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On the common risk explanation of the size-related premiums
– Early draft, comments welcome –

THIAGO DE OLIVEIRA SOUZA

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ABSTRACT

I provide further empirical evidence that a common (time-varying) risk omitted in the CAPM generates both the size and the value premiums. The portfolio sorts on value are normalized versions of the sorts on size and align better with the risk exposures. This allows the value premium to be marginally significant even in low risk states while the size premium is only significant in high risk states. This state dependence explains the out-of-sample $R^2$ of around 29% for forecasts of the returns on the SMB portfolio but only 7% for the HML portfolio compared to their historical means.

JEL Classification: G11, G12, G14.
Keywords: Size premium, Value premium, Risk, Conditional, Out-of-sample, Forecast.

*Department of Business and Economics, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark. Email: tsouza@sam.sdu.dk. I would like to thank Stefano Giglio, John Cochrane, Paulo Maio, Jonathan Berk, Christian Flor, Linda Larsen, Francisco Gomes, Jaime de Jesus, and the participants in the World Finance Conference 2016.
The size (SMB) and value (HML) factors on top of the excess return on the market in Fama and French (1996) are still the standard to explain the cross-section of average stock returns empirically despite the existence of recent extended factor models, such as in Fama and French (2015) or in Hou, Xue, and Zhang (2015). This guides a large body of theoretical research aiming at these factor premiums individually. And although the evidence in Davis, Fama, and French (2000), for example, challenges the characteristic explanation of these premiums, several models relate the premiums to at least partially independent risk dimensions that are assumed to be functions of the firms’ characteristics, such as their market values, price earnings ratios, the relation between their assets and their market values, and others.\(^1\)

This piecewise and characteristic driven approach also contrasts with the framework of Berk (1995), for which I provide further empirical support in a dynamic setting in this paper. Berk (1995) states that a common risk generates the size, the value, and indeed all the size-related premiums. Under this hypothesis, a theory for the cross-section of the average stock returns must describe risks that simultaneously generate the size and the value premiums, for example. Furthermore, the size-related characteristics of the firms tend to be related to, but are not the systematic risks that generate the premiums. More broadly, the results in the paper add support to the risk explanation (versus the characteristic explanation) of the size-related premiums in a discussion that goes back to at least Daniel and Titman (1997), and Davis et al. (2000).

The intuition in Berk (1995) is that given two firms, the one with riskier cash flows (and higher expected returns) will tend to have lower market value because its cash flows

\(^1\)For example, explanations for the value premium based on the firms’ characteristics include distress risk (Griffin and Lemmon, 2002); the real frictions on the firms’ investments (Zhang, 2005); the interaction between asset risk and financial leverage (Choi, 2013); sticky wages relative to output (Favilukis and Lin, 2016); or even systematic mispricing (Piotroski and So, 2012). On the other hand, examples of explanations for the size premium that consider the size characteristic include liquidity (Acharya and Pedersen, 2005); low information and market segmentation (Merton, 1987) with slow information diffusion (Hong, Lim, and Stein, 2000); difficult market making (Grossman and Miller, 1988); or the absence of institutional investors (Gompers and Metrick, 2001).
are discounted at a higher rate. This creates a (negative) relation between the market-related values and the expected returns. The relation corresponds to the size-related premiums, such as the size and the value premiums, in any misspecified asset pricing model.

To understand this, consider the beta representation of the conditional pricing equation with a single risk factor,

$$E_t[R^i_{t+1}] - R_f = \beta_{i,mv} E_t[\lambda_{mv,t+1}], \quad (1)$$

where $\beta_{i,mv}$ is the regression coefficient of the excess returns, $R^i_{t+1} - R_f$, on the risk premium, $\lambda_{mv,t+1}$. In a simplified one period formulation, the market value of equity is given by

$$ME_{i,t} = \frac{E_t[CF_{i,t+1}]}{E_t[R^i_{t+1}]}, \quad (2)$$

where $ME_{i,t}$ is the market value of equity in firm $i$ at time $t$, $E[CF_{i,t+1}]$ is the expected cash flow at period $t + 1$ for firm $i$, and $E[R^i_{t+1}]$ is the equilibrium return required for firm $i$ between times $t$ and $t + 1$, determined in Eq. (1). Considering that stock $s$ is riskier than stock $b$, $\beta_{s,mv} > \beta_{b,mv}$ in Eq. (1), its cash flow will be discounted at a higher rate, $E_t[R^s_{t+1}] > E_t[R^b_{t+1}]$, which is also the expected return on each stock in equilibrium. If the two stocks have the same expected cash flows, $E_t[CF_{s,t+1}] = E_t[CF_{b,t+1}]$, the market value of stock $s$ in Eq. (2) will be smaller than the market value of stock $b$, $ME_{s,t} < ME_{b,t}$. This creates the negative relation between market values and expected returns that corresponds to the size premium.

More generally, if the firms have different expected cash flows, rearranging Eq. (2) shows that the ranking on the ratio between expected cash flows and market values,

$$E_t[R^i_{t+1}] = \frac{E_t[CF_{i,t+1}]}{ME_{i,t}}, \quad (3)$$

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aligns perfectly with the ranking on expected returns. Berk (1995) argues that the book value of equity used to calculate the book-to-market (BM) of Fama and French (1996) is a proxy for expected cash flows, explaining the value premium. Hence, the portfolio rankings on value and size capture the same underlying risks. But the ranking on value is more efficiently aligned with the ranking on expected returns because it includes a proxy to control for differences in expected cash flows. In line with this explanation, Berk (1995) observes that the unconditional value premium is indeed larger than the unconditional size premium.

One of the key issues with the framework of Berk (1995) is that it imposes very few empirically rejectable restrictions on the portfolio returns in a static setting. This happens because the discount rates and the expected cash flows are unobservable and determine jointly the (observable) market price. This ability to explain almost “everything” limits the relevance of the eventual support that the model finds in the data. In order to address this point, I consider a simple dynamic version of the framework of Berk (1995) in the presence of time varying risk premiums (Cochrane, 2011). This formulation places a new set of restrictions on the joint behavior of the time series and cross sectional portfolio returns that I test in the data.

