_1p_{12}^* - k_1p_{12}^* + z_{11}^*]) f(u_1)du_1 - w[m_1 + d_1p_{12}^* - k_1p_{12}^* + z_{11}^*].
\]
Solving the first-order condition of retailer 1’s profit function with respect to \( p_{11}' \) and \( z_{11}' \), respectively, we can obtain
\[
p_{11}' = \frac{2(B_1-A_1) \left( m_1+k_1w+d_1p_{11}''+\mu_1 \right) - (B_1-z_{11}'')^2}{4k_1(B_1-A_1)}, \quad z_{11}' = \frac{B_1(p_{11}'-w)+A_1(w-s)}{p_{11}'-s},
\]
At this moment, retailer 2’s anticipated profit function in the view of retailer 1 is
\[
E[\pi_{12}^{n}(p_{11}', p_{12}', z_{12}')] = \int_{A_2}^{B_2} \left( p_{11}' [m_2 + d_2 p_{11}' - k_2 p_{12}' + \mu_2] + s (z_{12}' - u_2) \right) f(u_2) du_2 + \int_{A_2}^{B_2} \left( p_{12}' [m_2 + d_2 p_{12}' - k_2 p_{12}' + z_{12}'] \right) f(u_2) du_2 - w [m_2 + d_2 p_{11}' - k_2 p_{12}' + z_{12}'].
\]
Similarly, by taking the first-order condition of retailer 2’s expected function with respect to \( p_2 \) and \( z_2 \), respectively,
\[
p_{12}' = \frac{2(B_2-A_2) \left( m_2+k_2w+d_2p_{12}''+\mu_2 \right) - (B_2-z_{12}'')^2}{4k_2(B_2-A_2)}, \quad z_{12}' = \frac{B_2(p_{12}'-w)+A_2(w-s)}{p_{12}'-s}, \quad \text{Q.E.D}
\]
Proof of Corollary 1. According to the result obtained from Theorem 1, \( z_2 = \frac{aB_2(p_2-w)+aA_2(w-s)}{p_2-s} \), that is \( aB_2 \geq z_2 \). Notice that \( p_2^* = \frac{2a(B_2-A_2)(m_2+k_2w+d_2p_{12}'+\mu_2)-(3aB_2)^2}{4ak_3(B_2-A_2)^2} \), thus we have \( \frac{\partial p_2^*}{\partial a} = \frac{2k_2(B_2-A_2)(aB_2-z_2)(-3aB_2+z_2)}{4ak^3_3(B_2-A_2)^2} < 0 \). Therefore, \( p_2^* \) decreases with respect to \( a \), i.e., the higher the overconfident level is, the higher price retailer 2 charges, and vice versa. Q.E.D

Proof of Corollary 2. In case I, \( p_{11}' = \frac{m_1+k_1w+d_1p_{11}''+\mu_1}{2k_1} - \frac{(B_1-A_1)(w-s)^2}{4k_1(p_{11}'-s)^2} \). After simplification, we obtain
\[
p_{11}' - s = \frac{2k_1}{d_1} \left( p_{11}' - s \right) + \frac{(B_1-A_1)(w-s)^2}{2d_1(p_{11}'-s)\sqrt{d_1}} + \frac{2k_1}{d_1} s - s - \frac{m_1+k_1w+\mu_1}{d_1}. \]

Let \( y = p_{11}' - s \) and \( x = p_{11}' - s \), it can be rewritten as \( y = \frac{2k_1}{d_1} x + \frac{(B_1-A_1)(w-s)^2}{2d_1} + \frac{2k_1}{d_1} s - s - \frac{m_1+k_1w+\mu_1}{d_1} \).

Taking the first-order and second-order derivative of \( y \) with respect to \( x \), respectively, we can obtain
\[
\frac{\partial y}{\partial x} = \frac{2k_1}{d_1} \frac{(B_1-A_1)(w-s)^2}{d_1x},
\]
\[
\frac{\partial^2 y}{\partial x^2} = \frac{3(B_1-A_1)(w-s)^2}{d_1x^2}.
\]
Since \( \frac{\partial^2 y}{\partial x^2} > 0 \), then \( y \) is a strictly convex function with respect to \( x \). Thus we can obtain that \( p_{11}'_{-s} \) is strictly convex with respect to \( p_{11}' \). Q.E.D

Proof of Corollary 3. Since retailer 2’s decisions in case I are same with that in case II, we only analyze the relationship between \( E(\pi_{12}) \) and \( E(\pi_2) \). In the view of retailer 2, his anticipated profit is
\[
E[\pi_{12}^{n}(p_{12}', p_{12}', z_{12}')] = \int_{A_2}^{B_2} \left( p_{12}' [m_2 + d_2 p_{12}'' - k_2 p_{12}' + (1-a)\mu_2 + au_2] + s (z_{12}' - u_2) \right) f(u_2) du_2 + \int_{A_2}^{B_2} \left( p_{12}' [m_2 + d_2 p_{12}'' - k_2 p_{12}' + (1-a)\mu_2 + z_{12}'] \right) f(u_2) du_2 - w [m_2 + d_2 p_{12}'' - k_2 p_{12}' + (1-a)\mu_2 + z_{12}']
\]
\[
= (p_2'-w) (m_2 + d_2 p_{12}'' - k_2 p_{12}' + (1-a)\mu_2) - wz_{12}' + \]
\[ \frac{1}{B_2 - A_2} \int_{A_2}^{B_2} a p_2 u_2 du_2 + \frac{1}{B_2 - A_2} \int_{A_2}^{B_2} s(z_2^2 - au_2) du_2 + \frac{1}{B_2 - A_2} \int_{A_2}^{B_2} a p_2 z_2^2 du_2 \\
= \left( p_2^I - w \right) \left( m_2 - k_2 p_2^I + d_2 p_2^I + (1 - a) \mu_2 \right) - wz_2^I - \frac{(p_2^I - s)(z_2^2 + a^2 A_2^2 - 2a z_2^I (B_2 p_2^I - A_2 s))}{2a(B_2 - A_2)}. \]

