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On the Efficiency of Local and Global Communication in Modular Robots

Ricardo Franco Mendoza Garcia, Ulrik Pagh Schultz and Kasper Stoy

Abstract—As exchange of information is essential to modular robots, deciding between local or global communication is a common design choice. This choice, however, still lacks theoretical support.

In this paper we analyse the efficiency of local and global communication in modular robots. To this end, we use parameters to describe the topology of modular robots, develop a probabilistic model of local communication using these parameters and, using a model of global communication from literature, compare the transmission times of local and global communication in different robots.

Based on our results, we conclude that global communication is convenient for centralized control approaches and local communication is convenient for distributed control approaches. In addition, we conclude that global is in general convenient for low-connectivity configurations, such as chains, trees or limbs, and that local can be faster than global when communicating between distant modules which are not too far apart. Finally, we discuss the potential of flexible communication topologies, which can provide optimal topologies for many configurations, such as those we can split into bodies and limbs.

I. INTRODUCTION

Modular robots are robots built from many similar modules. Although single modules have limited uses, they are able to combine into more functional structures. The advantages of modular robots over robots made from a few special-purpose parts are as follows. First, their ability of assembling task-suitable structures makes modular robots more flexible. Second, their redundancy of modules makes them more robust. Third, their similarity between modules makes production of modular robots potentially cheaper [1].

We classify modular robots according to their topology or, in other words, according to the types of structures they form. While chain-type robots attach their modules in string or tree topology (see Fig. 2b), lattice-type robots arrange their modules in a 3D-virtual grid at discrete positions in space (see Fig. 2h). Mechanical attachment/detachment is achieved via connecting interfaces which, in general, also work as communication media.

As communication is essential to modular robots, deciding between local or global communication is a common design choice. While local communication allows only for hop-by-hop information diffusion (Fig. 1a), global communication allows for direct communication between all modules in the robot (Fig. 1b). Thus, for example, local communication is convenient to figure out the topology of the robot, and global communication is believed to be convenient for time-critical coordination between distant modules of the system [2]. However, is it true that global is always faster than local communication when coordinating distant modules? To the best of our knowledge, no theoretical support exists to answer this question for modular robotic systems.

In this paper we analyse the efficiency of local and global communication in modular robots. Our work is strongly motivated by a previous analysis addressing the same issues in distributed mobile robots (Yoshida et al. [3]), from which we could not extrapolate conclusions due to important differences between modular and mobile robots, such as:

- modules do not move in relation to their neighbours1,
- modules are not freely distributed in space but follow specific patterns (i.e., chains or lattices), and
- modular robots exhibit high spatial density of modules.

In spite of those differences, the previous analysis provided tools to build our models and to perform our comparisons.

We begin our analysis with a parametric description of the topology of modular robots and proceed to the creation of a probabilistic model of local communication based on this description. We can use this model to understand a wide range of scenarios in the robots under consideration, such as configurations with large numbers of modules, but also to analyse robots whose simulation models are not available. Our probabilistic model is then validated with simulations of three modular robots: CKBot [4], ATRON [5] and Odin [2], [6], whose selection was based on the availability of their simulation models.

1 This analysis ignores self-reconfiguration, a popular feature among modular robots. However, we still consider this analysis acceptable in systems undergoing morphological changes, because the rate of self-reconfiguration is typically one order of magnitude slower than communication.
Afterward, we compare the efficiency of local and global communication by using our local model and the global model proposed in [3] for distributed mobile robots, which is also suitable for modular robots. Our comparison is mainly based on the information transmission time from many-to-all modules in the robot, but it also considers other scenarios, such as one-to-many and many-to-many. In addition, the comparison covers four communication topologies in different configurations and at different workloads, and the topologies represent, in turn, six different modular robots: PolyBot [1], CKBot, M-TRAN [7], SuperBot [8], ATRON and Odin. In other words, the selected topologies are representative of a wide range of modular robots, which expand from low to high-connectivity systems.