Applying Eq. (1) to the market portfolio shows that the expected return on the market is large in the high risk states (with large $\lambda_{mv,t+1}$), and vice versa. Empirically, this means that the traditional ICAPM state variables in Welch and Goyal (2008) or Souza (2016), for example, can identify the state of the economy given a definition of a “high” level for the expected risk premium, $E_t[\lambda_{mv,t+1}]$, that I provide later. In particular, I consider the cyclically adjusted price earnings ratio (PE henceforth) of Shiller (2015) to identify the state of the economy. I find that both the deviations of the PE from its historical average, and the deviations from its most recent value (past year) are informative about
the risk state of the economy.\footnote{For instance, these two variables explain around 21% of the variation in the future 5 years market premium. The periods of high expected returns are the ones with low PE values and the ones with big drops in the PE relative to the previous year.}

Once the state of the economy is identified, it is possible to analyse the restrictions that the model places on the cross-section of returns in each state, in particular the size and value premiums, considering the returns on the SMB and HML portfolios of Fama and French (1996). The return spread between two portfolios $p_1$ and $p_2$, given Eq. (1), is

$$E_t[R_{t+1}^{p_1}] - E_t[R_{t+1}^{p_2}] = (\beta_{p_1,mv} - \beta_{p_2,mv})E_t[\lambda_{mv,t+1}].$$

(4)

In general, we cannot estimate the risk of each portfolio, $\beta_{i,mv}$, unless the risk premium, $\lambda_{mv,t}$, is observable. However, Eq. (3) implies that if the book-value of equity is a good proxy for the expected cash flows, the ranking on BM aligns with the ranking on risk exposures. In this case, the risk of the high BM portfolio, $p_1$, is larger than the risk of the low BM portfolio, $p_2$, with $\beta_{p_1,mv} > \beta_{p_2,mv}$ in every state. So the return on the HML portfolio (long on the high BM stocks and short on the low BM stocks) is large and positive in high risk states, and smaller but still positive in low risk states.

This is not the case for the SMB portfolio. The ranking on market size is only guaranteed to align with the ranking on expected returns when the expected risk premium, $E_t[\lambda_{mv,t+1}]$, is large enough. So the size premium is positive in the states of high risk, given that $\beta_{p_1,mv} > \beta_{p_2,mv}$, and zero otherwise, given that $\beta_{p_1,mv} = \beta_{p_2,mv}$. Intuitively, if the expected risk premium is zero, $E_t[\lambda_{mv,t+1}] = 0$, the risky stocks, $s$, and the safe stocks, $b$, have the same required return, $E_t[R_{t+1}^s] = E_t[R_{t+1}^b]$, even if $\beta_{s,mv} > \beta_{b,mv}$ in Eq. (1). Therefore, the ranking on market size only reflects differences in expected cash flows that are unrelated to the ranking on risk exposures, $\beta_{p_1,mv} = \beta_{p_2,mv}$. On the other hand, when the expected risk premium, $E_t[\lambda_{mv,t+1}]$, is very large, any differences in the
risk exposures of the stocks, $\beta_{s,mv} > \beta_{b,mv}$, results in a large difference in required returns, $E_t[R_{t+1}^s] >> E_t[R_{t+1}^b]$ in Eq. (1), that dominates any differences in expected cash flows. Hence, there is a threshold for the expected risk premium, $E_t[\lambda_{mv,t+1}]$, above which the risk exposures dominate the ranking on size. I define as “high” risk the states in which the expected risk premium, $E_t[\lambda_{mv,t+1}]$, is large enough so that the ranking on size aligns with the ranking on risk exposures, $ME_{s,t} < ME_{b,t}$ and $\beta_{p1,mv} > \beta_{p2,mv}$, generating a positive size premium.

This is precisely how I find the breakpoints for the two variables (PE and change in PE) mentioned earlier. The breakpoints for the variables are such that they minimize the in-sample mean squared error of the conditional forecast of the return on the SMB portfolio. The conditional forecast in the low risk states is zero, and the conditional forecast in the high risk states is equal to the historical conditional average of the size premium (in the previous high risk states). The economy is in a high risk state around 30% of the time according to this classification. In line with the predictions, the annual mean return on the SMB portfolio is close to 10.8% (4.00 t-statistics) in the high risk states and an insignificant −0.6% on average (−0.41 t-statistics) otherwise. On the same high risk states, the average return on the HML portfolio is 8.7% (3.31 t-statistics) and a marginally significant 3% on average (1.67 t-statistics) in the low risk states.

Furthermore, the model predicts that conditioning the forecast of the size premium on the state of the economy improves the forecast substantially compared to its unconditional mean: Both the risk alignment of the size portfolios and the risk premium, $\lambda_{mv,t+1}$, depend on the state of the economy. On the other hand, the ranking on value is aligned with the risk exposures in every state. So the value premium only depends on the state of the economy through the risk premium, $\lambda_{mv,t+1}$. Indeed, the out-of-sample forecasting exercise confirms this hypothesis. The out-of-sample $R^2$ for the forecast of the size pre-
mium is a little under 29% as early as 1967 in a sample that goes from 1927 to 2014.\textsuperscript{3} On the other hand, the out-of-sample $R^2$ is only 7% on average for the value premium with a sample split in each date between 1980 and 2005.

Finally, these results are not driven by a spurious selection of the years by the state classification procedure. This is especially important for the size premium because the significance of the unconditional size premium (at 10%) disappears removing only three of the 88 years between 1927 and 2014 from the sample. Any variable that classifies these three years as high risk states would seem to explain the size premium. But the same conclusions hold if I remove these three years from the sample.

The main contribution of the paper is to strengthen the evidence of a common risk explanation for both the size and the value premiums, confirming the predictions of Berk (1995) in a more restrictive dynamic setting. This adds to the literature on the discussion between the risk or characteristic explanations of the factors in Fama and French (1996), as Daniel and Titman (1997), Davis et al. (2000) and Chordia, Goyal, and Shanken (2015), for example. In terms of the asset pricing literature in general, my results reinforces the conclusion in Berk (1995) that the piecewise and characteristic driven explanations of the value and the size premiums could be misleading.

With respect to the literature on the pervasiveness of the empirical patterns in stock returns, such as Fama and French (2012) or Asness, Moskowitz, and Pedersen (2013), the paper provides further support to the existence of the conditional size premium documented in Souza (2016). This justifies the inclusion of the size factor in (conditional) factor models such as Fama and French (1996), Fama and French (2015) or Hou et al. (2015) for routine risk adjustment in empirical work.

Finally, the paper contributes to the literature on the out-of-sample forecasting of risk

\textsuperscript{3}The number varies depending on the year chosen to split the training and evaluation samples: The average out-of-sample $R^2$ is around 26% considering the split in each year between 1967 and 2005. The average out-of-sample $R^2$ is over 16% considering the split in any of the possible years from 1928 (with only one year in the training sample) until 2014 (one year in the evaluation sample).
premiums as in Kelly and Pruitt (2013), Ferreira and Santa-Clara (2011), Campbell and Thompson (2008), or Welch and Goyal (2008). We learn that the size premium is highly predictable out-of-sample and that the value premium is also predictable using the same method.

The rest of the paper is organized as follows: I present the model in section I; the data description and variables, and the in-sample and out-of-sample forecasting results in section II; and I summarize the paper in Section III.

I. Theoretical background

A. The pricing equation

In each period (t), an investor is born and lives only for another period (t + 1). The state of the economy is determined by the expected risk premium, $E_t[\lambda_{mv,t+1}]$, in Eq. (1) and the investor learns the state of the economy at birth. The economy is in a low risk state when the expected risk premium is below a certain threshold, $E_t[\lambda_{mv,t+1}] < \bar{\lambda}_{mv}$, and it is in a high risk state otherwise.

B. Cross-sectional predictions

There are two types of firms in the economy: Firms $s$ are riskier than firms $b$ with $\beta_{s,mv} > \beta_{b,mv}$ in Eq. (1). Their returns are given by $R^s$ and $R^b$.