Similarly, retailer 2’s actual expected profit is
\[
E[\pi_2^I(p_1^I, p_2^I, z_2^I)] = (p_2^I - w) \left( m_2 - k_2 p_2^I + d_2 p_1^I \right) - wz_2^I - \frac{(p_2^I - s)(z_2^2 + A_2^2 - 2a z_2^I (B_2 p_2^I - A_2 s))}{2a(B_2 - A_2)}. \]

Therefore, the difference of expected profit is
\[
\Delta = E[\pi_2^{II}(p_2^I, p_2^I, z_2^I)] - E[\pi_2^{II}(p_1^I, p_2^I, z_2^I)] \\
= (p_2^I - w) d_2 (p_2^I - p_1^I) + (p_2^I - w)(1 - a) \mu_2 - \frac{(p_2^I - s)(z_2^2 + a^2 A_2^2 - a(p_2^I - s)(z_2^2 + A_2^2))}{2a(B_2 - A_2)} \\
> \frac{A_2 + B_2}{2} (1 - a)(p_2^I - w) - \frac{(a - a^2)(p_2^I - s)(z_2^2 - aA_2^2)}{2(B_2 - A_2)} = \frac{1 - a}{a} \left[ \frac{a(A_2 + B_2)((p_2^I - w))}{2(B_2 - A_2)} \right] \\
= \frac{1 - a}{a} \left[ \left( p_2^I - s \right) \left( aA_2^2 - a^2 A_2^2 + aB_2 - A_2 \right) \right] \\
= \frac{1 - a}{a} \left[ \left( p_2^I - s \right) \left( -aA_2^2 + aB_2 - A_2 \right) \right] > 0. \]

Solving the first-order derivative of the difference with respect to \(a\), we can obtain
\[
\frac{d\Delta \pi}{da} = \frac{(B_2 + A_2)(p_2^I - w)}{2} - \frac{(1 - a)A_2^2}{B_2 - A_2} (p_2^I - s) - \frac{(1 - 2a)A_2^2}{2} (p_2^I - s) - \frac{(1 - a)(B_2 - A_2)(p_2^I - w)^2}{2(p_2^I - s)} \\
= \frac{(B_2 + A_2)(p_2^I - w)}{2} - \frac{(1 - a)A_2^2}{B_2 - A_2} (p_2^I - s) - \frac{(1 - a)(B_2 - A_2)(p_2^I - w)^2}{2(p_2^I - s)} + aA_2(p_2^I - w) + \\
\frac{a(B_2 - A_2)(p_2^I - w)^2}{2(p_2^I - s)} \\
\frac{(B_2 + A_2)(p_2^I - w)}{2} - \frac{(1 - a)A_2^2}{B_2 - A_2} (p_2^I - s) - \frac{(1 - a)(B_2 - A_2)(p_2^I - w)^2}{2(p_2^I - s)} + aA_2(p_2^I - w) + \\
\frac{a(B_2 - A_2)(p_2^I - w)^2}{2(p_2^I - s)} \\
= \frac{(1 - a)(B_2 + A_2)(p_2^I - w)}{2} - \frac{(1 - a)A_2^2}{B_2 - A_2} (p_2^I - s) - \frac{(1 - a)(B_2 - A_2)(p_2^I - w)^2}{2(p_2^I - s)} < 0. \]

Therefore, the difference of expected profit is a decreasing function with respect to \(a\). That is, the profit loss increases with the overconfident level. Q.E.D

**Proof of Corollary 4.** Since retailer 1 realizes retailer 2’s overconfident level in case I, it can be obviously derived that \(E(\pi_1^I) > E(\pi_1^{II})\).

We simplify retailer 1’s profit in case II, as shown in the following equation.
Similarly, in the benchmark model, retailer 1’s expected profit is

$$E(\pi_1^L) = (p_1^L - w)(m_1 + d_1p_2^L - k_1p_1^L) + \frac{(p_1^L - s)(x_1^L - A_1^L) + 2z_1(B_1p_1^L - A_1s) - 2z_1^2(p_1^L - s)}{2(B_1 - A_1)} - wz_1^L.$$ 

where

and

Note that $p_1^L = p_1^N$ and $p_2^L > p_2^N$, we can easily obtain $E(\pi_1^L) > E(\pi_1^N)$. Therefore, we have $E(\pi_1^L) > E(\pi_1^N) > E(\pi_1^N)$. Q.E.D.
References