Based on our results, we conclude that global communication is convenient for centralized control approaches and local communication is convenient for distributed control approaches. In addition, we conclude that global is in general convenient for low-connectivity configurations, such as chains, trees or limbs, and that local can be faster than global when communicating between distant modules which are not too far apart. Although our conclusions are deduced in the context of modular robots, they correlate to a large extent with those of Yoshida et al. for mobile distributed robots [3]. In that sense, we check the applicability of those conclusions to modular robots in order to provide the missing theory for design decisions. Finally, we discuss the potential of flexible communication topologies, which can outperform pure local or global approaches by providing optimal communication topologies to many configurations, such as those we can split into bodies and limbs.

II. RELATED WORK

PolyBot [1] is a chain-type robot with modules resembling cubes. Each module has two connecting interfaces, and the communication between modules is global. CKBot [4] is an evolution of PolyBot, and it preserves the topology and communication mode of its ancestor (Fig. 2a and 2b).

M-TRAN [7] is a hybrid between chain and lattice-type robot because its modules can form chains and lattices (Fig. 2c and 2d). Each module is made of two semi-cylindrical parts and has six connecting interfaces (three on each half). Communication between modules is local and global, and it runs over two separated physical media. SuperBot [8] is another hybrid between chain and lattice-type robot. Its topology is similar to that of M-TRAN, but communication between modules is only local.

ATRON [5] is a lattice-type robot with modules resembling spheres (Fig. 2e and 2f). Each module is divided into two halves and has eight connecting interfaces (four on each half). Communication between modules is local.

Odin [2], [6] is a lattice-type robot with cylindrical modules that connect to joints at the ends (Fig. 2g and 2h). Although each module has only two communication interfaces, joints act as hubs allowing for direct communication to a maximum of 12 neighbors. As Odin is implemented with a flexible network topology, inter-module communication can be local, global or anything in between [2].

III. TOPOLOGY DESCRIPTION

As we wanted to apply our analysis to many different modular robots, we began our work with a generic description of their topologies. That said, we considered modules and connecting interfaces as two separated entities, what allowed us to define a minimal set of parameters for topology description, as follows:

\[ n_i \], number of interfaces attached to one module,
\[ n_m \], maximum number of modules connected to one interface,
\[ n_{av} \], average number of modules connected to one interface, and
\[ p_{nb} \], probability of any module having neighbours, \( (n_{av} - 1)/n_m - 1 \).

\(^2\)Although we present this parameter in simple words, a more complete description would be: probability for all \( n_i \) interfaces connected to one module to have any of the remaining \( n_i(n_m - 1) \) slots also connected to other modules, \( \frac{n_{av} - 1}{n_m - 1} \)
IV. LOCAL COMMUNICATION MODEL

A. Assumptions and Considerations

We based our local communication model on the topology description presented in Sec. III. In addition, we modeled local communication as a diffusion process where, initially, only one module spread information through its interfaces but, afterward, more modules repeated and spread that information again. Fig. 4 shows this model. We further defined:

$I_{mod}$, an informed module in the structure and
$N_{mod}$, a non-informed module in the structure.

Also, as we wanted to explore the influence of different communication workloads, hardware capabilities, and numbers of modules, we introduced the following parameters:

- $p_{tx}$, information transmission probability of modules (or how likely is for a module to send a message at $\Delta t$),
- $r_c$, simultaneous reception capacity of modules (or how many messages a module is able to receive at $\Delta t$, before reception problems appear),
- $n_t$, total number of modules in the structure,
- $n_{rx}$, number of $I_{mod}s$ in the structure, and
- $r(t)$, ratio of $I_{mod}s$ at time $t$, $(n_{rx}/n_t)$.

$p_{tx}$ enabled us to account for communication workload as follows. At every time step $\Delta t$, each module can (or not) transmit a message which, in turn, can (or not) be relevant to our diffusion process. If a message is sent from an $I_{mod}$, our diffusion process progresses as shown in Fig. 4, but if a message is sent from an $N_{mod}$, the message is related to another process in the robot. A message originating from an $N_{mod}$, however, still counts as a received message to neighbour modules, which shows the importance of $r_c$.