PROPOSITION 1: The expected spread between the returns on stocks $s$ and $b$, $E_t[\text{Spread}_{t+1}] = E_t[R^s_{t+1} - R^b_{t+1}]$, increases with the expected risk premium, $E_t[\lambda_{mv,t+1}]$.

Proof. If there are only stocks $s$ in portfolio $p_1$ and only stocks $b$ in portfolio $p_2$, Eq. (4) gives
\[ E_t[\text{Spread}_{t+1}] = (\beta_{s,\text{mv}} - \beta_{b,\text{mv}})E_t[\lambda_{\text{mv},t+1}]. \] \hspace{1cm} (5)

Hence, the derivative of the spread with respect to the risk premium, \( \lambda_{\text{mv},t+1} \), is positive:

\[ \frac{\partial \text{Spread}_{t+1}}{\partial \lambda_{\text{mv},t+1}} = \beta_{s,\text{mv}} - \beta_{b,\text{mv}} > 0. \] \hspace{1cm} (6)

C. Size-related portfolio sorts and risk exposure

The state of the economy determines how well the ranking on size-related characteristics align with the ranking on risks.

C.1. Sorts on the market value of equity

PROPOSITION 2: The market value of equity on firm \( s \) is guaranteed to be smaller than the market value of equity on firm \( b \), \( ME_{s,t} < ME_{b,t} \), if and only if the expected risk premium, \( E_t[\lambda_{\text{mv},t+1}] \), is large enough. Otherwise, the ranking on size reflects differences in expected cash flows.

Proof. Eq. (2) implies that

\[ ME_{s,t} < ME_{b,t} \iff \frac{E[CF_{s,t+1}]}{E[R_{s,t+1}]} < \frac{E[CF_{b,t+1}]}{E[R_{b,t+1}]} \iff \frac{E[CF_{s,t+1}]}{E[R_{s,t+1}]} < \frac{E[R_{s,t+1}]}{E[R_{b,t+1}].} \] \hspace{1cm} (7)
And Eq. (5) implies that

\[
R^s_{t+1} = R^b_{t+1} + R^b_{t+1} = R^b_{t+1} + \text{Spread}_{t+1}
\]

\[= R^b_{t+1} + (\beta_{s,mv} - \beta_{b,mv})\lambda_{mv,t+1}. \tag{8}
\]

Therefore,

\[
ME_{s,t} < ME_{b,t} \iff \frac{E[CF_{s,t+1}]}{E[CF_{b,t+1}]} < 1 + \frac{(\beta_{s,mv} - \beta_{b,mv})}{E[R^b_{t+1}]} E_t[\lambda_{mv,t+1}], \tag{10}
\]

which shows that for any difference in expected cash flows, \(\frac{E[CF_{s,t+1}]}{E[CF_{b,t+1}]}\), there exists a large enough expected risk premium, \(E_t[\lambda_{mv,t+1}]\), that guarantees that the market value of stock \(s\) is smaller than the value of stock \(b\), given that \(\frac{(\beta_{s,mv} - \beta_{b,mv})}{E[R^b_{t+1}]} > 0\). If the expected risk premium is below the threshold given by

\[
\lambda_{mv} = E_t[\lambda_{mv,t+1}] > \left(\frac{E[CF_{s,t+1}]}{E[CF_{b,t+1}]} - 1\right) \frac{E[R^b_{t+1}]}{(\beta_{s,mv} - \beta_{b,mv})}, \tag{11}
\]

the portfolio ranking on size reflects differences in expected cash flows and not differences in expected returns.

Proposition 2 implies that if there are many firms, the ranking on market size bundles together firms with different risk exposures in general. For example, the portfolio of small stocks contains both risky firms, and safe firms with small expected cash flows. But as the expected risk premium increases above the threshold in Eq. (11) for more firms, \(E_t[\lambda_{mv,t+1}] > \lambda_{mv}\), the proportion of risky firms in the lowest size quantile increases. So a sort based on size aligns better with the stocks’ risk exposures when the economy is in a high risk state.

Empirically, this implies that the return on the SMB portfolio is significantly positive when the expected risk premium is above the threshold in Eq. (11) for a large proportion
of the firms. But in the low risk states, the dispersion in size relates mostly to differences in expected cash flows, so the return on the SMB portfolio should be zero.

C.2. Sorts on scaled price ratios

Proposition 2 suggests that if the expected cash flows of the risky firms, $s$, are too large compared to the cash flows of the safer firms $b$, the ranking on size does not align with the ranking on risks. Berk (1995) notes that a ranking on the market value of equity normalized by the expected cash flow solves this issue. So we may rank the stocks in terms of a scaled price ratio of the form $\frac{ME_i}{N_i}$, where $N_i$ is the normalization variable for stock $i$. As long as the normalization variable for the risky firm, $N_s$, is larger than the normalization variable of the safer firm, $N_b$, the ranking on the scaled price ratio is more efficient than the ranking on size.

PROPOSITION 3: The minimum expected risk premium, $E_t[\lambda_{mv,t+1}]$, that guarantees that the portfolio ranking on the scaled-price variable aligns with the risk exposure, $\frac{ME_s}{N_s} < \frac{ME_b}{N_b}$, increases with the ratio of the normalization variables $\frac{N_s}{N_b}$.

Proof. The normalized ranking is equivalent to (7), substituting each expected cash flow, $E[CF_{i,t+1}]$, by its normalized version, $E[CF_{i,t+1}]/N_i$:

$$\frac{ME_s}{N_s} < \frac{ME_b}{N_b} \iff \frac{E[CF_{s,t+1}]/N_s}{E[R_{t+1}^s]} < \frac{E[CF_{b,t+1}]/N_b}{E[R_{t+1}^b]}.$$  \(12\)

So the threshold for the risk premium necessary to align the risk exposures is similar to (11), but with the expected cash flows divided by the normalization variable:

$$\bar{\lambda}_{mv,norm} = E_t[\lambda_{mv,t+1}] > \left( \frac{E[CF_{s,t+1}]}{E[CF_{b,t+1}]} \frac{N_b}{N_s} - 1 \right) \frac{E[R_{t+1}^b]}{(\beta_{s,mv} - \beta_{b,mv})}.$$  \(13\)
Hence, the normalized threshold, $\bar{\lambda}_{mv,norm}$, increases with the ratio $\frac{N_b}{N_s}$:

$$\frac{\partial \bar{\lambda}_{mv,norm}}{\partial (N_b/N_s)} = \left( \frac{E[CF_{s,t+1}]}{E[CF_{b,t+1}]} \right) \frac{E[R_{t+1}^b]}{(\beta_{s,mv} - \beta_{b,mv})} > 0. \quad (14)$$

According to Eq. (14), ideally the normalization variables should be such that $N_s$ is much larger than $N_b$. But the ranking on risk and the scaled price ratios are aligned even if the expected risk premium, $E_t[\lambda_{mv,t+1}]$, is low as long as the normalization variables are good proxies for the expected cash flows. Eq. (13) implies that

$$\frac{N_b}{N_s} \approx \frac{E[CF_{b,t+1}]}{E[CF_{s,t+1}]} \Rightarrow \bar{\lambda}_{mv,norm} \approx 0. \quad (15)$$

Following this idea, it is possible to normalize the market value of the stock using the book value of equity, as the normalization variable $N_i$. This is how Fama and French (1996) obtain the BM variable used to sort the stocks into the BM portfolios that generate the HML portfolio. As long as the book value of equity is related to expected cash flows, the HML portfolio should have a better alignment with the risk exposures than the SMB portfolio. Therefore, the unconditional returns on the HML portfolio should be larger than the unconditional returns on the SMB portfolio.

On the other hand, the normalization can make the portfolio ranking less aligned with the risk exposures too. For example, when $\frac{N_b}{N_s} > 1$, we need a higher expected risk premium to align the normalized ranking with the risk exposures than we would need simply using the market size ranking: $\bar{\lambda}_{mv,norm} > \bar{\lambda}_{mv}$ in this case. Therefore, it is possible that the SMB portfolio aligns better with the risk exposures than the HML portfolio in the periods when the book value of equity is not a good proxy for expected cash flows. In these periods, we should observe that the excess returns on the SMB portfolio are larger than the ones on the HML portfolio.
D. The expected return on the market

PROPOSITION 4: The expected market return increases with the expected risk premium, \( E_t[\lambda_{mv,t+1}] \), considering that the market portfolio has a positive exposure to the risk premium, \( \beta_{m,mv} > 0 \).

Proof. Eq. (1) applied to the return on the market, \( R^m \), gives

\[
E_t[R^m_{t+1}] - R_f = \beta_{m,mv} E_t[\lambda_{mv,t+1}].
\]  

(16)

So the expected market return increases when expected risk premium increases, and vice versa.

II. Empirical tests

Let us first define two risk states: The states of high risk are the ones in which the expected risk premium is above the thresholds given in (11), \( E_t[\lambda_{mv,t+1}] > \bar{\lambda}_{mv} \), for a large number of stocks. In this case the risk exposures dominate the portfolio rankings on size and we should observe a size premium in the data.

Assuming that the expected risk premium is not readily measurable, for example because it is only in the agents’ private information sets, it is possible to indirectly identify it based on proposition 4: The periods of high risk premium risk tend to be periods of high expected return on the market too.

So the propositions 1 to 4 taken together have two broad empirical predictions that I test in this section. First, considering that the return spread between risky and safe firms increases with the expected market return (propositions 1 and 4):

1. The return on the SMB portfolio is large and significant if and only if the expected return on the market (and expected risk premium) is above a certain threshold
(proposition 2).

2. The return on the HML portfolio is large and significant in the same high risk states above, and smaller but still significant when the expected return on the market is not very high (proposition 3).

A. Data description and variables

I obtain the return data from Kenneth French’s data library on US stocks described in details in Fama and French (1993).\(^4\) The annual returns from 1927 to 2014 correspond to July in year \( t \) to the end of June in year \( t + 1 \). I collect the series of returns on the Fama/French SMB and the HML portfolios, and the market premium. I use annual data to avoid the short-term reversal in returns that generates the results in Vassalou and Xing (2004), for instance, as explained by Da and Gao (2010).

The Fama/French portfolios are constructed using six value-weighted portfolios double sorted on size and book-to-market. The SMB portfolio (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios. The HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios.

To forecast the market return, I use the cyclically adjusted PE of Shiller (2015) and Campbell and Shiller (1988).\(^5\) The ratio is constructed each year as the aggregate market value divided by the moving average of its past ten years of earnings adjusted for inflation.

A.1. The PE ratio

Although the PE series is relatively stable after 1926 the ratio seems to change over time.\(^6\) For instance, in the nineties it became more volatile and experienced a new

\(^4\)http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
\(^5\)The dataset is available at http://www.econ.yale.edu/ shiller/data.htm
\(^6\)See Figure 3 and Figure 4 in the appendix.
maximum. In order to make the ratio comparable over time, I adjust the PE values calculating the difference between the log-PE in each year $t$ and its historical average until $t$, dividing the result by the historical standard deviation until year $t$:

$$PE_{Adj,t} = \frac{\ln(PE_t) - \bar{\ln}(PE_t)}{\sigma_{\ln(PE_t)}}.$$  \hspace{1cm} (17)

I also calculate the change in the PE ratio

$$PE_{\Delta,t} = PE_{Adj,t} - PE_{Adj,t-1},$$ \hspace{1cm} (18)

and standardize it with respect to its own historical mean and standard deviation. So the two variables have the same order of magnitude. I standardize $PE_{\Delta,t}$ as in Eq. (17), but the adjustment almost doesn’t change the original variable.\(^7\)

A.2. The SMB portfolio returns

As explained before, portfolio sorts based on the market value of equity alone are not very efficient at capturing risk exposure variations in cross section. So the average return on the SMB portfolio tends to be small. In fact Table I shows that removing only two years out of the 88 in the sample reduces the significance of the size premium to 10%, and removing one extra data point renders the premium insignificant even at 10%.\(^8\)

A.3. The HML portfolio returns

The ranking based on a normalized market value of equity, as the book-to-market ratios used to construct the HML portfolio, produces portfolio sorts that are more aligned with the risk exposure variations in cross section. Indeed, the average return on the HML

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\(^7\)See Figure 5 in the appendix.
\(^8\)Figure 6 in the appendix shows these data points, and the box plot of the returns suggests that these three years could be “outliers”.

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Table I The table reports the mean, t-mean (ratio between the mean and the standard error), and the p-values for the returns on the SMB and HML portfolios. It also reports the number of years in each sub-sample (out of the 88 years in total). The sub-samples either include all the 88 years (SMB and HML), or removes the largest returns until the average becomes insignificant at 5% ($SMB_{-5}$ and $HML_{-5}$), or 10% ($SMB_{-10}$ and $HML_{-10}$).

<table>
<thead>
<tr>
<th></th>
<th>SMB</th>
<th>$SMB_{-5}$</th>
<th>$SMB_{-10}$</th>
<th>HML</th>
<th>$HML_{-5}$</th>
<th>$HML_{-10}$</th>
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</thead>
<tbody>
<tr>
<td>Mean</td>
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<td>2.32</td>
<td>1.95</td>
<td>5.00</td>
<td>2.44</td>
<td>2.16</td>
</tr>
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<td>1.77</td>
<td>1.54</td>
<td>3.33</td>
<td>1.83</td>
<td>1.64</td>
</tr>
<tr>
<td>p-value</td>
<td>0.02</td>
<td>0.08</td>
<td>0.13</td>
<td>0.00</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>Obs</td>
<td>88</td>
<td>86</td>
<td>85</td>
<td>88</td>
<td>80</td>
<td>79</td>
</tr>
</tbody>
</table>

portfolio of 5% per year is larger than the 3.4% on the SMB portfolio, and also more significant. Table I shows that even removing the 8 years with the highest returns on the HML portfolio from the sample, the average return on the remaining years is still significant at 10%.