In addition, we assumed that $\Delta t$ was long enough to process any information available to one module and that modules failed to successfully receive any information if more than one message arrived at $\Delta t$ (i.e., $r = 1$). Considering that the size of modular robots does not allow for any microcontrollers and that, therefore, communication ports are typically multiplexed to listen on many interfaces, the assumption of $r = 1$ is rather realistic. In spite of that, a better model could have considered the successful reception of at least one of the multiple arriving messages.

B. Analysis of Local Transmission Time, $T_{loc}$

To estimate the information transmission time from many to all the modules in the robot, we centered our attention on the $N_{mod}s$ of the diffusion process. We wanted to know how long it takes for all the modules in the robot to become $I_{mod}s$ (or the time required by $r(t)$ to become 1). If we could calculate the probability of an $N_{mod}$ to become $I_{mod}$, we could then formulate the growth rate of $I_{mod}s$ based on the number of remaining $N_{mod}s$ in the robot. To this end, we proceeded in four steps:

1) Boundary placement probability, $p_{bc}$: As only $N_{mod}s$ located around the growing blob of $I_{mod}s$ (see Fig. 4) were candidates to become $I_{mod}$, we needed the probability, $p_{bc}$, of any $N_{mod}$ to be in the boundary. This probability was equivalent to the ratio between the $N_{mod}s$ in the boundary (dependent on the size and shape of the blob) and the total number of $N_{mod}s$ in the robot at time $t$.

We tried to approximate $p_{bc}$ in two ways, one inversely proportional to the ratio of $N_{mod}$ at time $t$, $1 - r(t)$, and another proportional to the ratio of $I_{mod}$ at time $t$, $r(t)$.

The first way, which was expected to resemble better the real $p_{bc}$, led us to a complex solution (Lambert W-Function). Unfortunately, this solution required numerical methods to be solved, and the results diverged considerably from the simulation results in Sec. V. The second way, on the other hand, led us to a real solution and better results. That said, we approximated $p_{bc}$ as:

$$p_{bc}[r(t)] = ar(t) + b,$$

with $r(t) \in [r(0), 1]$. In addition, as we assumed that information diffusion was always initiated at a single module, $r(0) = 1/n_t$. To continue, we determined $a$ and $b$ as:
\[ a = \left( \frac{d - 1}{1 - r(0)} \right) p_{be0} \quad \text{and} \quad (2) \]
\[ b = \left( \frac{1 - r(0)}{1 - r(0)} \right) p_{be0}, \quad (3) \]

so that \( p_{be}[r(0)] = p_{be0} \) and \( p_{be}[1] = p_{be0} d \). This algebraic manipulation permitted us to control the final value \( p_{be}[1] \) with only one variable, \( d \), instead of \( a \) and \( b \). \( p_{be0} \) was then determined as:
\[ p_{be0} = \frac{n_i (n_{av} - 1)}{n_t}, \quad (4) \]

based on the topology description of Sec. III. Notice that (1) in combination with (4) captured topology and time dependencies of \( p_{be} \). Furthermore, as we assumed the beginning of propagation at one module, (4) was a good approximation for the real value of \( p_{be} \) at \( t = 0 \). At that time, the amount of \( \text{N}_m \) at the boundary were \( n_i(n_{av} - 1) \) in average, and the total amount of \( \text{N}_mod \) were \( n_t - 1 \). Notice also that \( p_{be0} \) and \( p_{be0} d \) were always kept below 1.

2) Information reception probability, \( p_{rx} \): To continue, we focused on the conditions for a particular \( \text{N}_mod \) to successfully receive information during a time step \( \Delta t \). To this end, we calculated the probability of this \( \text{N}_mod \) to be at the propagation boundary and to have \( x \) neighbours, as:
\[ p_1[r(t), x] = n_i(n_{m-1})C_x pnb^x (1-pnb)^{n_i(n_{m-1})-x} p_{be0}[r(t)], \quad (5) \]

with \( x = 0 \ldots n_i(n_{m-1}) \). Then, we calculated the probability that, in addition, \( y \) out of those \( x \) neighbours were transmitting information, as:
\[ p_2[r(t), x, y] = xC_y ptx^y (1-px)^{x-y} p_1[r(t), x], \quad (6) \]

with \( y = 0 \ldots x \), and that at least one out of those \( y \) neighbours transmitting information was an \( I_{mod} \), as:
\[ p_3[r(t), x, y] = (1 - 0.5^y) p_2[r(t), x, y]. \quad (7) \]