B. In-sample results

B.1. The SMB portfolio in sample

The model predicts that the expected return on the market portfolio is high when the economy is in a high risk state, and that the size-related anomalies should also increase in these years. But unless the risk is high enough, the size ranking that creates the SMB portfolio will not be well aligned with the risk exposures. So for the return on the SMB portfolio to be significant, the risk needs to be higher than a certain level. The empirical question in this section is: How large does the risk need to be for this to happen? Or in other words: How high does the expected return on the market need to be?

The return on the SMB portfolio should be insignificant unless the expected return on the market (i.e., the risk) is above a certain level. So my empirical strategy is to try a

---

9Figure 7 in the appendix shows these data points, and the box plot of the returns suggests only a negative “outlier” (instead of the positive ones for the SMB portfolio).
number of possible breakpoints, $PE_{BP}$ and $PE_{\Delta BP}$ respectively for the two forecasting variables $PE_{Adj,t}$ and $PE_{\Delta t}$, and check which pair of breakpoints results in the best forecast of the 1-year return on the SMB portfolio. The forecasted return on the SMB portfolio, using $I$ as an indicator function, is given by

$$E_t [SMB|PE_{Adj,t}, PE_{\Delta,t}] = SMB_t^Hi gh \times I_{PE_{Adj,t}<PE_{BP}, \text{or} \ PE_{\Delta,t}<PE_{\Delta BP}}. \tag{19}$$

So I forecast a return different from zero only when the forecasting variables are low enough (i.e., when the expected return on the market and the risks are high enough). When the expected return on the market is high enough, the forecast equals the “restricted” historical average of the return on the SMB portfolio, $SMB_t^Hi gh$. This average considers only the years in which at least one of the forecasting variables were below the breakpoints:

$$SMB_t^Hi gh = \frac{\sum_{t=1}^{T} (SMB_t \times I_{PE_{Adj,t}<PE_{BP}, \text{or} \ PE_{\Delta,t}<PE_{\Delta BP}})}{\sum_{t=1}^{T} I_{PE_{Adj,t}<PE_{BP}, \text{or} \ PE_{\Delta,t}<PE_{\Delta BP}}}, \tag{20}$$

where $T$ is the total sample size (88 years), and $SMB_t$ is the 1-year return on the SMB portfolio in year $t$. Finally, I choose the pair of breakpoints that minimize the mean squared error of the predictive regressions in sample:

$$MSE_{SMB} = \frac{1}{T} \sum_{t=1}^{T} \left( SMB_t^Hi gh \times I_{PE_{Adj,t}<PE_{BP}, \text{or} \ PE_{\Delta,t}<PE_{\Delta BP}} - SMB_t \right)^2. \tag{21}$$

The breakpoints that minimize the in-sample $MSE_{SMB}$ are $PE_{BP} = -0.55$ and $PE_{\Delta BP} = -0.85$. There are 31 (out of 88) years in which $PE_{Adj,t}$ or $PE_{\Delta,t}$ are lower than those breakpoints. The results in Table II strongly support the model’s predictions:
The size premium is very large (10.8% in annual terms) and strongly significant with a t-Mean, the ratio between the mean and standard error, equal to 4 in the high risk years. On the other hand, the premium is slightly negative (−0.6%) and insignificant (t-mean is −0.41) in the low risk years.

The next two columns in Table II, \( SMB_{H,-5} \) and \( SMB_{H,-10} \), show that the results are not only driven by what could be the outliers in the dataset.\(^{10}\) I remove the data points that make the unconditional average of the return on the SMB portfolio significant at 5% or 10% without re-estimating the model. I report these results in columns \( SMB_{H,-5} \) and \( SMB_{H,-10} \) respectively: The average return on the SMB portfolio decreases to 8.1 and 7.2 percent respectively, but remains strongly significant (t-Means of 3.91 and 3.72). Finally, the selected years are not concentrated around a certain period and both forecasting variables contribute for the identification of the high risk years (with \( PE_{\Delta,t} \) becoming more important towards the end of the sample).\(^{11}\)

**Table II** The table reports the mean return, t-Mean (the ratio between the mean and the standard error), and the number of years in each sample considering the SMB and HML portfolios. The subscript in each portfolio indicates the sample used, given by the indicator variable \( I_{PE_{Adj,t}<PE_{BP}, \text{ or } PE_{\Delta,t}<PE_{\Delta,BP}} \) used to identify high expected market return (and risk) years. “H” means that all the high risk years (i.e., \( I = 1 \)) were used. In “H−5” and “H−10” I remove from the high risk years, the ones that once removed from the full sample (88 years) render the average returns insignificant at 5% and 10% respectively. “L” means that only the years with low expected market return/risk were used (i.e., \( I = 0 \)).

<table>
<thead>
<tr>
<th>( SMB_H )</th>
<th>( SMB_{H,-5} )</th>
<th>( SMB_{H,-10} )</th>
<th>( SMB_L )</th>
<th>( HML_H )</th>
<th>( HML_{H,-5} )</th>
<th>( HML_{H,-10} )</th>
<th>( HML_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>10.8</td>
<td>8.1</td>
<td>7.2</td>
<td>-0.6</td>
<td>8.7</td>
<td>4.8</td>
<td>4.8</td>
</tr>
<tr>
<td>t-Mean</td>
<td>4.00</td>
<td>3.91</td>
<td>3.72</td>
<td>-0.41</td>
<td>3.31</td>
<td>1.98</td>
<td>1.98</td>
</tr>
<tr>
<td>Obs</td>
<td>31</td>
<td>29</td>
<td>28</td>
<td>57</td>
<td>31</td>
<td>26</td>
<td>26</td>
</tr>
</tbody>
</table>

----

\(^{10}\)As we see in Figure 6 in the appendix for instance.

\(^{11}\)We can see this in Figure 8 in the appendix.
B.2. The HML portfolio in sample

The Table II also shows that the significance of the return on the HML portfolio is less dependent on the risk level in the economy. I apply the same breakpoints $PE_{BP}$ and $PE_{\Delta BP}$ found earlier to identify the (same) years when the risk is high, but I analyse the return on the HML portfolio instead. In line with the model’s prediction, the average return on the HML portfolio increases when the expected market return is high: 8.7% (t-Mean of 3.31). But the average return is still large (3%) and just significant at 10% (t-Mean of 1.67) when the expected market return is lower as well. Removing the highest returns from the sample again has a minor impact on the significance of the premium in the high risk years (columns $HML_{H,-5}$ and $HML_{H,-10}$ in Table II).\(^{12}\)

C. The out-of-sample forecasts

One possible application of the model is in forecasting the return on the size related portfolios. The fitness of the out-of-sample forecasts is the best indication of the real utility or welfare gains arising from implementing the model. Furthermore, the out-of-sample forecast evaluation is particularly important to avoid any small sample biases. So in this section I focus on the recursive out-of-sample forecast implementation of the model, similar to the idea in Welch and Goyal (2008) for example.

In order to evaluate the out-of-sample forecasting performance of the model, I calculate a predictive $R^2$. The out-of-sample $R^2$ increases with the ratio between the mean squared forecasting error of the model that I use ($MSFE_{Model}$), and the one associated with the historical average ($MSFE_{Average}$):

\(^{12}\)The Figure 9 in the appendix shows that the return on the HML portfolio is less dependent on the risk level (compared with the SMB portfolio): There are more periods of significantly large returns due to a better alignment between the risk exposures and the normalized portfolio ranking.
\[ R^2 = 1 - \frac{MSFE_{Model}}{MSFE_{Average}} = 1 - \frac{\sum_t (y_t - \hat{y}_t)^2}{\sum_t (y_t - \bar{y}_t)^2}. \]  

(22)

The $R^2$ can take any value below 1, and negative values mean that the model’s forecast is less accurate than simply using the historical mean return.

C.1. The SMB portfolio out-of-sample

I consider all the possible split years, $s$, between 1928 (only one year in the training sample) and 2013 (only one year in the evaluation sample) to separate the sample into training and evaluation samples. Given a split year $s$, I use all the years before $s$ to find the in-sample optimal break points ($PE_{BP}$ and $PE_{\Delta,BP}$) minimizing the MSE in Eq. (21), and considering the forecast of the return on the SMB portfolio as in equations (19) and (20). I apply the estimated returns and breakpoints to Eq. (19) to obtain the forecast of the size premium for the next period (given the values of $PE_{Adj,t=s}$, and $PE_{\Delta,t=s}$). I then repeat the procedure recursively until the last forecast in 2013. Figure 1 shows that the out-of-sample $R^2$ is already very large as early as in 1967 (a little under 29%), with 40 years in the training sample, and 48 in the evaluation sample. Considering any of the possible breakpoints between 1928 and 2013, the average out-of-sample $R^2$ is over 16%. Between 1967 and 2005, the average $R^2$ is a little under 26%.\(^\text{13}\)

C.2. The HML portfolio out-of-sample

The superior risk alignment of the portfolio sorts based on the BM means that the return on the HML portfolio is less dependent on the risk level in the economy. So the historical average tends to be a better estimation of the return on the HML portfolio,\(^\text{13}\)

\text{As a comparison, Welch and Goyal (2008), Kelly and Pruitt (2013), and Ferreira and Santa-Clara (2011) usually obtain an out-of-sample $R^2$ below 13\% for other large portfolios.}
Figure 1. Out-of-sample $R^2$ by sample split date, 1-year returns on the SMB portfolio. The vertical axis shows the performance of the recursive out-of-sample forecast of the return on the SMB portfolio: $R^2 = 1 - \frac{\sum_t(y_t - \hat{y}_t)^2}{\sum_t(y_t - \bar{y}_t)^2}$. The value changes depending on the year (in the horizontal axis) used to split the sample into training and evaluation sub-samples.

compared to the SMB portfolio.

In order to forecast the out-of-sample return on the HML portfolio, I use the exact same breakpoints used to forecast the SMB returns. The difference is that there is no reason to expect a small and insignificant return on the HML portfolio when the risk (and the expected market return) is low. So the out-of-sample forecast of the return on the HML portfolio in time $t = s$ will be a modified version of the one used for the SMB portfolio in Eq. (19). At each period, the expectation of the return on the HML portfolio will be either “high” or “low”. This depends on how the values of $PE_{Adj,s}$ and $PE_{\Delta,s}$
observed in time \( t = s \) compares to the breakpoints \( PE_{BP} \) and \( PE_{∆BP} \):

\[
E_s \left[ HML_{s+1}|PE_{Adj,s}, PE_{∆,s} \right] = HML_s^{High} \times I_{PE_{Adj,t}<PE_{BP}, \text{ or } PE_{∆,t}<PE_{∆,BP}} \\
+ HML_s^{Low} \times \left( 1 - I_{PE_{Adj,t}<PE_{BP}, \text{ or } PE_{∆,t}<PE_{∆,BP}} \right),
\]

(23)

where \( HML_s^{High} \) is the average return on the HML portfolio in the years when the risk is high until time \( s \):

\[
HML_s^{High} = \frac{\sum_{t=1}^{s} (HML_t \times I_{PE_{Adj,t}<PE_{BP}, \text{ or } PE_{∆,t}<PE_{∆,BP}})}{\sum_{t=1}^{s} I_{PE_{Adj,t}<PE_{BP}, \text{ or } PE_{∆,t}<PE_{∆,BP}}},
\]

(24)

and \( HML_s^{Low} \) is the average return on the HML portfolio in the years when the risk is low:

\[
HML_s^{Low} = \frac{\sum_{t=1}^{s} \left[ HML_t \times (1 - I_{PE_{Adj,t}<PE_{BP}, \text{ or } PE_{∆,t}<PE_{∆,BP}}) \right]}{\sum_{t=1}^{s} (1 - I_{PE_{Adj,t}<PE_{BP}, \text{ or } PE_{∆,t}<PE_{∆,BP}})}.
\]

(25)

Accordingly, Figure 2 shows that it is more difficult to outperform the historical mean out-of-sample in the case of the return on the HML portfolio. In fact, the model only seems to outperform the historical mean after 1980. Considering any of the possible breakpoints between 1928 and 2013, the average out-of-sample \( R^2 \) is a little under 3%. Between 1967 and 2005, the average \( R^2 \) is a little over 2.8%, and between 1980 and 2005 it is around 7%.

D. PE predicting the market premium

I confirm the ability of the (adjusted) PE ratio, \( PE_{Adj,t} \), and its changes with respect to the previous years, \( PE_{∆,t} \), to forecast future market returns. As in previous return forecasting research, the accuracy of the forecast tends to increase with the return horizon
The vertical axis shows the performance of the recursive out-of-sample forecast of the return on the HML portfolio: $R^2 = 1 - \frac{\sum_t (y_t - \hat{y}_t)^2}{\sum_t (y_t - \bar{y}_t)^2}$. The value changes depending on the year (in the horizontal axis) used to split the sample into training and evaluation sub-samples.

The two variables are significantly related to the future 5-year returns on the market even if the relationship is less clear for the 1-year return. This result confirms that these two variables can be used to identify the high expected return years. This is important because it is the first part of the joint hypothesis that I test: The prediction of the model is that when the risk is high, then the expected returns on the market should also be high.

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14See (Kelly and Pruitt, 2013), or Campbell and Shiller (1988) for instance.
D.1. The predictive regression

I use the PE and its difference with the lagged PE to forecast the 1-year, and the 5-year excess returns on the market. I estimate regressions of the type

\[ MP_{t+1}^h = c_0 + c_1 \times PE_{Adj,t} + c_2 \times PE_{\Delta,t}, \] (26)

where \( MP^h \) is the excess return on the market over the next \( h = 1 \) or \( h = 5 \) years, \( PE_{Adj,t} \) is the adjusted PE ratio described in Eq. (17), \( PE_{\Delta,t} \) is the change in the adjusted PE ratio with respect with the previous year described in Eq. (18), and \( c_i \) are constants. On top of varying the return horizon that I forecast, I also vary the regressors including each one of them individually, or both for each return horizon forecasted.