Here we exploited the fact that, for an \( \text{N}_mod \) at the boundary, neighbours were approximately 50% \( I_{mod} \)s and 50% \( \text{N}_mod \)s. Finally, we calculated the probability that our \( \text{N}_mod \) was not transmitting information either, as:
\[ p_4[r(t), x, y] = (1 - ptx) p_3[r(t), x, y]. \quad (8) \]

After we considered, in general, all possible scenarios around an \( \text{N}_mod \), we focused on the proper conditions for information reception to occur, which we summarized with the probability:
\[ p_{rx}[r(t)] = \sum_{x=1}^{r_x} \sum_{y=1}^{x} p_4[r(t), x, y] + \sum_{x=r_x+1}^{n_i(n_{m-1})} \sum_{y=1}^{r_x} p_4[r(t), x, y], \quad (9) \]

and, as we assumed \( r_c = 1 \), (9) was arranged in the binomial theorem form and further simplified to:
\[ p_{rx}[r(t)] = \sum_{x=1}^{n_i(n_{m-1})} p_4[r(t), x, y = 1] = p_{be0}[r(t)] c, \quad (10) \]

with
\[ c = 0.5p_{nb}n_i(n_{m-1})(1-px)(1-p_{nb}ptx)^{n_i(n_{m-1})-1}. \]

3) Growth rate of \( r(t) \): By having \( p_{rx}[r(t)] \) for one \( \text{N}_mod \), we could then formulate the rate \( \Delta r(t)/\Delta t \) of newly generated \( I_{mod} \)s, as:
\[ \frac{\Delta r(t)}{\Delta t} = p_{rx}[r(t)](1 - r(t)), \quad (11) \]

with \( 1 - r(t) \) representing the ratio of \( \text{N}_mod \)s at time \( t \). To continue, as we realized from (1) and (10) that (11) was a differential equation of separable variables\(^3\), we further obtained the solution:
\[ r(t) = \frac{e^{(px0 c d t)} + r(0) d - 1}{e^{(px0 c d t)} + d - 1}. \quad (12) \]

4) Information transmission time, \( T_{loc} \): Finally, as we already had an expression for \( r(t) \), we could easily find an expression for the local transmission time, \( T_{loc} \), as the inverse function:
\[ T_{loc}(n_{rx}) = (p_{be0} c d)^{-1} \ln \left[ \frac{d(r(t) - r(0))}{1 - r(t)} + 1 \right], \quad (13) \]
\[ = (p_{be0} c d)^{-1} \ln \left[ \frac{d(n_{rx} - 1)}{n_t - n_{rx}} + 1 \right], \quad (14) \]

with \( r(t) \in [r(0), 1] \) or, the same, \( n_{rx} = 1 \ldots (n_t - 1) \).

C. Observations

As many processes like Fig. 4 could initialize simultaneously at different modules in the robot, this analysis also modeled transmission from many to all modules; without further modifications \cite{9}. The messages we previously considered not relevant to our diffusion, such as those coming from \( \text{N}_mod \)s, could then correspond to similar diffusion being carried out in parallel. Finally, although (14) prohibited calculations for \( n_{rx} = n_t \), that is, for information reaching all the modules in the robot, we could still evaluate transmission performance with \( n_{rx} = n_t - 1 \).

V. LOCAL MODEL VALIDATION

A. Setup

1) Simulation: We implemented the information diffusion process of Fig. 4 in the simulation models of three robots: CKBot, ATRON and Odin, and we also tried different configurations, such as: a chain of CKBot, and planes/cubes of ATRON/ODin. Even though chain configurations were also possible for lattice-type robots (see Fig. 2f), they would be topologically equivalent to a chain of CKBot. To continue, we utilized two simulators at different simulation stages: at

\(^3\text{In the form, } r(t)' = (a r(t) + b) c (1 - r(t)).\)
first, we generated the topological description of our robots with USSR [10] and, then, used this description to run information diffusion with OMNeT++ [11].