The results in table III show that \( PE_{Adj,t} \) is a very consistent predictor of the market excess returns. It significantly forecasts the market returns at both horizons, in single or multiple regressions. \( PE_{\Delta,t} \) also significantly forecasts the market excess returns over five years even if it fails to forecast the shorter one year horizon. Both variables have negative coefficients meaning that the periods of high expected market returns correspond to low values of the PE ratio, and to periods when the PE ratio experienced large drops (i.e., when \( PE_{\Delta,t} \) is very negative).

E. Time varying, badly estimated CAPM betas

One of the main difficulties in estimating the market risk exposure of the SMB and the HML portfolios is that they are managed portfolios (meaning that their compositions change periodically). The risk exposure of one portfolio formation period is not necessarily similar to another portfolio formation period. So a simple time series regression of the SMB or HML returns on the market return captures the average risk exposure of these portfolios over time. If the market risk of the SMB and the HML portfolios vary over
The table reports the results of using either $PE_{Adj,t}$ (the adjusted PE ratio), $PE_{\Delta,t}$ (the change in the adjusted PE ratio with respect with the previous year), or both to predict the excess returns on the market during the next $h = 1$ or $h = 5$ years. I run regressions of the type $MP^h_{t+1} = c_0 + c_1 \times PE_{Adj,t} + c_2 \times PE_{\Delta,t}$, where $MP^h$ is the excess return on the market over the next $h = 1$ or $h = 5$ years and $c_i$ are constants.

<table>
<thead>
<tr>
<th></th>
<th>$MP^{1y}$</th>
<th>$MP^{1y}$</th>
<th>$MP^{5y}$</th>
<th>$MP^{5y}$</th>
<th>$MP^{5y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PE_{Adj}$</td>
<td>-5.018*</td>
<td>-5.735**</td>
<td>-0.229***</td>
<td>-0.206***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.46)</td>
<td>(-2.72)</td>
<td>(-4.54)</td>
<td>(-3.99)</td>
<td></td>
</tr>
<tr>
<td>$PE_{\Delta}$</td>
<td>1.225</td>
<td>2.779</td>
<td>-0.146**</td>
<td>-0.0917</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.57)</td>
<td>(1.29)</td>
<td>(-2.64)</td>
<td>(-1.74)</td>
<td></td>
</tr>
<tr>
<td>Const.</td>
<td>10.39***</td>
<td>8.320***</td>
<td>10.50***</td>
<td>0.526***</td>
<td>0.449***</td>
</tr>
<tr>
<td></td>
<td>(4.53)</td>
<td>(3.75)</td>
<td>(4.60)</td>
<td>(9.24)</td>
<td>(7.77)</td>
</tr>
<tr>
<td>N</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>84</td>
<td>84</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0548</td>
<td>-0.00786</td>
<td>0.0619</td>
<td>0.191</td>
<td>0.0673</td>
</tr>
</tbody>
</table>

In addition, the size related sorts align also better with the market risk exposures when the market risk increases. This implies that the SMB portfolio should have larger market risk when the market premium is high. So their CAPM betas should be larger when the market premium is large. It is possible that the change in composition induced on the portfolio sorts by the variation in the market risk of each portfolio generates the size related “excess” returns. I investigate this hypothesis in this section.
E.1. Annual frequency data

The results in Table IV suggest that there is in fact a difference between the SMB and the HML portfolios exposures to market risk in high and low risk years. This is consistent with the hypothesis that some of the high risk years correspond to increases in the market risk. I use yearly data to estimate the portfolios exposures to market risk. But I split the sample in high and low risk years according to the classification that I obtained in the previous sections. Next I use either the full sample (reported in column All), only the high risk years (High risk), or only the low risk years (Low risk) to estimate the CAPM equation:

\[ R_{i,t} = \alpha_i + \beta_i \times MP_t + \epsilon_t, \]  

(27)

where \(\alpha_i\) and \(\beta_i\) are the coefficients of the regression of the return on the SMB or the HML portfolios \(R_{SMB,t} = SMB_t\) or \(R_{HML,t} = HML_t\) on the market premium \(MP_t\).

Table IV shows that the betas of the SMB and the HML portfolios tend to be higher when the risk is higher (columns “High risk” compared to “Low risk”). But the intercepts are also significant for both portfolios in the high risk years, and insignificant in the low risk years. The main difference between the returns on the SMB and HML portfolios appears when we consider the full sample. While there is no evidence of an unconditional excess return on the SMB portfolio, the HML portfolio earns a significant unconditional excess return.

Even considering a higher market risk exposure in high risk years, there is still strong evidence that the CAPM is unable to explain premiums of around 8% and 7.8% per year for the SMB and the HML portfolios respectively. In the remaining 57 out of 88 low risk years there isn’t strong evidence that the SMB or the HML portfolios earn excess returns.
So the exercise does not provide any evidence that the change in the composition of the portfolios due to their exposure to market risk alone generates the returns unexplained by the CAPM.

E.2. Forward looking weekly data

It is possible that the procedure above using annual frequency data still bundles together portfolio formations with very different market risk exposures. So my next step is to obtain and analyse the coefficient estimations for each individual portfolio formation period.

The SMB and HML portfolio formations in year $t$ last between July in year $t$ to the end of June in $t+1$. So each year I use forward looking weekly data between these two dates to estimate the coefficients in Eq. (27). Therefore, I obtain a pair of coefficients $\alpha_{i,t}$ and $\beta_{i,t}$ for each year $t$, and each portfolio $i$ (HML or SMB). In Table V I report the average of these estimated coefficients over different groups of years. Table V shows that the results are essentially the same as the one obtained from yearly data in Table IV. The biggest difference is that the average betas are all insignificant. The noisier weekly estimations may explain this fact. The SMB and HML portfolios have unexplained weekly excess returns of 12bps and 8.4bps respectively (6.4% and 4.5% weekly compounded in annual terms). The excess returns are again statistically and economically significant in the high risk years, but not in the low risk years. The unconditional excess returns on the SMB portfolio are insignificant, but they are significant on the HML portfolio.

F. Idiosyncratic risk

Finally, Table VI shows no evidence that the returns on the size portfolios are proportional to their idiosyncratic risks. The variance of the return on the SMB and the HML portfolios do increase in years of high expected return on the market. But the
Table IV The yearly CAPM $\alpha_i$ and $\beta_i$ are the coefficients of the regression of the return on the SMB or the HML portfolios $R_{SMB,t} = SMB_t$ or $R_{HML,t} = HML_t$ on the market premium $MP_t$: $R_{i,t} = \alpha_i + \beta_i \times MP_t + \epsilon_t$. The estimations use yearly data considering either the full sample (All), only the high risk years (High risk), or only the low risk years (Low risk). I also report the number of years in each of these samples (Obs.) and the adjusted in sample $R^2$.