2) Analytical Model: As we began validation with simulations of the robots in different configurations, we had then to extract the parameters introduced in Sec. III and IV from the simulation models and introduce them into our analytical model. These parameters are shown in Table I.

Based on experimentation we found that for $d$, the multiple of $p_{tx}$ to $ln(2)$ and $3$, $d = 2.5 \times (p_{tx})^{-1} \times \min \{n_t, \frac{n_m}{m}, \frac{n_t}{m} \}$ was a good value in most cases. Nevertheless, as this formulation ended up with too high values for $d$ in CKBot ($n_t = 2$ and $n_m = 2$), we simply used $d = 2.5 \times \min \{n_t, \frac{n_m}{m}, \frac{n_t}{m} \} = 2.5$ in this robot.

B. Results

For all robots and configurations, we considered three different workloads, $p_{tx} = 0.05, 0.2$ and $0.3$. In addition, we ran 100 simulation cycles for each workload and, every time, we randomly selected a seed module from which to begin propagation. Fig. 5 shows the simulation results compared to those of the analytical model. Notice that we did not use $T_{loc}(r_{tx})$ for our validations. Instead, we used $r(t)$ to ease data acquisition from simulations. However, once $r(t)$ was validated, so was $T_{loc}$.

Finally, even if not shown in Fig. 5, our model could not make accurate predictions of the information diffusion process at high $p_{tx}$ (i.e., the model was faster than simulation). Nonetheless, we realized that this issue did not have important consequences for our conclusions, because they never relied in optimistic local communication. As we will see in our results, local communication improved and degraded in performance when going from low to high $p_{tx}$, so a better model would have simply made this change in performance to occur earlier.

VI. GLOBAL COMMUNICATION MODEL

A. Assumptions and Considerations

In this analysis we used the global communication model developed for distributed mobile robots in [3]. This model was also suitable to modular robots, because the only difference with mobile robots was that the physical global medium (see Fig. 1b) could be either wired or wireless.

For this model, Yoshida et al. assumed that global communication was based on time-division multiple access (TDMA) to a common medium where, at first, a module was assigned a time slot by either a central manager or a token passing method and, then, its message immediately reached all modules in the robot. Fig. 6 shows this model.

In addition, this model assumed that a time slot was long enough to send all the information available from one module and that such a time slot had the same duration as the time step, $\Delta t$, defined for the local communication model.

B. Analysis of Global Transmission Time, $T_{glo}$

Based on hyper-geometric and conditional probabilities, together with the expected value of the information transmission time for global communication, Yoshida et al. concluded that the information transmission time, $T_{glo}$, was defined by the equation:

$$T_{glo}(n_{tx}) = \sum_{i=n_{tx}}^{n_t} \frac{i^{-1}C_{n_{tx}-1}}{n_t C_{n_{tx}}} \times i$$ (15)
Fig. 6. Global communication model proposed by Yoshida et al. in a robot made of 20 modules. This model is based on time-division multiple access (TDMA) to a common medium where, at first, a module is assigned a time slot by either a central manager or a token passing method and, then, its message immediately reaches all modules in the robot.

Fig. 7. Information transmission time, $T_{glo}(n_{tx})$, for global communication when $n_t = 10, 100$ and $1000$. Notice how $T_{glo}(n_{tx})$ approaches $n_t$ as soon as more than a few modules attempt to access the global medium.

where $n_t$ represented the total number of modules in the structure and $n_{tx}$ represented the number of modules sending information. Fig. 7 shows $T_{glo}(n_{tx})$ for three structures made of 10, 100 and 1000 modules.

C. Observations

As every time one module accessed the common medium its information was globally broadcasted, the number of receiving modules was not as important as the number of transmitting modules in the system ($n_{tx}$).

Also, as the only assumption was modules with access to a global medium, a single curve of $T_{glo}(n_{tx})$ could be used in different modular robots as long as $n_t$ remained unchanged.