<table>
<thead>
<tr>
<th></th>
<th>(All) SMB</th>
<th>(All) HML</th>
<th>(High risk) SMB</th>
<th>(High risk) HML</th>
<th>(Low risk) SMB</th>
<th>(Low risk) HML</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
<td>1.1</td>
<td>2.5</td>
<td>8.0**</td>
<td>7.8**</td>
<td>-2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(1.10)</td>
<td>(3.19)</td>
<td>(2.77)</td>
<td>(-1.60)</td>
<td>(2.57)</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>0.27***</td>
<td>3.7</td>
<td>0.32**</td>
<td>0.09</td>
<td>0.23**</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(4.13)</td>
<td></td>
<td>(3.19)</td>
<td></td>
<td>(2.99)</td>
<td>(0.88)</td>
</tr>
<tr>
<td>Obs.</td>
<td>88</td>
<td>31</td>
<td>57</td>
<td>31</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.16</td>
<td>0.12</td>
<td>0.23</td>
<td>0.002</td>
<td>-0.007</td>
<td>-0.010</td>
</tr>
</tbody>
</table>

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

t statistics in parentheses

Table V The weekly CAPM $\alpha_i$ and $\beta_i$ are the coefficients of the regression of the return on the SMB or the HML portfolios $R_{SMB,t} = SMB_t$ or $R_{HML,t} = HML_t$ on the market premium $MP_t$: $R_{i,t} = \alpha_i + \beta_i \times MP_t + \epsilon_t$ in general. The estimations use weekly data and I obtain one pair of coefficients for each year that corresponds to each portfolio formation period. I report the average of these coefficients ($\bar{\alpha}_i$ and $\bar{\beta}_i$) for each portfolio SMB and HML. The average of the coefficients corresponds to either the full sample (All), only the high risk years (High risk), or only the low risk years (Low risk). The number of years in each of these samples is in “Obs.”.

<table>
<thead>
<tr>
<th></th>
<th>(All) SMB</th>
<th>(All) HML</th>
<th>(High risk) SMB</th>
<th>(High risk) HML</th>
<th>(Low risk) SMB</th>
<th>(Low risk) HML</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\alpha}_i$</td>
<td>2.5</td>
<td>5.0*</td>
<td>12**</td>
<td>8.4*</td>
<td>-2.4</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>(1.10)</td>
<td>(2.24)</td>
<td>(2.77)</td>
<td>(2.57)</td>
<td>(-0.94)</td>
<td>(1.10)</td>
</tr>
<tr>
<td>$\bar{\beta}_i$</td>
<td>3.7</td>
<td>4.4</td>
<td>2.4</td>
<td>6.7</td>
<td>4.4</td>
<td>-4.9</td>
</tr>
<tr>
<td></td>
<td>(1.62)</td>
<td>(-0.26)</td>
<td>(0.60)</td>
<td>(0.93)</td>
<td>(1.57)</td>
<td>(-1.36)</td>
</tr>
<tr>
<td>Obs.</td>
<td>88</td>
<td>88</td>
<td>57</td>
<td>57</td>
<td>31</td>
<td></td>
</tr>
</tbody>
</table>

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

t statistics in parentheses
average return on the portfolios increase disproportionately. The mean-variance return on the SMB and HML portfolios increase significantly in the years of high expected market return. For example, the variance of the return on the SMB portfolio increases by the same factor as the variance on the market premium. But the average return on the SMB portfolio changes from negative to a large positive value.

**Table VI** The table shows the number of years, the mean, the variance and the ratio between the mean and variance of the market premium (MP), and the return on the SMB and HML portfolios. I split the sample with years of high or low expected return on the market.

<table>
<thead>
<tr>
<th></th>
<th>High Expected return on market</th>
<th>Low Expected return on market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs</td>
<td>Mean</td>
</tr>
<tr>
<td>MP</td>
<td>31</td>
<td>8.74</td>
</tr>
<tr>
<td>SMB</td>
<td>31</td>
<td>10.81</td>
</tr>
<tr>
<td>HML</td>
<td>31</td>
<td>8.71</td>
</tr>
</tbody>
</table>

**III. Summary**

The paper contributes to the literature in a few ways. First, it provides stronger empirical support for the common risk explanation of the size related anomalies given by Berk (1995). In addition, the paper contributes to the discussion on the size premium. In one hand, the paper contributes for the out-of-sample forecast of the size premium obtaining an $R^2$ or around 26%. On the other hand, the paper also contributes to solve the controversy over the very existence of the size premium by establishing that the premium does exist, in line with the results in Souza (2016).
REFERENCES


Figure 3. The historic values of the Shiller’s PE ratio. The picture shows the log of the Shiller’s PE ratio, $ln(PE)$, and the historical standard deviation and average (on the y axis) up to each each year (on the x axis).
Figure 4. Adjusted and unadjusted Shiller’s PE ratio. The picture shows the log of the Shiller’s PE, $\ln(PE)$, and the standardized difference from the historical mean divided by the historical standard deviation: $PE_{Adj} = \frac{\ln(PE) - \bar{\ln(PE)}}{\sigma_{\ln(PE)}}$. 
Figure 5. Adjusted and unadjusted changes in the Shiller’s PE ratio. The picture shows the change in the log of the Shiller’s PE, \( \ln(PE) \), with respect to the previous period, and its standardized difference from the historical mean divided by the historical standard deviation: 

\[
PE_{\Delta,t} = \frac{(PE_{Adj,t} - PE_{Adj,t-1}) - PE_{Adj,t-1}}{\sigma_{PE_{Adj CHANGE}}}
\]
Figure 6. The returns on the SMB portfolio. The vertical axis in both panels are percentage returns on the SMB portfolio. On the left I mark the data points that, if removed from the sample, make the significance of the average return on the SMB portfolio drop below 5%, 10%, or negative. The box plot on the right panel suggests that the three points responsible for the significance of the size premium are in fact outliers.
Figure 7. The returns on the HML portfolio. The vertical axis in both panels are percentage returns on the HML portfolio. On the left I mark the data points that, if removed from the sample, make the significance of the average return on the HML portfolio drop below 5%, 10%, or negative. The box plot on the right panel suggests that the positive returns are not outliers (if anything, there is one negative outlier).
Figure 8. Returns on the SMB portfolio in high risk years. The vertical axis shows the percentage returns on the SMB portfolio, and the horizontal axis corresponds to the year. The picture displays all the data points in the sample, showing which points correspond to the high risk years. The year can be classified as high risk years if they have low PE (low $PE_{Adj,t}$), big drops in the PE with respect to the previous year (low $PE_{Δ,t}$), or both.
Figure 9. Returns on the HML portfolio in high risk years. The vertical axis shows the percentage returns on the HML portfolio, and the horizontal axis corresponds to the year. The picture displays all the data points in the sample, showing which points correspond to the high risk years. The year can be classified as high risk years if they have low PE (low $PE_{Adj,t}$), big drops in the PE with respect to the previous year (low $PE_{\Delta,t}$), or both.