Finally, although $T_{glo}(n_{tx})$ approached its maximum value as soon as more than a few $n_{tx}$ modules were to send information (see Fig. 7), such value could never be more than $n_t$ time slots.

VII. COMPARISON OF TRANSMISSION TIMES

A. Setup

In this section we compared $T_{loc}(n_{rx})$ given by (14) with $T_{glo}(n_{tx})$ given by (15); hence the importance of assuming time units with equal duration. As opposite to Sec. V, where we compared an analytical model with simulations, in this section we did so with two analytical models.

For the comparison we used the parameters from Table I, which were extracted from the robots and configurations mentioned in Sec. V. In addition, we introduced a model of M-TRAN with parameters to account for a cubic configuration: $n_s = 6$, $n_m = 2$, $n_{av} = 1.5$, $d = 27.8$ and $n_t = 100$. The introduction of M-TRAN is an example of analysis of a robot for which we did not have a simulation model. For all cases, we considered local communication under four workloads: $p_{tx} = 0.05, 0.1, 0.2$ and 0.3.

B. Results

As we wanted to compare transmission times from one-to-all the modules in the robot, we plotted $T_{loc}(n_{rx})$ for $n_{rx} = \{1... (n_t – 1)\}$, and we overlapped $T_{glo}(n_{tx})$ for $n_{tx} = \{1, 10, n_t\}$ with horizontal lines. In this way, we could visualize local diffusion times together with global broadcasting times and, eventually, identify points where local became slower than global or viceversa (intersections between $T_{loc}$ and $T_{glo}$). Fig. 8 shows the results of our comparison.

VIII. DISCUSSIONS AND CONCLUSIONS

Although we considered only small values for $p_{tx}$, such as 0.05, 0.1, 0.2 and 0.3, we believe they represent reasonable communication workloads. If $p_{tx}$ was high, for example a value near to one, modules would hardly execute anything but the communication process. There would not be room for any other task, such as control and sensing.

To continue, as local-based information diffusion does not reach all modules in the robot before the $T_{loc}$ curves touch the right side of the plots, we interpret Fig. 8 as follows:

- Global is more convenient than local when few modules broadcast information globally, and they do not
expect feedback from others modules in the robot. This statement resembles the modus operandi of centralized control approaches.

- **Global is in general more convenient than local** in low-connectivity configurations, such as chains and limbs, regardless of the number of modules exchanging information.

- **Local is more convenient than global** when many modules exchange information, and this information does not need to reach all the modules in the robot. This statement resembles the modus operandi of distributed control approaches.

- **Local is in general more convenient than global** when receiving (destination) modules are relatively near to transmitting modules, regardless of the number of modules exchanging information. Notice that local-based diffusion propagates information to significant percentage of modules before global broadcasting even starts.

Previous observations also reveal the potential of flexible communication topologies [2], which can provide local, global or extended buses on demand (see Fig. 1). Consider the structure of Fig. 9, where we assemble a dog-like configuration. In this case, global buses in the limbs could provide a convenient topology for centralized control, and local buses in the spine could do the same for distributed control between limbs’ centralized controllers and a sensor module in the head.

Since our models are simplified versions of more complex processes, we do not dare to interpret their results quantitatively. However, as models are refined, we should observe better performance in both processes (local and global), and so we anticipate that previous observations will hold.

IX. SUMMARY AND FUTURE WORK

In this paper we analysed the efficiency of local and global communication in modular robots. From our results, we concluded that global communication is convenient for centralized control approaches and local communication is convenient for distributed control approaches. In addition, we concluded that local is in general convenient for low-connectivity configurations, such as chains, trees or limbs, and that local can be faster than global when communicating distant modules which are not too separated.

Our results also uncovered the potential of flexible communication topologies, which can outperform pure local or pure global approaches by providing optimal communication topologies to many configurations, such as those we can split into bodies and limbs.

As future work, we want to analyse how to establish optimal communication topologies, such as the one shown in Fig. 9. This challenge demands, among others, topological knowledge to determine where to establish local/global communication buses and to avoid the creation of communication loops. Since Odin is implemented with a flexible communication topology, it can be used as experimental platform for validation of future analyses.

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